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THE PRINCIPLES OF ELECTRICAL ENGINEERING AND THEIR APPLICATION

BY
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VOL. II
APPLICATION

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NOTE

FOR the scope of this book the reader is referred to Vol. I. I take this opportunity of expressing my indebtedness to the various Firms mentioned in the text who have supplied for publication practical information and to Mr E. J. Kipps, M.Sc., M.Inst.E.E. for having assisted me in correcting the proofs.

GISBERT KAPP.

BIRMINGHAM,
August, 1919.

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PRINCIPLES OF ELECTRICAL ENGINEERING

CHAPTER I

THE ARMATURE

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The Electromotive Force induced in a Rotating Armature.

The conductors are bars or wires placed parallel to the shaft on or near the surface of the armature core and so connected as to form a continuous winding. If the connection between any two bars is made by a conductor carried through the interior of the core we speak of a ring wound armature. The whole winding is then a continuous helix round the cylindrical ring forming the core. If the connection is made not through the core, but across the end faces we speak of a drum wound armature. The core may also in this case be of ring shape, that is to say, have an internal space free from iron, but this merely on account of saving of metal and especially of admitting air for cooling the core, which gets heated by hysteresis. Each conductor together with the one to which it is immediately connected forms one turn of the winding. In a ring wound armature only one conductor of each turn is active, because the conductor passing through the interior of the core is shielded by it from the magnetic influence of the poles of the field magnet. In a drum wound armature both conductors in the turn are exposed to this influence and therefore both are "active conductors." In studying the e.m.f. generated by the passage of the conductors before the polar faces we may either start with the conception that the e.m.f. is due to change of linkage or to the rate at which lines

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of force are cut. (Vol. I, page 203.) In the one case we shall have to count turns, in the other active conductors. Although modern machines are very seldom ring wound it is as well to avoid any ambiguity and for this reason we shall not speak of turns, but of active conductors. Then any type of winding is brought under the same formula.

Let D be the diameter and L the length of the armature core, z the total number of active conductors and $2p$ the number of poles. In a two pole machine $p = 1$ and as that is the simplest case we study it first. Let in a ring armature every turn of the helix be connected to a segment of the commutator, then it will be obvious that by placing brushes at diametral points of this commutator the current

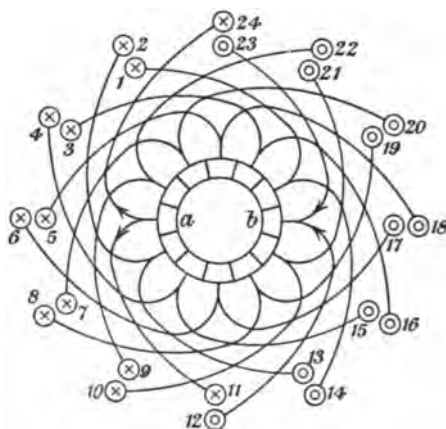


FIG. 1.

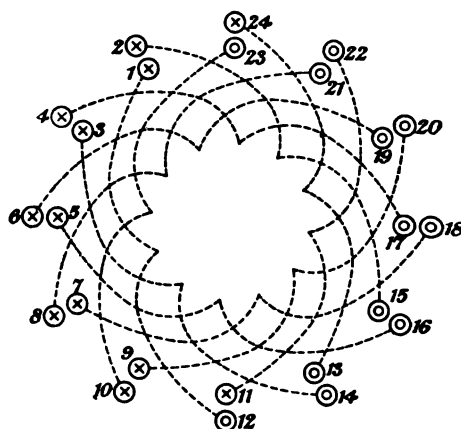


FIG. 2.

going in at one brush and coming out at the other will divide equally through the winding, each of the two current components passing through $z/2$ active wires. On one side of the dividing line the currents flow one way and on the other they flow the opposite way through the active conductors. In a drum wound armature the usual arrangement is to place the wires in two layers. Numbering the wires consecutively we would have all wires bearing an odd number in the lower or inner layer and wires bearing even numbers in the outer layer. The wires having even numbers or a certain proportion of them are connected to the segments of the commutator. Here again it can easily be seen by making a sketch of the winding that with brushes placed diametrically on the commutator, the current divides evenly between the active wires, flowing down one side and

up the other side of the dividing line. Fig. 1 is a sketch of the front or commutator end of a drum wound armature, having 24 active conductors. Only the end connections at the front are shown; those at the back are shown dotted in Fig. 2, but the front connections are left out to avoid complication. The two figures must therefore be considered together. Let a current be sent through the armature so that it enters at the front end of bar 24 and leaves at the front end of bar 12. This comes to the same thing as a current entering at segment *a* and leaving at segment *b* of the commutator. Following the course of the current we find that it is downward in bar 24 and also in bar 11. A downward current is indicated by a cross and an upward current by a little circle or dot. The current flowing down in bar 11 crosses at the back to bar 22 where it flows upward, then crosses in front to bar 9 where it flows downward and so on. In the same way the upward current in bar 12 is supplied from bar 1 where the current flows downward and by tracing the currents in this way all round the winding it is seen that in all bars to the right of the vertical dividing line the current flows upward and in all bars to the left of that line it flows downward. Conversely, if an e.m.f. is induced upward in all bars on the right and downward in all bars on the left, the resultant current will flow as indicated by the dots and crosses and enter the commutator at the brush touching segment *a* and leave it at that touching segment *b*. The brush at *b* is positive, that at *a* is negative. By the "right hand rule" we find that with counter-clockwise rotation the e.m.f.'s will be as assumed if the armature revolves in a polar cavity having north on the right and south on the left. Owing to the series connection of half the bars on either side of the dividing line the total e.m.f. is the sum of the e.m.f.'s in the individual bars. The arrangement of the bars in two layers is merely a matter of convenience in mechanical construction; the electrical effect must obviously be the same with a one layer arrangement, since the total flux cut by each bar is the same in either case. In deducing the formula for the e.m.f. we shall assume a one layer arrangement as this makes the calculation a little more simple.

The e.m.f. induced in any one bar is BLv , where B is the induction in the place where the bar happens to be at the time, L the length and v the circumferential speed. The induction need not be the same all over the polar surface, but however it may vary from place to place it has at any definite place a definite value. Let this be

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B_1 at bar 1, B_2 at bar 2, and so on. Let the corresponding e.m.f.'s be e_1, e_2 , and so on.

Let b represent the distance between neighbouring bars. The circumference is then bz , and if u is the speed in revolutions per second, the linear speed is $v = bzu$. The e.m.f. in bar 1 is then $e_1 = B_1Lv$ or $e_1 = B_1Lbzu$.

B_1Lb is that part ϕ_1 of the flux which enters the armature within the spaces appropriate to bar 1. We may therefore write $e_1 = \phi_1zu$; $e_2 = \phi_2zu$, and so on.

The sum of all such values of e contains the common factor zu , and the sum of all the partial fluxes is the total flux emanating from one pole. Let this be Φ . We thus find the e.m.f. of a two pole machine by the formula

$$E = \Phi zu$$

This is in c.g.s. measure. To get it in volts we divide by 10^8 , and if we express the magnetic flux in units of 10^6 or megalines we obtain

$$E = \Phi \frac{z}{100} u \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Figs. 1 and 2 illustrate the simplest case of a two circuit winding in a two pole armature. Machines are also constructed with four, six or more poles and armature windings which consist of two or more parallel circuits. Multipolar and multicircuit windings will be considered under the section "Armature Windings" later, but for the present we need only state the conditions that such windings must fulfil. These are: The same total e.m.f. must be induced in all the parallel circuits. The e.m.f. induced in all conductors forming one of the parallel circuits must at any moment act in the same sense, so that the total e.m.f. shall be the sum of the individual e.m.f.'s.

In a machine of $2p$ poles the armature winding may be so arranged that it forms an equal number of parallel circuits. In this case we speak of a simple parallel winding and the number of active conductors in series, instead of being $z/2$ for which formula (1) has been found, is only $z/2a$ where $a = p$. The e.m.f. is correspondingly reduced to

$$e = \frac{z}{100a} \Phi u$$

We may also wind the armature so as to form only two parallel circuits, but in this case the $z/2$ wires of each circuit will be acted

on not by two poles only, but by $2p$ poles. The e.m.f. will therefore be p times as great, or

$$e = \frac{pz}{100} \Phi u$$

Finally it is possible to so subdivide the wires that there are for a $2p$ pole machine $2a$ parallel circuits. The e.m.f. will then be decreased because of the decrease in the number of conductors in each parallel circuit and increased because of the increased number of poles. We thus find the general expression for the e.m.f. with any type of armature winding

$$E = \frac{p}{a} \Phi \frac{z}{100} u \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since p , a and z are constants of the machine we may comprise them in one symbol

$$\epsilon = \frac{pz}{100a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and then write the e.m.f. formula in the simple form

$$E = \epsilon \Phi u \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This shows that in any machine the e.m.f. induced in the armature is simply proportional to the product of flux and speed.

The Torque exerted by an Armature. Let I be the total current passing through the armature and i the current in one conductor, so that $I = 2ai$; then the force in dynes acting on one conductor is from first principles (Chapter XI, Vol. I)

$$F = BL \frac{i}{10}$$

if i is given in amperes. Let, irrespective of the type of winding, c be the distance between neighbouring conductors, so that $c = \pi D/z$, and let F_1 , F_2 , etc., be the forces of individual conductors. We have then

$$F_1 = B_1 L \frac{i}{10} = B_1 L c \frac{i}{10c} = \phi_1 \frac{i}{10c} = \phi_1 \frac{iz}{10\pi D}$$

In the same way for the next conductor $F_2 = \phi_2 \frac{iz}{10\pi D}$

and
$$F = F_1 + F_2 + \dots = \Phi \frac{iz}{10\pi D}$$

This is the force in dynes due to one pole. As there are $2p$ poles the total circumferential force is $2p\Phi \frac{iz}{10\pi D}$ dynes and the torque

is
$$\frac{D}{2} \times 2p\Phi \frac{iz}{10\pi D}, \text{ or } T = \frac{2p\Phi iz}{20\pi} \text{ dyne cm.}$$

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To get the torque in kg. m. we divide by $981,000 \times 100$ and if Φ is given in megalines we multiply by 10^6 . Since $i = \frac{I}{2a}$ we obtain

$$T = \frac{p}{a} \frac{z}{6160} \Phi I \quad (5)$$

Put

$$\tau = \frac{p}{a} \frac{z}{6160}$$

then

$$T = \tau \Phi I \text{ kg. m.} \quad (6)$$

This equation shows that the torque is simply proportional to the product of flux and current.

The relation between the coefficients ϵ and τ is

$$\epsilon = 61.6\tau. \quad (7)$$

Relation between Torque, Output and Speed. Combining (4) and (7)

$$E = 61.6\tau\Phi u$$

From (6)

$$I = \frac{T}{\tau\Phi}$$

$$EI = 61.6Tu$$

Let $U = 60u$ be speed in revs./min., then the relation between power in watts and torque in kg. m. is

$$P = 1.027 TU \quad (8)$$

This relation takes no account of internal losses and is therefore only approximate. This being the case we may replace it by the simple rule easily remembered:

$$\text{Torque in kg. m.} = \frac{\text{Power in watts}}{\text{Revs. per minute}} \quad . . (9)$$

Armature Windings. The length of an end connection between two bars defined by the number of intermediate bars is called the winding step. With the bars numbered consecutively it is obvious that the step both at the back and at the front must be an odd number, for otherwise no odd numbered bars would be included in the armature circuit. The front and back steps need however not be equal. In a two pole armature having 100 conductors both steps might be 51. Going round clockwise and starting down bar 100 we would come to the rear end of bar 51, then up this bar and from the front end again in a clockwise direction a step of 51 would bring us to bar 102, but as there are only 100 bars in all, we come to bar 2. The same bar can be reached in two steps if

they are of different length and taken in different direction. Thus we may have a positive back step of 51 taken as before in a clockwise direction and a negative front step taken counter-clockwise of 49. Or we might have a back step of + 49 and a front step of - 47. In either case the difference is 2, that is to say, starting from bar 100 we come to bar 2. We might also have two positive steps of 49 and thus reach bar 98; or we might have a positive back step of 51 and a positive front step of 47 and again reach bar 98. These different possibilities all fall under the two conditions, namely

$$y_B + y_F = Z \pm 2 \quad \text{and} \quad y_B - y_F = \pm 2$$

where y_F and y_B denote front and back step respectively.

In each turn of two steps the winding creeps either forward or backward by 2. Such a winding for 24 conductors and a backward creep of 2 has been shown in Figs. 1 and 2, but if we attempt to draw a diagram of this kind for an armature having 100 conductors we shall find it not only very laborious, but also quite useless, since the many lines crossing would render the whole drawing illegible. It is much easier to represent a winding by means of a table. In the case of 100 conductors and two forward steps of 49 resulting in a backward creep of 2 such a winding table might be written as follows:

WINDING TABLE OF TWO POLE DRUM ARMATURE HAVING 100 BARS

Front step = +49. Back step = +49. Creep = -2.

Line	No.	B	No.	F	No.	B	No.	F	No.	B	No.	F	No.	B	No.	F	No.	B	No.	F	No.
1	100		49		98		47		96		45		94		43		92		41		90
2	90		39																		
⋮																					
10	10		59		8		57		6		55		4		53		12		61		100
																	2		51		

In this table F stands for front and B for back step. It is not necessary to write out the full table. By arranging the bar numbers in ten columns the numbers read downward in each column differ by 10, and it is easily seen that ten lines include all the 100 bars; in other words the winding closes only after each bar has been connected up. We have here a "single re-entrant winding."

Electrically the same effect is obtained by using a positive back step of 49 and a negative front step of 47; the creep being in this

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case + 2. The winding table is given below, but the columns marked B and F are omitted.

WINDING TABLE OF TWO POLE DRUM ARMATURE HAVING 100 BARS
Front step = - 47. Back step = + 49. Creep = + 2.

Line	No.	No.	No.	No.	No.	No.	No.	No.	No.	No.	No.
1	100	49	2	51	4	53	6	55	8	57	10
2	10	59									
...											
10	90	39	92	41	94	43	96	45	98	47	100

That this winding fulfils the condition of a proper series connection between the active bars can easily be seen by writing out the winding table in full and assuming a certain position for the brushes. Let for instance one brush be on the commutator segment connected to the front ends of bars 49 and 2. The other brush being diametrically opposite will touch the segment connected to the front ends of bars 99 and 52. The polar diameter must be a line at right angles to the neutral axis and pass through the points 25 on one side and 75 on the other. With the usual arrangement of a polar arc spanning about 120° there would be on either side of the brush axis 33 or 34 conductors in which an e.m.f. is induced. It would be say, upward in bars 8 to 42 and downward in bars 58 to 92. The other bars being at the moment between the polar edges contribute nothing to the e.m.f., but by following the active bars through the winding table it will be seen that those on one side of the neutral axis all act in one, and those on the other side of that line in the opposite direction.

Multipolar Lap Winding. If the armature is to be used in a four pole field, it is obvious that the winding step must not be much different from a quarter of the circumference in order that the e.m.f.'s in two bars in series may be properly added. Let there be 200 conductors, then steps of + 49 and - 47 would again be appropriate. The winding table in a shortened form is given here. Assume that at a given moment the centre of one north pole is opposite bar 25, then the centre of the other north pole will be opposite bar 125. The south poles will be opposite bars 75 and 175. From north to south the distance measured in mechanical degrees is 90 and in electrical degrees 180. The brush axes must be midway

between the polar axes so that there shall be no e.m.f. in the bars undergoing commutation. The brush positions are thereby fixed. They must be somewhere in the neighbourhood of bars 50, 100, 150 and 200. There is no e.m.f. in the bars contained within the groups 193 to 7; 43 to 57; 93 to 107; and 143 to 157. In all the other bars there is at the moment an e.m.f., namely, downward in groups 58 to 92 and 158 to 192, and upward in groups 8 to 42 and 108 to 142.

In the table bars at the moment under commutation are joined by a hyphen and printed in italics.

ABBREVIATED WINDING TABLE OF FOUR POLE DRUM ARMATURE
HAVING 200 BARS

Front step = +49.		Back step = -47.		Creep = +2.					
1	200	49— 2	51	4	53	6	55	8	57
5							95	48	97— 50
6	50	99— 52	101	54	103	56	105	58	107
10							145	98	147—100
11	100	149—102	151	104	153	106	155	108	157
15							195	148	197—150
16	150	199—152	1	154	3	156	5	158	7
20	190	39 192	41	194	43	196	45	198	47—200

By writing out the winding table in full and marking each number with the direction in which the e.m.f. acts (whether up- or down-ward) it will be found that all acting bars are coupled in the right sequence. A current entering at the segments connected to bars 50 and 52 has two paths; one by way of bars 101, 54, etc., to bar 100, and the other by way of 97, 48, 95, etc., to bar 2. Similarly a current entering at 150 and 152 splits into two parts, one going by way of 1, 154, 3, etc., to 200, and the other by way of 197, 148, 195, etc., to 102. We thus get four circuits as shown hereunder, the place where the current enters being marked as the negative and where it leaves as the positive brush.

Negative Brushes in contact with	Positive Brushes in contact with
{ 50 current flows to	{ 2
52 " "	100 }
and	
{ 150 " "	102 }
152 " "	200 }

Bars 2 and 200 are at the same potential and are touched by one and the same brush. Also bars 100 and 102 are under one brush. We have thus four brushes, two at which the current enters and two at which it leaves the winding. There are four parallel circuits, each carrying one quarter the total current.

The same type of winding, *i.e.*, a forward and backward step with a creep of 2 can be applied to an armature for any number of poles. There must be $2p$ brushes and there are $2p$ parallel circuits. This kind of winding is called *parallel winding* or "*lap winding*" to indicate the condition that each turn overlaps the preceding turn by the creep.

Multipolar Wave Winding. There is another type of multipolar winding in use, the so-called *series*, or "*wave winding*." In this type all steps are forward in direction. For the reason already mentioned both back and front steps must be odd numbers, but they need not necessarily be equally long. If they differ they must do so by an even number. In going once round the armature we must not hit the starting point as otherwise the winding would close with only $2p$ bars. Here also we must have some creep. The smallest possible creep is 2. If this is adopted we have

$$z = 2py \pm 2$$

where

$$y = \frac{y_B + y_F}{2}$$

the mean between the two steps. If both are equal, y is an odd number, but if they are unequal y may be an even number. We shall now study such a six pole wave winding, but as it will be necessary to write out the whole of the winding table, we shall assume a rather smaller number of bars so as not to make the investigation too laborious. The principle remains the same whether z be large or small. We assume a six pole armature and 50 conductors. Only a little more than half the total number of bars are at any moment active. Let bar 50 be under the centre of the north pole, then bar 25 will be under the centre of the south pole. Bar 50

and its immediate neighbours on either side will be active. Let the e.m.f. be directed downwards in bar 50. In bar 25 and its immediate neighbours on either side the e.m.f. will then be directed upwards. The same will be the condition in bar 8 and its immediate neighbours and in bar 42 and its immediate neighbours. We have thus three groups of bars in which there acts an e.m.f. towards a front connection. Similarly there are three groups of bars (namely, those under the north poles) in which the e.m.f. acts towards a back connection. If we write out these numbers in two lists, one referring to an upward and the other to a downward e.m.f. we can by going through the winding table affix to each connection a number indicating the number of bars actually active between any starting point such as the front connection 13-20 and any other front connection. In the following winding table these numbers are indicated in *italics*. The columns headed F and B signify front and back connections respectively.

WINDING TABLE OF SIX POLE WAVE WOUND DRUM ARMATURE OF 50 BARS

Back step = +9.				Front step = +7.				Creep = -2.			
F		B		F		B		F		B	F
<i>4</i>	50	<i>5</i>	9	<i>6</i>	16	<i>7</i>	25	<i>8</i>	32	<i>9</i>	41
<i>10</i>	48	<i>11</i>	7	<i>12</i>	14	<i>12</i>	23	<i>13</i>	30	<i>13</i>	39
<i>13</i>	46	<i>13</i>	5	<i>13</i>	12	<i>13</i>	21	<i>13</i>	28	<i>13</i>	37
<i>13</i>	44	<i>13</i>	3	<i>13</i>	10	<i>13</i>	19	<i>13</i>	26	<i>12</i>	35
<i>12</i>	42	<i>11</i>	1	<i>10</i>	8	<i>9</i>	17	<i>8</i>	24	<i>7</i>	33
<i>6</i>	40	<i>5</i>	49	<i>4</i>	6	<i>3</i>	15	<i>2</i>	22	<i>1</i>	31
<i>0</i>	38	<i>0</i>	47	<i>0</i>	4	<i>0</i>	13	<i>0</i>	20	<i>0</i>	29
<i>0</i>	36	<i>0</i>	45	<i>0</i>	2	<i>0</i>	11	<i>0</i>	18	<i>1</i>	27
<i>2</i>	34	<i>3</i>	43	<i>4</i>	50	<i>5</i>	9	<i>6</i>	16	<i>7</i>	25

It will be noticed that there are no active bars between 36 and 38 or 2 and 4, or 18 and 20. The corresponding commutator segments may therefore all be connected by the brushes. Similarly the commutator segments connected to 10, 12; 26, 28; 44, 46 may also be connected by the other set of brushes since their potential is the same, namely, that produced by the 13 active bars intervening between the former and latter groups. Let a current enter at the segment connected to bar 20. It flows two ways, one is down to bar 29 and the other across the front connection and down in bar 13. Going forward through the winding table we arrive at the front connection 23-30 when the full e.m.f. corresponding to 13

active bars is obtained; going backward we arrive at 26-19, where again the full e.m.f. is obtained. Between 23-30 and 19-26 there is no potential difference. One pair of brushes set either 60 or 180 degrees apart is all that is required to bring in and take out the current since there are only two parallel paths through the winding. It is, however, permissible and generally advisable to use six brushes because the length of the commutator may thereby be reduced.

Multipolar Series-parallel Winding. If in a 50 bar six pole armature we use equal steps front and back of 9 we obtain in going once round a creep of + 4. This winding is represented in the table below. Italics as before indicate the number of actually active bars between any two points. The highest number reached now is only 6, but there are two regions in the table where these numbers occur and two regions where 0 occurs. To utilise the winding fully we must therefore introduce the current at no less than two points and take it off at two points at least. In actual practice 6 brushes in all would be used as in a plain lap winding. The table shows that now there are four parallel paths through the winding, whilst each path is under the influence of three pairs of poles in series. Hence the name *series-parallel winding*. It is also called "*Arnold Winding*" after its inventor the late Professor Arnold. It falls under the general winding formula

$$z = 2py \pm 2a \quad \dots \quad (10)$$

where $2a$ is the number of parallel circuits. It is also applicable to a two pole machine, but in any case the brushes must be wide enough to cover at least a segment.

WINDING TABLE OF SIX POLE SERIES-PARALLEL DRUM WOUND
ARMATURE OF 50 BARS

Back step = +9.				Front step = +9.				Creep = +4.			
F		B		F		B		F		B	F
3	50	4	9	5	18	6	27	6	36	6	45
6	4	6	13	6	22	5	31	4	40	3	49
2	8	1	17	0	26	0	35	0	44	0	3
0	12	0	21	0	30	0	39	0	48	1	7
2	16	3	25	4	34	5	43	6	2	6	11
6	20	6	29	6	38	6	47	6	6	5	15
4	24	3	33	2	42	1	1	0	10	0	19
0	28	0	37	0	46	0	5	0	14	0	23
1	32	2	41	3	50	4	9	5	18	6	27

The general winding formula (10) may be written

$$n = py \pm a$$

where $n = z/2$ is the number of turns counted all round the armature. If n and y have a common divisor d such as a , or $a/2$, or $a/3$, we can write $n' = n/d$, $y' = y/d$, $a' = a/d$ and obtain $n' = py' \pm a'$ which is also a correct winding formula, but for an armature of only z/d bars. If then n and y have a common divisor the winding closes too soon. To obtain a single re-entrant winding $z/2$ and y must not have a common divisor. Take for example a 10 pole armature with about 630 or 640 conductors to be so connected as to form six parallel circuits. With equal front and back steps of 63 the number of bars would be

$$10 \times 63 + 6 = 636$$

$$\frac{z}{2} = 318 \text{ and } y = 63. \text{ But these two numbers have the common}$$

divisor 3 and consequently the winding would close after only 212 bars have been connected. A mean step of 63 is therefore inadmissible. But if we make the front step 63 and the back step 65 we get a mean step of 64 and then the number of bars becomes

$$10 \times 64 - 6 = 634$$

$$\frac{z}{2} = 317 \text{ and } y = 64. \text{ These two numbers have no common}$$

divisor and the winding will only close after all the bars have been connected.

The series-parallel system of armature winding occupies an intermediate position between the plain wave and the plain lap winding and may be used in cases where neither of these would be satisfactory. In a machine of fairly large output but very low speed the plain lap winding requires an inordinately large number of conductors, whilst the plain wave winding, where the large total current is divided between only two parallel circuits, results in too large a current in each bar. This leads to unsatisfactory commutation. There is no hard and fast limit to the current that can be satisfactorily commutated, but 250 amperes per bar may be considered as rather a high value and if possible 200 amperes as an upper limit is preferable. As an example take a motor to develop 500 h.p. at 40 r.p.m. with 500 volts impressed. This motor would require about 800 amperes. It would be a 16 pole machine, each pole giving a flux of 14 megalines. By reference to (2) it will be found

that plain lap winding would require about 5000 conductors and is therefore out of the question. Plain wave winding would require only 626 conductors, but then each conductor would have to carry 400 amperes and the commutation of so large a current would certainly be unsatisfactory. By adopting a series-parallel winding a good design is obtained. We would arrange for six parallel circuits making p/a in (2) $8/3 = 2.66$. The number of conductors then becomes 1930, which is quite practicable with an armature of about 2.8 m. diameter. The current per bar is only 133 amperes, which gives very good commutating conditions. The mean winding step would be 121 and the commutator would have 965 segments.

There is another way of solving such a problem. We can use three interleaved, but otherwise independent plain wave windings each containing 626 active conductors. The commutator would have in all 939 segments. In either case the brushes must be wide enough to cover fully three segments.

Armature Slots. If the armature core is a smooth cylinder the mechanical support of the conductors presents some difficulty. Insulated driving pins may be used projecting into the layer of the winding and in addition the winding must be held down by a number of bands so as to resist lifting by centrifugal force. The use of driving pins is objectionable on account of difficulties of insulation and the holding down of the conductors by binding hoops becomes mechanically difficult with increasing diameter and beyond a certain size and speed impracticable. Thus the smooth armature core is now a thing of the past and slotted cores are almost exclusively employed. The advantage of housing the conductors in slots is not only mechanical, but also electrical, inasmuch as the insulating covering of the wires is hardly at all subjected to a dynamic force arising from the current in the conductor. It is, of course, subjected to centrifugal force, but this is a steady pressure and not nearly so injurious to the insulation as the circumferential force due to the current which in a smooth armature pulsates at each pole and which in the aggregate equals the force corresponding to the full torque of the machine. In a smooth armature the torque has to be transmitted through the insulating covering of the conductors; in a slotted armature very little of it is transmitted to the conductor itself, but nearly all the forces act on the teeth between the slots. At first sight it might seem incomprehensible that a conductor

can be relieved of mechanical stress and yet perform its function as a producer of e.m.f. Both the mechanical force and the e.m.f. depend on and are proportional to the induction. The explanation why the mechanical force acting on the conductor in a slot is very small is simple enough. The flux emanating from the polar surface crowds through the teeth and very little goes through the air in the slot, the reluctance of which is enormously greater than that of the teeth. Hence the mechanical force acting on the conductor in the slot is small; it is $\frac{i}{10} B_s$ dynes per cm. if B_s is the slot induction.

This is only a very small fraction of the induction that would obtain with a smooth core and the mechanical force is reduced to a corresponding extent. It is only when the tooth induction is very great that any appreciable number of lines are squeezed out from the teeth into the slots, but even then the induction is very much less than would be the case with a smooth core, so that practically speaking conductors housed in slots are only subjected to centrifugal but not to any appreciable extent to electrodynamic forces. How is it then that an electromotive force is produced in a conductor situated in so weak a field that we may disregard the dynamic effect? The answer is that the e.m.f. is the product of two things, namely, strength of field and speed of cutting. In a smooth core armature the strength of field is that of the air-gap induction and the speed of cutting is the circumferential speed of the armature. In a slotted armature the slot induction is enormously reduced, but the speed of cutting is enormously greater than the circumferential speed of the armature. Whatever the arrangement of the winding, a coil advancing through 180° (electrical) must cut through all the lines of force emanating from one pole and the product of speed of cutting and induction must be constant. Hence the e.m.f. generated in the conductor is the same whether the conductor lies in a slot or on the surface of the core. In the slot the induction may be only one hundredth part of that in the air gap, but then the lines of force snap across the slot with 100 times the circumferential speed of the armature.

The torque is due to the magnetic forces acting not between conductors and fixed poles directly, but to those acting between the tips of the teeth and the fixed poles. The teeth are made into electromagnets by the currents in the slots and the torque is that due to attraction and repulsion between physical magnets.

A great variety of slot shapes are used. Fig. 3 shows a few typical examples. For armatures of small diameter (see type *a*) the coils in the slots may be held down against centrifugal force

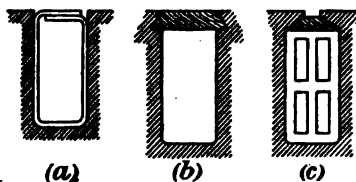


FIG. 3.

by binding hoops applied after the winding is finished. A band of insulating material (such as tape with mica pasted on) is first applied and over this is wound a single layer of steel wire, the ends being secured by brazing on to a clip. Such bands are applied

every few inches along the length of the completed armature. Banding is only practicable for small armatures; with increasing diameter the bands would require to be so heavy as to cause waste of power owing to eddy currents in them. In moderate size and large armatures the conductors are secured in the slots by closing the slot as shown in *b* and *c*. After the coil sides are laid in the slot wooden wedges are driven along the grooves left in the upper part of the teeth. In *b* is shown a so-called "open slot" whilst *c* shows a "semi-closed slot." The slots are lined with two or more layers of paper, press span, empire cloth or other suitable insulating material. The lining is folded over at the top as shown in *a*. Each slot may contain only two coil sides or bars; or it may contain any greater number; in *c* are shown four bars per slot.

By writing out the winding table for any type of winding it will be seen that the full voltage of the machine exists between any top bar in a slot and that lying immediately beneath it. Hence it is advisable to put some extra insulation in the shape of a separator between the top and bottom group of bars in each slot. The same applies if the winding does not consist of bars, but of coil sides containing a number of individual wires. The insulation between wire and wire need not be stronger than that naturally provided by the cotton covering of the wire, but between the upper and lower coil side some extra insulation either in the shape of a strip of insulating material or heavy taping of the coil side as a whole must be provided.

Space required for Insulation. In machines up to 600 volts the insulating lining of the slot takes up about 0.8 mm. to which must be added an allowance of about 0.4 mm. for roughness

of the stampings, leaving a clear space for the insulated bars or coils 1.2 mm. less than the designed width of the slot. In a radial direction the clear space is reduced by about 2.4 mm. owing to the folding over of the lining and the separator between top and bottom conductors. Bars are insulated individually by mica and paper wrapping which adds from twice $\frac{1}{4}$ to $\frac{1}{2}$ mm. to the width and height of the bar. The group of wires forming a coil side is insulated by wrapping with empire tape and ordinary cotton tape to a thickness of about $\frac{1}{2}$ mm. The cotton covering of the round wires adds $0.25 + 0.04 d$ to the diameter of d mm. Thus a 3 mm. round wire has, when double cotton covered, a diameter of 3.37 mm.

End connections. The individual conductors, whether wires or bars, in one slot must be connected with those in another slot by conductors passing across the end face of the armature core.

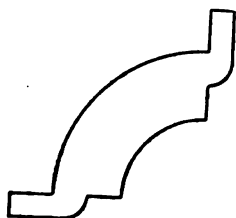


FIG. 4.

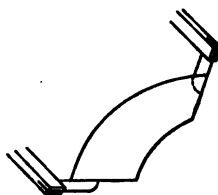
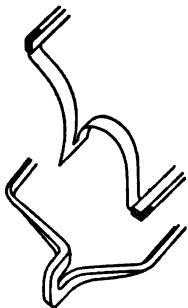


FIG. 5.

In most cases these conductors are merely the wires or bars themselves bent into a suitable shape so as to cross each other without the danger of coming into contact and forming a short circuit. In some cases separate connectors are used. This is convenient if in a bar armature the conductors are not placed one above the other, that is in two layers, but all at the same radial distance. Fig. 4 shows one type of connector for a four pole armature. The connectors consist of quarter circle segments of sheet copper provided with tags as shown. These are bent over, one forward and the other backward at a right angle and the segments are arranged spirally round an insulated supporting pulley so that the tags point outwards on either side of the flanges and their plane goes through the axis of the armature. The bars are provided with saw cuts at their ends into which the tags are inserted and sweated up. Fig. 5 is a perspective view of one segment with its two bars attached.

A more usual form of end connector is shown in Fig. 6. It consists of a rectangular strip of copper with a slot cut out down its middle. The one half is bent to the right and the other to the left with the ends turned up so as to stand radially, ready for insertion into the saw cut of each bar. To prevent short circuit between the connectors they are individually taped, the slot cut out in each strip providing the necessary room for the taping. In Fig. 5 each connector must also be insulated from its neighbours either by taping or by the insertion of insulating segments, preferably by both. In most modern machines the conductors are arranged in two layers and



Figs. 6 & 7.

then the simplest plan of making the end connections is to bend the conductor itself in the manner shown in Fig. 7. The conductor may be either a single bar or a group of bars taped together, or a coil side consisting of a number of insulated wires taped together so as to present a rectangular outline as shown. Special moulds or formers are used for giving the correct shape to the bars or to the wound coil. This type of winding is sometimes called an "*Eickemeyer Winding*" since it was an American engineer, Mr Eickemeyer who first used moulded coils. A more usual name is "*diamond winding*" or "*grid winding*" by reason of the upper and lower layers of the connecting members of the winding crossing in the form of a grid. An advantage of this winding is that the space left between the crossings of the conductors provides access for air and thus assists the dissipation of heat.

Chord Winding. If the winding step is as nearly as possible equal in length to the distance between the centres of two neighbouring poles we have a "*diametral winding*." The brushes being necessarily also placed diametrically it follows that commutation of current in top and bottom bars in the slot takes place simultaneously. The change of linkage in the coils at one time short circuited by the brushes is therefore greater than would be the case if it were possible to so arrange matters that whilst the current in a top bar reverses no change takes place in current strength in the bottom bar in the same slot. The current in the bottom bar must of course also reverse, but it need not reverse precisely at the same moment as

in the top bar; it may reverse a little sooner or a little later. It will be shown in the section on commutation that by avoiding simultaneity in reversal in top and bottom bars a better commutation is obtained. For the present we are merely concerned with the question how the commutation of the currents within a slot may be split up into two stages. This may be done by applying a so-called "*chord winding*." Instead of making the step equal to the polar pitch it is shortened by a certain amount so as to span a chord instead of a diameter and therefore less than 180 electrical degrees. The chord must, of course, remain long enough to embrace the whole of the polar arc including an allowance for "*fringing*," by which is meant the spreading of the lines of force a little distance to the outside of the polar edges. This fringe of the field obviously depends on the air gap, that is the distance between top of teeth

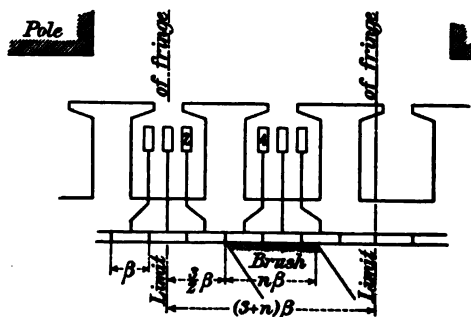


FIG. 8.

and polar surface. It will be more with a large and less with a small air gap and can be estimated from a drawing of the magnetic circuit. In machines intended always to run in the same direction there is some advantage in placing the commutation zone within the fringe of the field, which is done by shifting the brushes forward in a generator and backward in a motor, but if the machine is intended to run either way, as for instance in a reversible motor, the brushes must stand in the neutral space, that is midway between neighbouring polar edges as shown in Fig. 8. This illustration represents a toothed armature with six conductors per slot. Only the upper conductors are shown. At the moment to which the diagram refers the tooth between conductors 2 and 4 is about to come under the influence of the fringe if the movement is from right to left, or has

just emerged from it if the movement is from left to right. The commutator segments may therefore be short-circuited by the brush, there being no flux through the tooth between the conductors 2 and 4. Let β be the width of a commutator segment reduced to the air gap radius, then it will be seen that with three conductors side by side in the slot the distance between the toe of the brush and the line representing the limit of the fringe must not be less than $\frac{3}{2}\beta$. With four conductors side by side in a slot it would be $\frac{4}{2}\beta$; and generally with c conductors or groups of conductors it would be $\frac{c}{2}\beta$. The same applies to the heel of the brush and the other fringing limit, so that if $b = n\beta$ is the width of the brush covering n segments we have for the minimum distance between the two fringing lines

$$(c + n)\beta$$

In this case the commutation takes place quite clear of the influence of either pole and the commutating conditions are the same whichever way the machine runs. If used as a generator the commutation will be improved by shifting the brushes forward, if used as a motor by shifting them backward. Here we have tacitly assumed diametral winding so that the bar to which 2 and 4 are connected and which lies at the bottom of a slot 180 electrical degrees distant is also free from magnetic influence. If $S/2$ denotes the number of commutator segments corresponding to a pole pitch and m the number of segments under one pole and its fringes we may with diametral winding use a length of polar arc of $P = m\beta$ including fringes, where

$$m \leq \frac{S}{2} - c - n \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If now without reducing the polar arc we were to adopt chord winding, the result would be that the lower bar connected to 2 and 4 would during the commutation of these two bars lie within the field of an opposite pole. An e.m.f. would be induced in this lower bar and produce a short circuit current whilst the segments connected to 2 and 4 are still under the same brush. Good commutation would be impossible. It is therefore necessary to shorten the polar arc to such an extent that the lower bar only enters the polar fringe after the commutation of the bars 2 and 4 has been completed. This means that the distance from the toe of the brush to the limit of the fringe has to be at least equal to the amount by which the winding step has been shortened. To insure that a lower and upper

bar shall not commutate simultaneously we must shorten the winding step by at least $\frac{c}{2}$ segments so that the width of the neutral space must be at least

$$c\beta + n\beta + \frac{c}{2}\beta$$

The neutral space must therefore contain at least $\frac{3}{2}c + n$ segments. We have therefore with chord winding

$$m \leq \frac{S}{2} - \frac{3}{2}c - n \quad . \quad . \quad . \quad (11 a)$$

In this formula

m = number of commutator segments covered by one pole.

$\frac{S}{2}$ = number of commutator segments contained within a pole pitch.

n = number of segments covered by the brush.

c = number of coil sides or bars placed side by side in one slot.

Spacerelations between Flux, Sense of Rotation, e.m.f., Current and Torque in the Armature. The quantitative relation between flux and e.m.f. produced at a given speed of rotation can be calculated from (4), but this equation gives no indication as to the direction in which the e.m.f. acts. This necessarily depends on the sense in which the winding progresses round the armature and on the direction in which the armature rotates. To facilitate the study of the working condition of a dynamo it is convenient to adopt certain purely conventional standards as to polarity of brushes, progression of the winding in the armature and direction of the magnetising helix in a field magnet coil. The conventions we adopt are the following.

Field Coil. The magnetising helix on a cylindrical core shall be wound in such sense that the direction of the flux produced shall be the same as the axial progression of the current through the coil.

Polarity of Brushes. In a generator the current issues at the positive brush. In a motor the current enters at the positive brush.

Progression of Armature Winding. This shall be such that a current flowing across a diameter in a two pole armature shall magnetise the armature so as to produce a south pole at the point

of entry and a north pole at the point of exit. The same applies to multipolar armatures, the direction of the different fluxes coinciding always with the radial direction of the different currents. The drum winding shown in Fig. 1 complies with this rule. A current entering at the front end of bar 24 and leaving at the front end of bar 12 passes vertically downwards through the armature and, as will be seen by the dots and crosses, magnetises the armature in a vertical direction, producing north polarity at the lower, and south polarity at the upper end of the brush axis. As a matter of convenience to get the two halves of each end connection as nearly equal as possible the actual position of the brushes is not on the true brush axis 24-12, but advanced by 90 electrical degrees in a counter-clockwise direction. In any diagrammatic representation of the armature we shall, however, always assume that the brushes are placed in the neutral axis.

Having settled on the progression of armature winding we can easily find a rule for the space relation between the direction of the following quantities: Flux, rotation and induced e.m.f. Let the flux produced by the field magnets be from left to right in Fig. 1 and let the armature revolve clockwise. By applying the right hand rule it will be found that in all wires on the right of the brush axis the direction of the induced e.m.f. is upwards as shown by the dots and in all the wires on the left of the brush axis it is downwards as shown by the crosses. The resultant of all these individual e.m.f.'s is an e.m.f. directed vertically downwards. If we reverse the direction of rotation the induced e.m.f. would be in the opposite direction, that is, directed vertically upwards. In either case the direction of induced e.m.f. is found by rotating the flux through 90 electrical degrees in the direction of rotation. To facilitate the analysis of multipolar machines this rule may also be expressed as follows: A brush is positive if the bar connected to the segments on leaving it advance towards a north pole.

A current as indicated by the dots and crosses flows vertically downwards through the armature. Let again the main flux be directed from left to right then by applying the left hand rule we find that all the wires on the left of the brush axis exert a downward and all the wires on the right of that axis an upward force. The torque is therefore counter-clockwise. This also follows from the fact that the machine acts as a generator and must therefore absorb mechanical power, which with clockwise rotation is only possible

if the torque acts counter-clockwise. To get the direction of the torque we draw an arrow to represent the current through the armature and imagine it to be a lever pivotted at its tail. Then apply the field with its proper direction to the head of the arrow and note which way the lever will turn. This is the direction of the torque.

These various rules will be found useful in analysing the working of any C.C. dynamo machine. The rules are diagrammatically illustrated in Fig. 9. A current is represented by an arrow with a narrow closed head, an e.m.f. by an arrow with open head, a flux

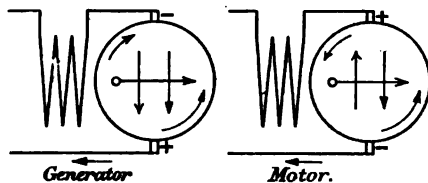


FIG. 9.

by an arrow with open head and a little circle at the tail. The torque due to the interaction of flux and current is indicated by a curved arrow with a closed narrow head and direction of rotation by a curved arrow with open head. The diagram on the left represents a series excited generator, that on the right a series excited motor.

Equipotential connections. In a lap wound multipolar armature all positive brushes should be at the same potential and all negative brushes should be at the same potential. This would indeed be the case if the flux issuing from any north pole were absolutely the same as that entering any south pole. Such absolute symmetry of the magnetic circuit cannot be obtained in practice; either by reason of inequality of the material and dimensions or by reason of some slight eccentricity between the armature and the polar cavity small differences in flux are inevitable and the result is that the potential difference between any pair of positive and negative brushes may not be absolutely the same. There will then be slight potential difference among the positive brushes and also among the negative; and since all positive brushes are coupled in parallel and all negative are also coupled in parallel there is a tendency for the production of circulating currents through the

armature and those brushes which are in parallel connection. These currents have a magnetising effect which in part restores the balance of e.m.f. between brushes of equal sign, and are therefore weaker than corresponds to the original inequality of strength of poles, but whatever their magnitude may be, they would increase the loading of the brushes unless an alternative path were provided. This is done by the so-called "equipotential connectors" which are conductors joining parts of the winding which ought to be at the same potential. Sometimes these equipotential connectors are placed at the back of the commutator and joined to segments of nominally equal voltage, but more frequently they are placed at

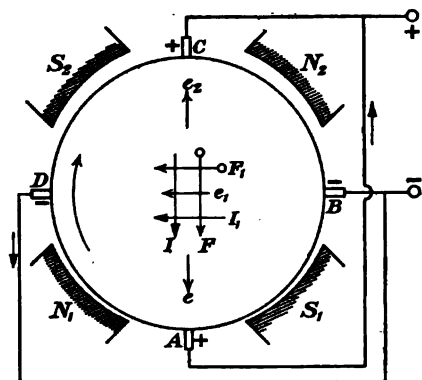


FIG. 10.

the other end of the armature and consist of a number of circular copper rings insulated from each other and connected by tags to the equipotential points of the winding. Thus in a six pole machine there may be a dozen rings each having three tags set 120 mechanical degrees apart. In an eight pole machine each ring would require four tags and so on.

The rectifying effect of these

equipotential connectors and consequent weakening of the circulating currents can most easily be shown in the case of a four pole machine, but it is also to some extent present in a machine of a larger number of poles, as was pointed out by Mr Hawkins in *The Electrician*, 1914, pp. 901 to 905. In Fig. 10 it is assumed that magnetic dissymmetry has resulted from the bearings being worn down so that the centre of the armature is lower than the centre of the polar cavity, but electrically any other cause, such as spongy castings in the upper poles, will have the same effect. Let A , B , C and D represent the potential of the four brushes and let the system be symmetrical as regards the vertical so that $D = B$. With clockwise rotation we find by the rule given in the previous section that the C and A brushes are positive but A is more positive than C and in consequence there will be a downward e.m.f. e , causing a current to circulate externally from A to C as shown by the arrow along the

connecting wire. Internally this current I is from C to A . It magnetises the armature producing a field F . Since a large part of the magnetic path of this field is in air it may be considered as proportional to the current I , so that we write $F = \kappa I$, where κ is a constant depending on the dimensions and the winding of the armature. For our present purpose it is not necessary to determine this constant. It may, however, be noted that this field is feeble as compared to the field produced by the physical magnet system. The field F produces by virtue of rotation of the armature an e.m.f. e_1 , acting internally from right to left and making brush D slightly positive in comparison with brush B . The result is a circulating current, $I_1 < I$, which in turn produces a horizontal field $F_1 < F$. Since the armature conductors cut through this field also, an upward e.m.f. e_2 is produced by it which opposes the original e.m.f. E due to want of magnetic symmetry. We have therefore

$$E - e_2 = e \quad \text{and} \quad I = \frac{e}{\rho}$$

where ρ is the resistance of the armature and brush contacts taken over the vertical diameter. Using the previous notation we write

$$\begin{aligned} e_1 &= \epsilon F u \text{ or } e_1 = \epsilon \kappa I u \text{ or } e_1 = \frac{\epsilon \kappa e u}{\rho} \\ I_1 &= \frac{e_1}{\rho} \text{ or } I_1 = \frac{\epsilon \kappa e u}{\rho^2} \text{ and } F_1 = \frac{\kappa^2 \epsilon e u}{\rho^2} \\ e_2 &= \epsilon F_1 u \text{ or } e_2 = e \left(\frac{\epsilon \kappa u}{\rho} \right)^2 \text{ or } E - e = e \left(\frac{\epsilon \kappa u}{\rho} \right)^2 \\ e &= E \frac{\rho^2}{\rho^2 + (\epsilon \kappa u)^2} \\ I &= \frac{E}{\rho} \left(\frac{\rho^2}{\rho^2 + (\epsilon \kappa u)^2} \right) \end{aligned}$$

If it were not for the reaction produced by the circulating current I , its magnitude would be E/ρ . The actual magnitude is reduced by the factor in brackets. Writing the equation for I in the form

$$I = E \frac{\rho}{\rho^2 + (\epsilon \kappa u)^2}$$

it will be seen that the lower the armature resistance the smaller is the circulating current resulting from a given dissymmetry of the magnetic field. Generally speaking, low armature resistance means a large circulating current, but in a four pole machine the fact that the two brush axes are in quadrature, produces a compen-

sating effect, so that the circulating current is sensibly reduced. The theory of the compensating effect of the circulating current in a machine of six or more poles is more complicated and the effect is not so marked. For a detailed investigation the reader is referred to the article quoted.

With a plain series winding no equipotential connections are required or possible since there are only two parallel circuits.

Exciting Force required by Armature Teeth. The slots have generally parallel sides so that the teeth flanks are inclined to each other making the tooth taper with the narrow part at the root. The induction must therefore increase from the top of the tooth down to its root and the ampere-turns required per unit length of tooth must also increase towards the root. The calculation of the total ampere-turns for one tooth becomes therefore rather complicated and it is rendered still more so by the fact that with the high induction customary in modern machines some of the flux is squeezed out from the tooth flank through the air of the slot and thus the increase of induction as we proceed from the head to the root of the tooth is a little smaller than would be the case if the flux were strictly confined to the metal. An ingenious method for determining the total exciting force required for tapered teeth has been published by Mr W. B. Hird*, but we shall here follow a simpler and generally used method which is sufficiently accurate for the immediate purpose, namely the determination of the exciting force required for the teeth. It is only an approximate method, but since the exciting force for the teeth is only a small fraction of the total exciting force required in the machine even a sensible error in this small component does not materially affect the total. The method generally used in practice is to assume the tooth with parallel flanks and its thickness to be that of the real, that is tapered, tooth $\frac{1}{4}$ the way up from the root. This assumption gives approximately the same exciting force as the more rigorous method. Let A_1 be the area of the tooth $\frac{1}{4}$ the way up from root and A_2 the area of the slot plus that part of the tooth occupied by insulation and air ducts. Let the total core length be L cm., the axial length of all the air ducts provided for internal ventilation of the armature

* "The reluctance of the teeth in a slotted armature," *J.I.E.E.*, vol. xxix. p. 933. See also Miles Walker, "The predetermination of the performance of dynamo-electric machinery," *J.I.E.E.*, vol. liv. p. 245, and S. Neville, in *Papers on Designs*, etc. Sir Isaac Pitman and Sons, 1919.

core be a , then the length of iron plus paper insulation is $L_1 = L - a$, and the net length of iron is $L_2 = 0.89L_1$, since about 11 per cent. of the axial length of the core packets is required for the insulation. With a slot width of s and a tooth width of t , a quarter the length up from the root, we have

$$A_a = Ls + (L - L_2)t$$

$$A_t = L_2 t$$

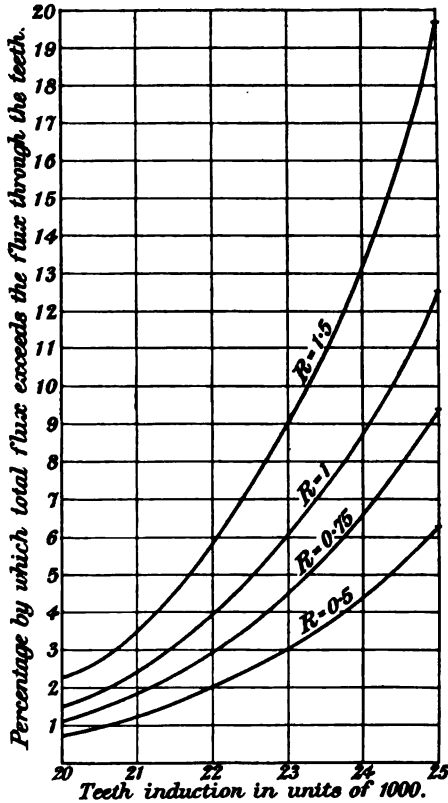


FIG. 11.

To produce an induction B_t through the tooth we require $X = 2lx$ ampere-turns, where x is taken from the table on page 282, Vol. I, and twice the length l of one tooth must be counted since the flux must go down one tooth and up another. The flux actually passing through a tooth is $\phi_t = A_t B_t$. The same exciting force X drives

some flux through the area A_a and the induction is $B_a = 1.25 \frac{X}{2l}$, or $B_a = 1.25x$. The flux through tooth and air is

$$\phi = A_t B_t + A_a 1.25x$$

$$\phi = A_t B_t \left(1 + \frac{A_a 1.25x}{A_t B_t} \right)$$

Let $R = A_a/A_t$ be the ratio between the area of air and metal, then

$$\phi = \phi_t \left(1 + R \frac{1.25x}{B_t} \right) \quad . \quad . \quad . \quad . \quad (12)$$

The second term in the bracket shows the proportion by which the total flux through the tooth ϕ_t is increased by the flux which is squeezed out through the air.

The relation (12) may conveniently be represented by curves for different ratios of air to iron area. In Fig. 11 the abscissae represent true teeth induction in units of 1000 lines per sq. cm. and the ordinates the percentage increase of flux due to the lines of induction which pass through air. Thus with 23,000 tooth induction and equal cross sectional area through air and metal there would be an increase of 6 per cent. Some engineers speak of real and "apparent" tooth induction, meaning by the latter term the induction that would obtain in the metal if the air carried no flux. In the case of 6 per cent. increase the apparent induction would be 24,380 and with wider slots and greater induction its apparent value may reach as high as 28,000. As such a figure gives obviously a wrong impression and has in fact no physical meaning it is better not to speak of apparent induction, but state the true induction and the percentage in reference to it which is carried through the air.

CHAPTER II

THE FIELD MAGNET SYSTEM

Type of field magnet—Advantage of multipolar fields—Materials and mechanical construction—Ratio of length to diameter of armature—Unbalanced magnetic attraction—Size of shaft—Critical speed in turbo-dynamos—Bearing friction.

Type of Field Magnet. Many different shapes of field magnets have from time to time been developed, but of all these only two types have survived. For very small machines the simple horseshoe form as shown in Fig. 97 of Vol. I, and for all other machines a so-called ironclad type in which the yoke surrounds the magnet cores. In the horseshoe type the armature is at one end of the cores and the yoke at the other. Its advantages are a short yoke, an easily accessible commutator and brushes and simple mechanical construction. There is, however, the disadvantage of a strong stray field surrounding the whole machine and in any but very small machines an excessive weight. Where the core is tangential to the armature the external surface of the poles is necessarily greater than the useful polar surface opposite the armature and since the full magnetic driving force obtains at these large surfaces the magnetic leakage is correspondingly great, necessitating the provision of heavier cores than would be the case if the external leakage were reduced to a minimum. A leakage field of from 25 to 45 per cent. is the result, whereas in the ironclad type where the axes of the magnets are radial to the armature the leakage field is generally of the order of 15 to 25 per cent. If the machine is very small, the extra cost of material is not important in comparison with the cost of labour and as this is decreased on account of the simplicity of mechanical construction the adoption of the horseshoe type may be financially the right policy.

Advantage of Multipolar Fields. The ironclad principle, that is a closed yoke ring surrounding the armature completely, can, of course, also be applied to a two pole machine and Fig. 12 *a* shows such a construction to scale for a 75 kw. dynamo running at 700 r.p.m. Its armature has a diameter of 42 cm. and a gross length of 38 cm. With a magnet system (technically termed the

"frame") of dynamo steel the weight of magnetic material in frame and armature is 1600 kg. Fig. 12 *b* refers to a machine of the same output and speed, but constructed as a four pole dynamo. The armature has the same diameter, but its gross length need only be 22 cm. and the weight of magnetic material is only 740 kg., that is, less than half that of the two pole design. The saving in weight is in this case not due to any difference in magnetic leakage (this is of the order of 20 per cent. in both machines), but to the fact that the yoke section is reduced by 50 per cent. and the length of the yoke ring by 20 per cent. The reduction of yoke section is a natural consequence of employing four magnetic circuits instead

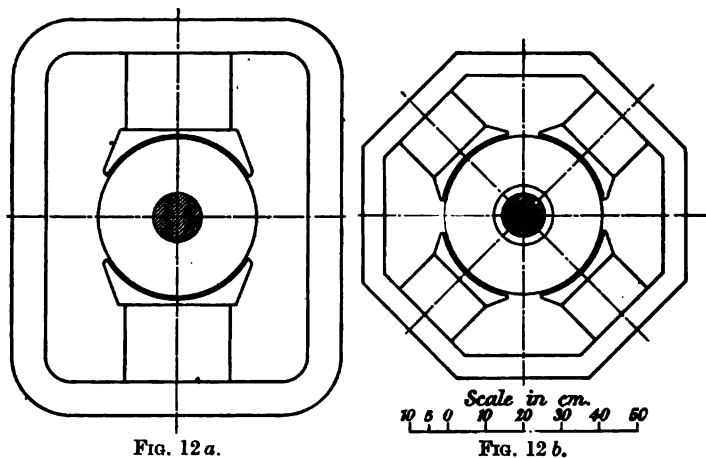


FIG. 12 a.

FIG. 12 b.

of two. Each of these has only to carry half the flux as compared with the two pole design. Hence we get a lighter machine.

Generally speaking the larger the number of poles the lighter is the machine, but weight is not the only consideration. The cost of labour increases with the number of poles, because there are more parts to be handled. There is also the difficulty of providing sufficient room on the armature for the greater number of poles so that the diameter may have to be increased. This may again increase the weight and also bring the circumferential speed beyond a safe limit and there may also be difficulty in commutation. In the case illustrated by Fig. 12 it is doubtful whether a six pole design would be cheaper, although it would certainly be lighter than the four pole design. Each case must be judged on its own merit by working out comparative designs.

Materials and Mechanical Construction. In a frame three parts may be distinguished, namely pole shoes, magnet cores and yoke. Since the yoke is not wound its cross section has no influence on the amount of exciting wire required, but the cores which are surrounded by the exciting coils have an influence. The better the material of the cores magnetically, the smaller is the amount of exciting copper and the use of cast iron for the cores is only advisable in small machines, where the cost of a little more copper may be compensated by the advantage of casting poles and yoke together. In medium sized and large machines the cores must be of dynamo steel or built up of sheet steel so as to reduce as much as possible the perimeter of the exciting coils. The yoke may be of cast iron if the greater weight is not objectionable, but the difference in cost between dynamo steel and cast iron is not of great importance. Roughly speaking dynamo steel is twice or two and a half times the price of cast iron for equal weight, but as the latter material must be used with a cross section at least twice that of dynamo steel there may not be any saving in using the magnetically inferior material. It may, however, become necessary to use cast iron for the yoke for a slow speed machine having many poles and a large diameter yoke. In such a case the section required for the flux may with dynamo steel be less than mechanically desirable to get a sufficiently stiff construction and then the designer is justified in using the magnetically inferior material because it is mechanically the more suitable.

The pole shoes are expansions of the magnet cores and may be cast in one with them, or be separate pieces attached after the coils are in place. With armatures having wide slots it is advisable to build up the pole shoes of sheet steel so as to avoid the generation of eddy currents in the polar faces by reason of the armature teeth sweeping past them. The danger of eddy currents will obviously be the greater the larger the slot and the smaller the air gap. With a slot opening no larger than the air gap, solid polar faces may be used; with a slot opening of $1\frac{1}{2}$ times the air gap a laminated face is advisable and with a slot opening equal to twice the air gap imperative.

With a view to reducing the amount of exciting copper the section of the magnet core should be circular, since the circle is the shape of minimum periphery for a given area. If a rectangular section is unavoidable, as with laminated cores, the rectangle should

for the same reason not depart too much from the square. Whatever the section of the core, the joint with the yoke must have a surface appropriate to the material of the yoke. Hence with a cast iron yoke the core must either be provided with an expanded flange by which it is bolted to a faced surface of the yoke, or it must be let into the metal of the yoke a sufficient depth to provide a total contact surface about double the cross sectional area of the magnet. With a cylindrical core this means that the core must be let into the yoke for a distance of one quarter diameter. This is, of course,

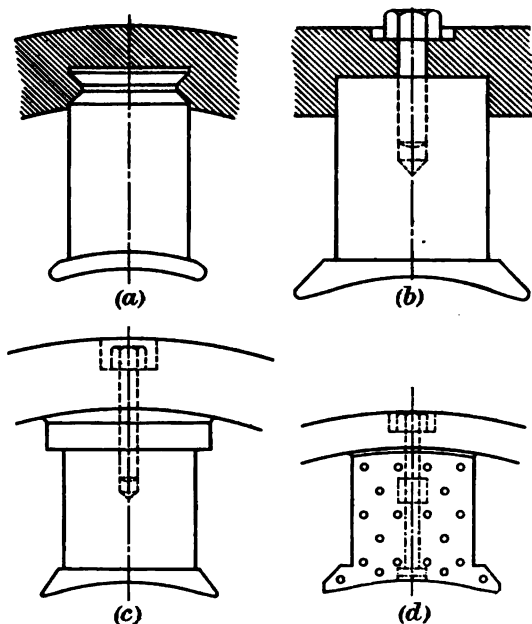


FIG. 13.

impossible with laminated magnets, so that in this case the yoke must be of dynamo steel. Fig. 13, *a* to *d*, show some typical methods of attachment. In *a* the magnet cores are cut from rolled bars, and are grooved at one end so that when the yoke is cast round the ends there shall be a solid connection and good contact. The frame is then bored out and the pole shoes, which are cut from a turned ring or may be packets of stamped plates, are attached by countersunk screws. The pole shoes must be detachable so as to admit the threading of the exciting coil over the core. In Fig. 13 *b*

the core is detachable from the yoke and may therefore be cast in one piece with the pole shoe, but generally it is more convenient to use separate pole shoes as these are more easily handled than the heavy cores. In Fig. 13 *c* the core is provided at its outer end with a flange giving about double the contact surface as compared with the cross section of the core so as to reduce the induction at the joint to a value appropriate to cast iron. In Fig. 13 *d* the yoke is of dynamo steel and no extension of the contact surface is required since magnetically the materials of core and yoke are equally good. The core is built up of sheet steel with stout plates on either side and riveted together as shown. The plates are stamped with a rectangular hole in the centre for the insertion of a steel key and this is tapped for the insertion of the fastening bolts. In an alternative method shown in chain dotted lines the bolt goes through to the pole face and ends with a countersunk head.

Ratio of Length to Diameter of Armature. From a manufacturing point of view it is advisable to vary rather the length of the armature than its diameter for different sizes of machines, since by this policy the number of blanks of different diameters that have to be stocked is reduced. There is, however, a limit to the variation in length of armature that is permissible. This limit is imposed by the economy in exciting copper. It has already been pointed out that the most economical section of a magnet core is either circular or square. The axial length of the pole shoe may exceed the depth of the core by a certain amount provided the radial thickness of the shoe is made sufficiently large to allow an even flow of lines from the core to the polar face. If the axial length of the shoe is great, its thickness must be correspondingly increased, thus making the diameter of the yoke greater and also increasing magnetic leakage. Practical experience has shown that an axial length of shoe exceeding by 50 per cent. the depth of the core is admissible, so that with cylindrical cores the length of armature may be 50 per cent. greater than the diameter of the core. The rectangle of the pole face may therefore have sides in the ratio $1:1\frac{1}{2}$, or it may be a square. With a polar angle of 120 degrees this means that the length of armature may vary between $\frac{2}{3}$ of the pole pitch for square pole shoes and $\frac{1}{2}$ of the pole pitch for pole shoes of rectangular shape. Departure from these proportions is, of course, permissible and in some cases may even be necessary, but as a general guide to the relation between

armature diameter D to armature length L as affected by the number of poles these rules will be found useful. We thus get the following table for the upper and lower limit of L/D :

Number of poles	4	6	8	10	12
$\frac{L}{D}$ upper limit	0.78	0.52	0.39	0.31	0.26
$\frac{L}{D}$ lower limit	0.52	0.35	0.26	0.21	0.17

Unbalanced Magnetic Attraction. The magnetic attraction between polar face and armature core can be calculated by formula (93) (Vol. I), and if the field were absolutely symmetrical all these forces acting radially would cancel out and the armature shaft would be stressed merely by the combining bending moment due to the weight of the armature and the twisting moment due to the torque. If the armature is not absolutely central then the reluctance within the air gap will not be the same all round and the induction will be different. It will be a maximum where the armature surface is nearest to the pole and a minimum at the opposite end of the diameter where it is farthest. The opposing attractive forces no longer balance and the resultant produces an additional bending moment on the shaft. Similar conditions prevail between any opposite poles intermediate to those where the discrepancy of air space is greatest, with the general result that the shaft has to withstand a bending moment which may be comparable with that due to weight. The unbalanced magnetic attraction must obviously depend on the amount of eccentricity in relation to the normal air gap and also on the degree of saturation. If absolute saturation could be attained then it will be clear that the reduction of the air gap under one pole could not result in any increase of induction since the teeth being already saturated cannot pass any more lines. Neither could an increase of air gap cause a reduction of the number of lines. Absolute saturation can only be obtained with an infinite exciting force and hence a little more air reluctance can have no influence. These are, of course, impossible conditions, but their consideration shows that in calculating the unbalanced magnetic attraction we must take account of the degree to which the teeth are saturated. The problem is somewhat complicated; not only is the attractive force different at every point of the circumference, and acting in a different direction, but the saturation is also different. To bring the problem within reach of easy mathematical treatment

we shall assume that the armature is shortened in the ratio of pole pitch to length of polar arc, so that without alteration in the total attracting surfaces or the induction, every point of the circumference (therefore also any part in the neutral zones) is subjected to an attractive force acting radially. We shall also as a preliminary assume that the teeth and other iron have negligible reluctance and that the air gap alone has reluctance proportional to its length. This means that we assume the characteristic of the magnetic circuit of the armature to be a straight line. This assumption is not correct, if we retain the air gap as actually existing, but it becomes very nearly correct if we so increase the air gap that the increased magnetic reluctance compensates

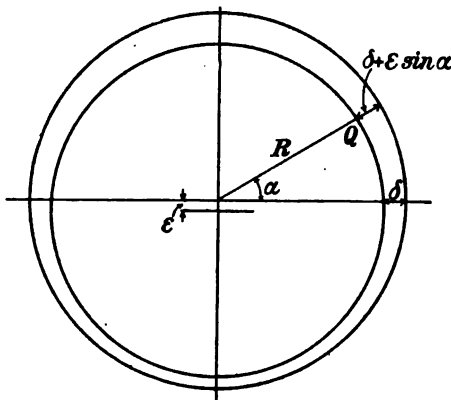


FIG. 14.

for the actual magnetic iron reluctance which we have neglected. The problem then resolves itself into the determination of what this increased air gap must be in order that with the same absolute (not relative) eccentricity the unbalanced magnetic attraction shall be that of the actual machine.

Let R be the air gap radius, δ the normal air gap and ϵ the eccentricity. The two circles in Fig. 14 represent armature and continuous polar cavity. Let the induction with the normal air gap across the horizontal diameter be B , then the induction at Q will be $B \frac{\delta}{\delta + \epsilon \sin \alpha}$ and the attractive force of an element $RLd\alpha$ will be according to formula (93), Vol. I,

$$dF = \frac{RLd\alpha}{8\pi} B^2 \left(\frac{\delta}{\delta + \epsilon \sin \alpha} \right)^2$$

The horizontal component of this force cancels against that of an element on the other side of the vertical centre line, but the vertical component has the same direction as that of the other element. It is found by multiplication with $\sin \alpha$. Since ϵ is very small as

compared with δ the bracket may be written

$$\left(\frac{\delta}{\delta + \epsilon \sin \alpha}\right)^2 = 1 - 2 \frac{\epsilon}{\delta} \sin \alpha$$

$$dF \sin \alpha = \frac{RLB^2}{8\pi} \left(\sin \alpha - 2 \frac{\epsilon}{\delta} \sin^2 \alpha\right) d\alpha$$

Integrating this expression between the limits $\alpha = 0$ and $\alpha = \pi$, we obtain the force with which the armature is pulled upwards by the top half of the polar cavity

$$\text{upward pull} = \frac{RLB^2}{8\pi} \left(2 - \frac{\epsilon}{\delta} \pi\right)$$

Similarly we find the downward pull of the bottom half of the polar cavity

$$\text{downward pull} = \frac{RLB^2}{8\pi} \left(2 + \frac{\epsilon}{\delta} \pi\right)$$

The resultant pull, or unbalanced magnetic attraction, is

$$F = \frac{RLB^2}{8\pi} \cdot 2\pi \frac{\epsilon}{\delta} \text{ dynes}$$

$2\pi RL$ is the total cylindrical surface of the shortened armature which, as above explained, is equivalent to $2pA$ where A is the area of the actual polar face,

$$F = \frac{2pAB^2 \epsilon}{8\pi \delta}$$

Let $f = \frac{A}{8\pi} B^2$ be the magnetic attraction of one pole if the induction and air gap have normal values, then with a relative eccentricity of ϵ/δ we have

$$F = 2pf \frac{\epsilon}{\delta} \dots \dots \dots (13)$$

To get F in kg. we must insert f in kg. by formula (94) of Vol. I,

$$f = \frac{A}{24 \cdot 6} \left(\frac{B}{1000}\right)^2$$

In a ten pole machine with a relative eccentricity of 10 per cent. the unbalanced magnetic attraction would be equal to the magnetic attraction of one pole. This refers to a machine in which the reluctance of iron is negligible, that is, a machine worked at a magnetisation well below the knee of the characteristic. Generally machines are worked well above the knee and then a correction of (13) becomes necessary. We must determine the dimension of a fictitious air gap which shall represent the increase of iron reluct-

ance due to saturation. In Fig. 15 is shown the characteristic of an armature with the working point b above the knee. The straight line OB_a connects induction and exciting force required for air with the normal air gap δ and the curve OB connects induction and total exciting force for the centrally placed armature. If now the air gap is reduced on one side by the amount ϵ , the air characteristic is OB_{a_1} and the curve lies higher as shown by B_1 . With an increased air gap we get the characteristic B_2 . In the usual way of drawing a characteristic the abscissae represent exciting force, but since for a given flux the exciting force required by the air is proportional to the air gap, the horizontal scale in Fig. 15 is supposed

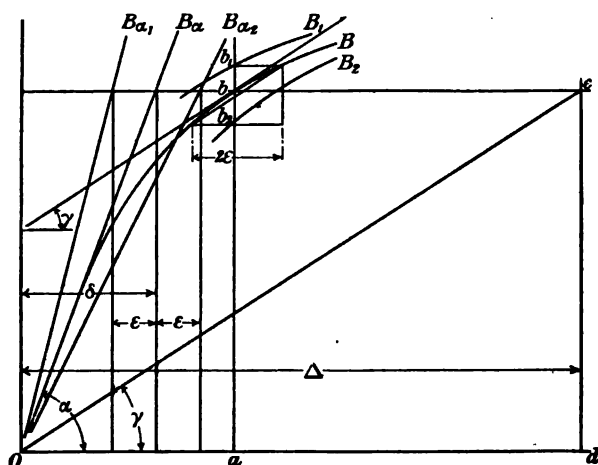


FIG. 15.

to be so chosen that abscissae represent air gap. Oa (measured with the correct scale for ampere-turns) gives the exciting force required to produce in the properly centred armature the gap induction ab ; on the side of the narrow air gap the same exciting force produces the gap induction ab_1 ; and on the side of the wide air gap it produces the gap induction ab_2 . Draw horizontal lines through b_1 and b_2 so as to intersect the normal characteristic, then the horizontal distance between the two points of intersection shows the difference in excitation if measured in the usual scale of ampere-turns, and if measured in the air gap scale it shows difference of air gap, that is very nearly 2ϵ . The vertical distance between the B_1 and B_2 curves is b_1b_2 , and the sloping line joining the working points

corresponding to narrow and wide air gap is parallel (or nearly so) to the tangent drawn to the normal characteristic B at the working point b . We have now the following relations

$$ab = B, \quad ab_1 = B_1, \quad ab_2 = B_2$$

$$\tan \alpha = \frac{B}{\delta}, \quad \tan \gamma = \frac{B_1 - B_2}{2\epsilon}$$

Draw through O a line parallel to the tangent through b and prolong it so as to cut the horizontal through b in c . The height dc is then also equal to $B = ab$ and the line Oc may be considered as the air characteristic of the machine with a much increased air gap Δ , so that $\tan \gamma = \frac{B}{\Delta}$.

We thus find $\Delta \tan \gamma = \delta \tan \alpha$
and the fictitious air gap is

$$\Delta = \delta \frac{\tan \alpha}{\tan \gamma} \quad \dots \dots \dots (14)$$

The construction for finding Δ is therefore quite simple. Draw the characteristic of the armature and to the working point draw a

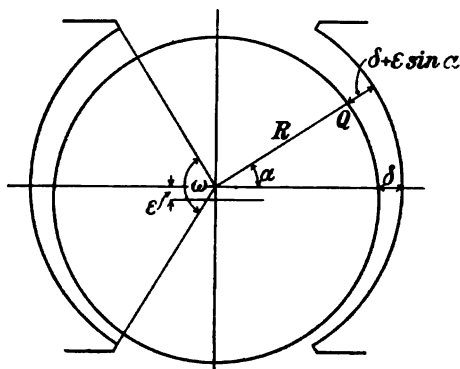


FIG. 16.

tangent. Draw a parallel to this tangent through the origin and determine its point of intersection with a horizontal through the working point. The ratio between the abscissae of these two points namely Od/Oa is the ratio in which the true air gap δ must be increased to find the air gap Δ . By inserting Δ instead of δ in (13) we get the true unbalanced attraction. It will be noticed that the higher the working point lies on the characteristic the smaller is

the angle γ and the greater is Δ . High saturation of armature teeth therefore reduces the unbalanced magnetic attraction.

In deducing formula (13) we have used the expedient of reducing the length and increasing the width of the polar faces so as to get a continuous polar cavity. This is permissible in a multipolar machine, but it is not permissible in a two pole machine. In this case we must not extend the integration over the whole circumference, but restrict it to the polar angle. In Fig. 16 is shown a two pole field with the armature displaced in the direction of the brush axis by the eccentricity ϵ . In the polar axis there can be no unbalanced magnetic attraction because the same number of lines must pass out of a north pole that goes into a south pole, but in the direction of the brush axis there will be an unbalanced force if there is a displacement in this direction. Using the previous notation we have for the vertical component of the attractive force of an element at Q

$$dF \sin \alpha = \frac{RLB^2}{8\pi} \left(\sin \alpha - 2 \frac{\epsilon}{\delta} \sin^2 \alpha \right) d\alpha$$

L being in this case the total (not the reduced) length of the armature. By integrating between the limits $-\frac{1}{2}\omega$ and $+\frac{1}{2}\omega$, that is over the total polar angle ω , we obtain the downward attractive force due to the right hand pole and as that of the left hand pole is equal in amount we simply double the result to get the total unbalanced magnetic attraction,

$$F = \frac{RLB^2}{8\pi} 2 \int_{-\frac{\omega}{2}}^{+\frac{\omega}{2}} \left(\sin \alpha - 2 \frac{\epsilon}{\delta} \sin^2 \alpha \right) d\alpha$$

$$F = \frac{2\pi RLB^2}{8\pi} \left(\frac{\omega}{\pi} - \frac{\sin \omega}{\pi} \right) \frac{\epsilon}{\delta} \text{ dynes}$$

$2\pi RL = S$, the total cylindrical surface of the armature; and $\frac{SB^2}{8\pi}$ is the magnetic attraction of such a surface subjected in every point to the gap induction B . To get F in kg. we use formula (94) in Vol. I and obtain

$$F = \frac{S}{24 \cdot 6} \left(\frac{B}{1000} \right)^2 \left(\frac{\omega}{\pi} - \frac{\sin \omega}{\pi} \right) \frac{\epsilon}{\delta} \text{ kg.}$$

If the working point is on the straight part of the characteristic, δ is the actual normal air gap; if the working point is above the knee,

which is usually the case, then we have to substitute for δ the enlarged value $\Delta = \delta \frac{\tan \alpha}{\tan \gamma}$, the angles α and γ being taken from the characteristic as explained with reference to Fig. 15.

The final equation of the unbalanced magnetic attraction on the armature of a two pole machine with the relative eccentricity ϵ in the direction of the neutral axis may therefore be written as follows:

$$F = \frac{S}{24 \cdot 6} \left(\frac{B}{1000} \right)^2 \frac{\epsilon \tan \gamma}{\delta \tan \alpha} a \text{ kg.} \quad . \quad . \quad . \quad (15)$$

In this equation B is the normal gap induction for a central armature, S is the total cylindrical surface of the armature and a is a factor depending on the polar angle ω . For convenience in determining the attractive force this factor $a = \frac{\omega}{\pi} - \frac{\sin \omega}{\pi}$ is here tabulated:

Polar angle in degrees	130	120	110	100
Value of $a \quad \dots \quad \dots$	0.47	0.39	0.31	0.24

In some machines advantage is taken of the unbalanced attraction by placing the armature slightly above the true central position and thus relieving the bearings of part of the weight of the armature.

Size of Shaft. In determining the diameter of the armature shaft not only the weight and the torsional effort to be transmitted must be taken into account, but also a possible unbalanced magnetic attraction which may be additional to the weight. With turbo-machines there is the further possibility of resonance being set up between the natural vibration of the shaft considered as an elastic beam and the speed of rotation. On account of these various influences it is not a certain limit of initial stress in the material, but rather the initial flexure which determines the size of the shaft. A rough, but only very approximate empirical formula for the diameter of the shaft is

$$d = \frac{1}{3} \sqrt[3]{RL} \quad . \quad . \quad . \quad . \quad (16)$$

where R is the radius of the armature, L its gross core length and l the distance between centre of armature and centre of the nearest bearing. If these dimensions are inserted in cm. the diameter on the shaft results also in cm., but the formula is also valid for inch measure.

A more exact formula may be deduced on the basis of a limited flexure. If the bearings are properly centred, but the shaft may bend, then we have the same condition as with an eccentric armature and any initial flexure tends to increase under the unbalanced magnetic attraction. Hence it is important to make the shaft stiff enough that under the weight of the armature it shall not appreciably bend. That there must be some bending is obvious, but it should be a small amount so that the flexure shall be only a small fraction of the air gap distance. The relation between bending moment, moment of inertia and diameter of the shaft is found in every text-book on mechanics and need therefore not be discussed at length in this place. Taking the modulus for steel at 2×10^6 in metric measure we find that with a bearing span of l metres and a force of P kg. attacking the shaft in the middle between the bearings the following relation obtains

$$d^4 = \frac{2 \cdot 13 P l^3}{f} \quad . \quad . \quad . \quad . \quad . \quad (16a)$$

where d is the diameter of the shaft in cm. and f the flexure in mm. This formula errs, if anything, rather on the safe side, because 2×10^6 is rather a low modulus and the assumption that the load is concentrated in one point is not correct. In reality the load is distributed over the length of the armature or rotor, but where the actual amount of load P is necessarily uncertain (as accuracy of workmanship in setting up the bearings influences the unbalanced attraction which is one of the components of P) it would not be wise to aim at cutting down the size of the shaft to the least possible value. If the armature is appreciably to one side of the centre between the bearings the flexure will be slightly diminished. Let l_1 and l_2 be the distances to the bearings, then we have

$$d^4 = \frac{2 \cdot 13 P l^3}{f} \left(\frac{2l_1}{l} \right)^2 \left(\frac{2l_2}{l} \right)^2 \quad . \quad . \quad . \quad . \quad (17)$$

Critical Speed in Turbo-dynamos. A difficulty is sometimes met on account of excessive vibration in turbo-dynamos, but only at certain speeds. The explanation is as follows. When the rotor is at rest the weight on the shaft causes a compression in its upper fibres and an extension in the lower, resulting in a certain flexure. Whether this be large or only minute is immaterial. What matters is that when the rotor is turned through 180 degrees the fibres which were formerly in compression are now in tension and *vice*

versa. The flexure has now the opposite sign. If the speed is low, no swing can be set up, but at a certain (generally fairly high) speed a swing may be set up. We have here the case of a mass m threaded on to an elastic beam which is subjected to a bending force travelling round it at the speed of rotation. The mass is thereby set swinging and although the initial amplitude of the swing may be exceedingly minute it may become appreciable if there is resonance between the natural period of the swing and the speed of rotation. The periodic time of a mass m swinging under the controlling force c is

$$T = 2\pi \sqrt{\frac{m}{c}}$$

The controlling force c is defined as the ratio between bending force and flexure. Taking the force in kg. and flexure f in metres, we have $c = P/f$ and

$$100f = \frac{P^2}{EJ48}$$

where E is the modulus and J the moment of inertia. The 48 in the denominator refers to the load P being centralised. In a turbo-dynamo the load is distributed over an appreciable distance (namely, the axial dimension of the rotor), and in this case we may substitute 60 for 48, so that the controlling force is

$$c = \frac{60JE}{l^3} 100$$

The mass m is in units of 9.81 kg. The periodic time of the free swing is therefore

$$T = 0.628 \sqrt{\frac{ml^3}{60JE}}$$

Since $J = \frac{\pi d^4}{64}$ and $E = 2 \times 10^6$ we find

$$\frac{1}{T} = \frac{3.86d^2}{\sqrt{ml^3}}$$

where l is in metres and d in cm.

If the speed is $1/T$ revolutions a second we have perfect resonance and the machine will come to grief. If the speed is appreciably higher the working will be satisfactory, but to reach the higher speed the machine must pass the condition of resonance. In running up this may be done so quickly that there is not time enough for resonance to develop, but in stopping, the machine may not pass the critical condition quickly enough and then there is danger of

the shaft breaking. To avoid all such danger it is best to make the shaft stiff enough so that the critical speed of resonance shall be well above any speed which may be reached in practical working conditions. If U_m is the maximum possible number of revolutions per minute we have as the condition of safe working under all circumstances

$$U_m < \frac{231d^2}{\sqrt{ml^3}}$$

or approximately

$$U_m < \frac{23d^2}{\sqrt{Wl^3}} \dots \dots \dots (18)$$

where W is the weight of the rotor in tons, l the distance of centres of bearings in metres and d the diameter of the shaft in centimetres.

It should be noted that U may be greater than the normal working speed, in the case that the governor loses control of the engine. If for some reason it is impossible to provide a shaft so large that even the runaway speed shall be smaller than the critical speed, then the shaft should be dimensioned so as to make the critical speed sensibly less than the normal working speed and some provision should be made to enable the machine to stop quickly by letting it work on an external resistance after the driving power has been cut off. This is, however, but a makeshift and can hardly be considered good practice.

Bearing Friction. Let p represent the specific pressure on a bearing in kg. per sq. cm. of the area A represented by the product of diameter and length of journal; let further v represent the linear speed of the journal in m./sec., then experience shows that as long as the product $p \times v$ is not larger than 15 or 20 there will not be any undue heating. It is also found that the loss of power in the journal is hardly at all affected by the specific pressure. It is merely a function of speed and area. Experiment also shows that frictional drag on the journal increases with the square root of the speed and the power is therefore proportional to the root of the cube of the speed. The formula commonly used to calculate the loss of power in watts by journal friction is

$$P = 0.4 \, dlv \sqrt{v} \dots \dots \dots (19)$$

where d = diameter of journal in cm.

l = length of journal in cm.

v = linear speed of journal in m. per second.

Some engineers take 0.35 instead of 0.4 as the coefficient in (19), but in all cases the formula refers to ring lubrication where the oil is continuously supplied to the upper part of the journal. By the motion of the surface fresh oil is continuously packed between it and the brass so that the journal floats on a thin layer of oil which is continuously squeezed out and renewed. If the speed is so low that the process of packing in fresh oil cannot keep pace with the rate at which the oil is squeezed out, the film of oil becomes very thin and then we approach the condition of friction between solid surfaces and the loss increases with the specific pressure. Also if the bearing is cold the film of oil does not flow so readily and there is more liquid friction. It will thus be seen that the calculation of the frictional loss in the bearings can only be approximate, but the above formula gives a fair average value. Moreover bearing friction is only a very small fraction of the other losses so that extreme accuracy in its determination is not very important.

CHAPTER III

THE COMPLETE DYNAMO

Characteristic curves—Leakage coefficient—Magnetisation characteristic—Armature reaction—Methods of excitation—Calculation of exciting coils—Condition of self-excitation—The output coefficient—Temperature rise—Commutation losses—Commutation—Reactance voltage—Exciting force required for interpoles—Advantages of interpole commutation—Compensating windings—Some special types of dynamo.

Characteristic Curves. Any two co-related quantities in the working of a dynamo can be represented by a so-called characteristic. Thus in a motor we may co-relate current and torque, or torque and speed; in a generator current and external resistance, or terminal e.m.f. and current, or e.m.f. and exciting force and so on. Of such possible curves the one connecting exciting force and magnetic flux, the so-called “magnetisation characteristic,” may be considered as fundamental, since all other characteristics can be obtained from it. The general principle on which the magnetisation characteristic is obtained has been discussed in Vol. I, Chapter X, but only for a smooth armature core and a constant leakage coefficient. We also neglected any influence that the current in the armature winding might have on the field magnets, in other words we determined the flux for a machine running on open circuit. To determine the characteristic for a machine with toothed armature we make use of the relation discussed at the end of Chapter I; and to assign the leakage its correct value we must have recourse to experiment.

Leakage Coefficient. The leakage coefficient expressed as a percentage of the useful flux through the armature is not a constant, but if the leakage flux is expressed by the product of a factor and the exciting force required for the armature, experiment shows that this factor can be considered constant for any particular machine. How such an experiment can be made will be shown later; for the present we are only concerned to note that the leakage flux may be expressed by

$$\Phi_{\lambda} = \lambda X_a (20)$$

where λ is the leakage factor and X_a is the exciting force required to overcome the reluctance of air gap, armature teeth and armature core. Strictly speaking λ should be experimentally determined for each size of machine of one and the same general type, but as it is necessary to predetermine the characteristic before the machine is actually built, we may arrive at the value of λ from its value for a machine of the same type but different size by the following reasoning. The leakage field being through air is proportional to the magnetic driving force as indicated by the formula. For one and the same driving force it is inversely proportional to the reluctance of the space surrounding the magnet cores and poles. However complicated the actual leakage field may be, its reluctance for similar shapes must be inversely proportional to the linear dimensions of similar machines. This follows immediately from the conception of reluctance as the ratio of a length to an area. We thus find that the leakage factor of machines of the same type (that is geometrically similar) is proportional to a linear dimension, say for instance to the square root of the product of diameter and length of armature. Let the indices 1 and 2 refer to two machines of the same type, then (even if the ratio of D/L is not precisely the same in both) we can find the leakage factor of one machine from that of the other, since

$$\frac{\sqrt{L_1 D_1}}{\sqrt{L_2 D_2}} = \frac{\lambda_1}{\lambda_2}.$$

On the other hand with the same magnetic loading the useful flux is proportional to the square of the linear dimension, so that if the exciting force in the larger of two machines would be increased in the proportion of the linear dimension, the percentage of leakage would not be increased. Generally speaking the exciting force need not increase quite at the same rate as the linear dimensions of machines of the same type, so that the percentage leakage in large machines is less than in small machines.

Magnetisation characteristic. To obtain the magnetisation characteristic (also called saturation curve) of a machine if its working drawings are available we proceed as follows. Determine the tooth area A_t under one pole, the section through the teeth being taken $\frac{1}{4}$ the way up from the root. To find the air gap area take the mean between the area of the polar face and the top of the teeth, no deduction being made for paper insulation. Let this

area be A_a . The cross sectional area of the other parts of the magnetic circuit are simply scaled off from the drawing, as are also the different mean lengths. Assume a value of tooth induction and take from the curve Vol. I, Fig. 121, or the table on p. 282, the corresponding exciting force per cm.; this multiplied with double the tooth length gives X_t , the exciting force required for the teeth. The flux passing through the teeth is Φ_t , and the total useful flux Φ from one pole is then found by using formula (12). This flux has to pass through the air gap and the excitation required for it, namely X_a , can be found as already shown. The same flux also passes through the armature core and as the mean length of path can be measured off the drawing we find the corresponding excitation X_c . We now have $X_a = X_t + X_a + X_c$, that is the total exciting force required by the armature and also that which produces the leakage flux

$$\Phi_\lambda = \lambda X_\lambda$$

The flux carried by the field system is

$$\Phi_m = \Phi + \Phi_A$$

and by using the curves in Vol. I, Fig. 96 or Fig. 121, for laminated field magnets, we can determine the different components of the exciting force X_m required for the field and by adding this to that required for the armature we find the total exciting force

$$X = X_a + X_m$$

which must be applied to produce the useful flux Φ . This calculation repeated for different values of B_t gives a series of related values of X and Φ , from which the magnetic characteristic of the machine can be plotted. Fig. 17 shows such a curve OBc . To the excitation $X=Oa$ corresponds the working point B . According to the scale of ordinates the curve may represent either total useful flux or induction in the air gap.

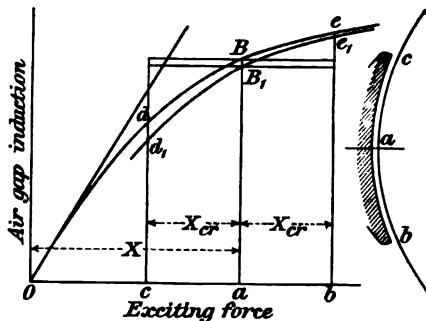


FIG. 17.

Armature reaction. If the machine is running at constant speed the ordinate in Fig. 17 may to an appropriate scale also represent the e.m.f. induced in the armature and appearing at the

brushes if the machine is on open circuit. If it is producing current the brush voltage will be lowered by the effect of armature resistance and also by a magnetising effect due to the armature current. This effect is comprised under the term armature reaction. The armature may be considered as an electromagnet with the exciting winding distributed over its surface. If a current flows through this winding it will magnetise the armature and produce a flux in the surrounding space. The brush axis becomes the polar axis of this self-induced armature field and this in a two pole machine stands geometrically at right angles to the polar axis of the field magnets if the brushes are exactly midway between the poles. In this case the volume of current flowing say downwards between the brush and the edge of a north pole is exactly the same as that flowing upwards between the brush and the edge of the south pole, so that no magnetic action is produced on the main field and the excitation which produces it is neither increased nor diminished by the armature current.

Now let the brush be shifted by a certain distance c . The volume of current flowing one way on one side of the brush will now be increased by the amount Δc ampere wires, where Δ is the linear current density per cm. of circumference. On the other side of the brush the volume of current will be diminished by an equal amount. The two sides no longer balance and the effect is that an exciting force of $2c\Delta$ is applied to the field magnet system by the armature current. If in order to improve commutation the brushes are shifted forward in a generator and backward in a motor it will be found on applying Ampere's rule that in either case this additional exciting force acts in opposition to that applied by the coils of the field magnet and for this reason we speak of the "back turns" of the armature. The applied exciting force is reduced by the back turns. In a multipolar machine the effect is the same, but to get a numerical expression for the back turns it will be convenient to define the shift of the brushes not as a linear dimension but as an angle, and to give the relation of this angle to the angular distance between the axes of two neighbouring field poles. Let the pole pitch be τ then $\frac{c}{\tau} = \kappa$ is the ratio between the two angles and the back turns expressed in ampere wires are $X_b = 2\kappa\tau\Delta$. Using the same notation as in Chapter I, we have

$$\Delta = \frac{zi}{\pi D}; \quad \pi D = 2p\tau; \quad i = \frac{I}{2a}; \quad \Delta = \frac{zI}{4p\tau a}$$

The exciting force produced by the armature current in the axis of any pair of + and - brushes is therefore

$$\Delta\tau = \frac{zI}{4pa} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Since $X_b = 2\kappa\Delta\tau$ we get for the armature back turns the expression

$$X_b = 2\kappa \left(\frac{zI}{4ap} \right) \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The quantity in brackets is the exciting force acting in the different brush axes of the armature. It drives flux outward at the brush where the current leaves the armature and inward where the current enters the armature. Midway between two brushes no flux passes either way, but the farther we go from the mid position the stronger is the magnetic force driving lines either into or out of the armature, the force increasing proportionately with the distance from mid pole. At the polar edges it is a maximum and corresponds to the current volume expressed in ampere wires under the polar surface. If m is the ratio of pole width to pole pitch (m is generally of the order $2/3$ corresponding to a polar angle of 120 electrical degrees) the self-induced flux under the polar edges corresponds to an exciting force of

$$\begin{aligned} X_{cr} &= m\tau\Delta \\ X_{cr} &= m \left(\frac{zI}{4ap} \right) \quad . \quad . \quad . \quad . \quad . \quad (23) \end{aligned}$$

This exciting force does not directly influence the main field since it acts in a direction electrically in quadrature to the main exciting force, that is to say, across it. For this reason we speak of "armature cross turns" and their numerical value is very much greater than that of the back turns and may even be equal to a considerable fraction of the main field ampere-turns. The effect of the cross turns is to deform the shape of the main field, making the induction greater under one and less under the other polar edge. If the working point lies very high on the characteristic the influence of the cross turns is small. The characteristic is only slightly curved and has but a small inclination to the horizontal so that the addition of X_{cr} to X or its subtraction does not very greatly alter the induction. It is greater at one edge than at the other, but the curve being almost a straight line the average is practically unaltered. If, however, the working point is somewhere near the knee the reduction of induction at one side is not fully compensated by the increase on

the other and then the cross turns have the result of slightly lowering the flux. This will be seen from Fig. 17. The sketch on the right of the characteristic shows a polar face and the part bc of the armature immediately beneath it. The total exciting force at b is $X + X_{cr}$, at a it is X and at c it is $X - X_{cr}$. The exciting force at intermediate points is changed by a corresponding fraction of X_{cr} . To find the induction at any point of the polar face we must therefore add to or deduct from X this fraction of X_{cr} and take the corresponding ordinate on the characteristic $OdBe$. Thus for the polar edge b the induction is be ; for the polar edge c it is cd . Since the total flux is the integral of induction and differential of surface we find that the flux is proportional to the area of $cdBeb$ and the mean induction corresponding to the excitation Oa is the height of a rectangle of equal area. Let this be aB_1 , then B_1 is a point of the characteristic as modified by the cross turns. Other points for larger and smaller values of X , but for the same armature current, are found in the same way giving the curve $d_1B_1e_1$. To each armature current I and therefore to each value of X_{cr} corresponds a particular curve $d_1B_1e_1$, and by repeating the construction we find for each electric loading of the armature a characteristic which does not spring from the origin as does the characteristic for the electrically unloaded machine. The back turns having to be deducted also reduce the effective flux. Thus both the back and cross turns have the effect of slightly lowering the characteristic, so that the e.m.f. induced in the loaded machine is slightly less than in the unloaded. With the brushes not too far from the neutral axis and the working point well above the knee the difference is, however, not very great. It can be compensated by an increase of excitation.

Methods of Excitation. In a motor the power required for excitation is necessarily derived from some external source and in this case we speak of a "separately excited" machine. A generator may also be separately excited as for instance in the case of the machines at a central station which are excited either from the main bus bars or from some exciter serving all machines. Single generators are, however, generally self-exciting. The magnet winding may be arranged to carry the full current and is then put in series with the armature and external circuit, or the magnet winding may be arranged as a shunt to the armature, or both methods may be employed simultaneously. The technical terms for these three methods are "series machine," "shunt machine" and "compound machine."

Calculation of Exciting Coils. In a shunt machine the excitation depends on the voltage. Let e be the voltage available for one magnetic circuit and the number of turns inter-linked with it be n . Let further l be the mean length of the turns in metres, q the cross sections of the wire in square mm., r the resistance of the n turns and i the current. Call ρ the specific resistance, then $r = \rho \frac{nl}{q}$ if l is in metres and q in sq. mm. For the machine heated up by the current ρ may be taken as 0.02. The following relations are self evident:

$$X = ni; \quad i = \frac{e}{r}; \quad X = e \frac{50q}{l}; \quad q = \frac{Xl}{50e}$$

For a given voltage and perimeter the exciting force is simply proportional to the section of the wire and does not depend on the number of turns. It is therefore impossible to increase excitation of a shunt machine by winding more wire on to the coil; on the contrary, by increasing the winding depth we increase slightly the perimeter and thereby reduce the excitation. The power absorbed by the n turns is $ei = P$, or

$$P = X \frac{e}{n}$$

This diminishes as the number of turns is increased. The efficiency of the machine is increased by incurring a greater capital outlay for exciting copper, but there is a limit to this improvement by the available winding space and the necessity of providing some air space round the coils for cooling. If Q is the total winding space available we have $(sf) Q = nq$ where (sf) is the space factor which depends on the size of wire and the kind of insulation employed. For covered wire and the usual double cotton covering (sf) may be taken from the following table:

Diam. of wire in mm.	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Space factor	0.32	0.47	0.53	0.57	0.6	0.62	0.63	0.64	0.65	0.65

By introducing Q into the above equations we obtain

$$P = \frac{X^2 l}{50 (sf) Q} \text{ watts, } Q = \frac{X^2 l}{50 (sf) P} \text{ sq. mm.} \quad . \quad (24)$$

The different methods of excitation are shown in Fig. 18; a represents a series machine, b a shunt machine and c a compound machine. The fourth figure, d , is also a compound machine, but with this slight variation that the shunt is excited from the terminals. This is called a "long shunt" to distinguish it from Fig. c , which is called

a "short shunt." In the long shunt the series coils contribute a trifle more to the excitation and the shunt coils a trifle less than in the short shunt but the general effect of producing an approximately constant voltage at the terminals is the same.

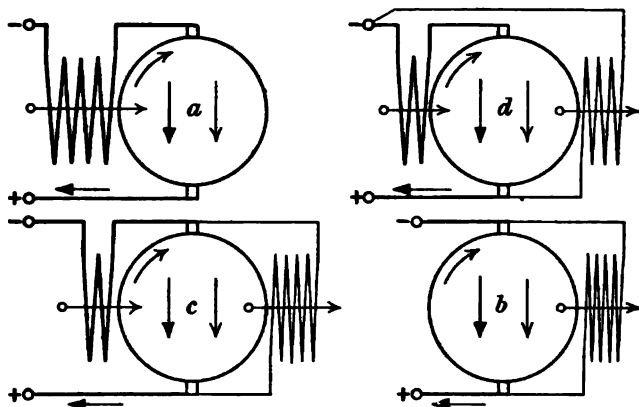


FIG. 18.

Condition of Self-excitation. Whatever method of self-excitation is adopted the e.m.f. induced in the armature must be equal to that required to drive the exciting current through the exciting circuit and armature, including in the latter an equivalent resistance to represent armature reaction. It has been shown that armature reaction lowers the characteristic of magnetisation. At constant speed it also lowers the terminal voltage. Armature resistance does the same and although the lowering of voltage is not strictly proportional to armature reaction we may as an approximation assume it to be so and then its effect may be represented by assuming the armature resistance to be a little greater than its true ohmic value. If, for instance, the ohmic drop of voltage be 3 per cent. due to true armature and brush resistance at full load and the effect of armature reaction be a lowering of the magnetic characteristic by 2 per cent., then we can replace the real machine by an ideal machine having no armature reaction, but an armature resistance which causes a drop of 5 per cent. Let in Fig. 19 the curve represent either working flux or, for a constant speed of n r.p.s., the induced e.m.f. in a generator as a function of the exciting force X . Let R be the total resistance in the exciting circuit. In a series machine R contains the resistance of the external circuit; in a shunt machine that of any field rheostat that may be included.

In all cases it contains also the resistance of the exciting coils and of the armature, the latter augmented so as to represent the effect of armature reaction. The exciting force is $X = in$ and $\Phi = X \tan \alpha$. The sloping line OQ represents this relation and the working point must lie somewhere on this line. It must also lie somewhere on the characteristic; its position is therefore given by the point of intersection between the two lines. If there is no point of intersection the machine cannot self-excite; the condition of self-excitation is therefore $\alpha < \alpha_0$,

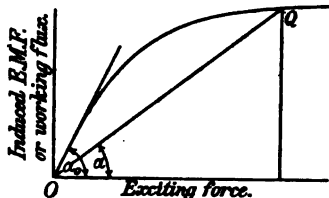


FIG. 19.

where α_0 is the slope of the characteristic at the origin. To find the numerical value of $\tan \alpha$ we use the relations $ni \tan \alpha = \Phi$ or $ne \tan \alpha = \Phi R$ and $e = \epsilon \Phi u$. By combining we obtain

$$\tan \alpha = \frac{R}{ne u}$$

n is the number of turns in one magnetic circuit, that is double the number of turns on one pole, and ϵ is a constant of the design, its numerical value is by (3)

$$\epsilon = \frac{pz}{100 a}$$

The only variables in the above equation are therefore R , the ohmic resistance of the exciting circuit, and u the speed in r.p.s. A shunt machine will fail to excite if the speed is below a certain critical value; a series machine running at constant speed will fail to excite if the resistance in the external circuit is above a certain critical value. The condition for self-excitation may therefore be written in the general form

$$\frac{R}{ne u} < \tan \alpha_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

In deducing the inequality (25) we have made the assumption that armature reaction may be considered as equivalent to a certain increase in armature resistance. This is obviously permissible in the case of a shunt machine because on open circuit the armature current is very small; it is also permissible in the case of a series machine with interpoles, because the armature reaction is small at all loads and the approximation in replacing its effect by that of additional resistance is fairly close, but the approximation becomes rather bad in the case of a series machine when the brushes have

to be shifted so as to obtain good commutation. In this case there is considerable armature reaction and its effect depends on the position of the working point, that is on the shape of the characteristic, so that some inaccuracy is introduced by assuming that armature reaction can be represented by additional armature resistance. The argument in relation to Fig. 19 remains valid all the same, but instead of basing it on the characteristic of magnetisation as we may do in a shunt machine it must be based on the external characteristic. This matter is treated in the next chapter.

The Output Coefficient. One of the considerations which limits the output of a machine is rise of temperature produced by power losses which are a certain fraction of the total power. The better the ventilation the greater may be these losses with a given temperature rise and if all machines had the same efficiency the output could then simply be considered as a function of the rate at which heat can be dissipated by the ventilating action of the revolving armature. The assumption of equal efficiency is, of course, inadmissible, but leaving this point for a later correction we may estimate roughly the ability of a machine to dissipate heat as the product of surface and speed. The surface may be taken as proportional to diameter and length of armature, the speed as proportional to diameter of armature and r.p.m., so that the output on this reasoning would be represented by the product of square of diameter of armature, length of its core and speed multiplied with a coefficient which is not a constant, but depends on the size of the machine and thus takes account of the fact that the bigger machine has also the higher efficiency.

Thus the error due to the assumption of equal efficiency for all machines can be corrected by adopting for this coefficient values obtained from experience.

Denoting the speed in r.p.m. by U and retaining the previous notation for diameter and length of armature core in metres the output in kw. may be calculated from

$$P = CD^2LU \quad . \quad . \quad . \quad . \quad . \quad (26)$$

In this formula C is the *output coefficient*. It is, as already stated, not a constant, but may vary between wide limits. Its numerical value does not only depend on the size of the machine, but also on the voltage, the efficiency demanded, the temperature rise permitted and finally also on the skill of the designer. For voltages

up to 500 or 600 volts and the usual temperature rise the value of C in very rough approximation may be taken as a function of the armature diameter according to the formula

$$C = 0.7 + 2\sqrt{D}$$

where D is in metres. At first sight it might seem that formula (26) is purely empirical, but this is not the case; there is a scientific basis for it as will be seen by the following reasoning. The output is the product of e.m.f. and current, or $P = EI10^{-3}$ if, as in (26), the power is to be stated in kw. Reckoning flux in c.g.s. lines we have

$$E = \frac{p}{a} \Phi z u 10^{-8} \quad \text{and} \quad I = 2ai$$

$$P = 2p\Phi zi u 10^{-11}$$

Let B represent the average induction reckoned not only over the polar faces, but over the whole of the armature surface, so that

$\Phi = B\tau L$ where $\tau = \frac{\pi D}{2p}$ then, since $u = \frac{U}{60}$ and $zi = \Delta\pi D$ we can write

$$P = \frac{BL}{60} \pi D zi U 10^{-11} = \frac{\pi^2}{60} D^2 L B \Delta U 10^{-11}$$

where D and L are in cm. If reckoned in metres we have

$$P = 0.165 D^2 L B \Delta U 10^{-5}$$

$$P = 0.165 \left(\frac{B}{1000} \right) \left(\frac{\Delta}{100} \right) D^2 L U$$

This is the same expression as (26) if we put

$$C = 0.165 \left(\frac{B}{1000} \right) \left(\frac{\Delta}{100} \right) \quad . \quad . \quad . \quad (27)$$

The output coefficient therefore depends on the product of B and Δ . B is a measure of the magnetic loading and Δ , the number of ampere-wires per cm. circumference, is a measure of the electric loading of the machine. With a polar angle of 120 electrical degrees the actual induction under the pole is 50 per cent. greater than the average induction B reckoned as if the flux were evenly distributed over the whole pole pitch. The latter is in modern machines of the order of 6 to 7000, which gives $B_{\max.}$ of the order of 10,000. The current density with efficient cooling arrangements and special commutating devices may be of the order of 200 to 250 ampere-wires per cm., so that in machines of moderate and large size output coefficients of 3 to 4 can be obtained and even more for intermittent

service, or if special provision for very energetic cooling is made, since then both the magnetic and electric loading may be pushed a little further than above stated.

In (26) no other dimensions than diameter and length of armature core enter and it may thus appear that as long as the value of D^2L remains unaltered, neither the ratio of length to diameter nor the number of poles matter. This is however not the case. Although the number of poles does not directly enter into (25) it does so indirectly since it influences the limits for B and Δ in (27). Thus in a two pole machine having an armature of small diameter and great length B cannot be so large as in a six pole machine of larger diameter and smaller length, because there is not sufficient sectional area in the core to carry all the flux which corresponds to a large induction in the air gap. Also the current density per linear cm. of circumference must be smaller in the two pole machine, not only because there can be no internal ventilation of the armature core, but also, as will be seen from (22), because the cross turns and consequent distortion of the field would become excessive with an amount of electric loading which in a six pole machine would be quite admissible. Hence an indiscriminating application of the output coefficient should be avoided. It is a useful figure when comparing different machines, but the comparison must be made on common sense lines.

Temperature rise. In the armature the causes of heating are hysteretic and eddy current losses and loss due to resistance. In the field there may be some eddy current loss in the pole shoes, but as previously pointed out, this can be reduced to a negligible amount by reducing the ratio between slot width and air gap or by using laminated pole shoes, and need therefore not be further considered. The other loss is due to the resistance of the exciting coils. The heat generated in armature and field must be carried away by the surrounding air or in machines with forced ventilation by a stream of air blown through the casing which encloses the machine. The latter method is often employed in railway motors because these must for obvious reasons be totally enclosed and then a separate fan may be used to blow air through them. In stationary machines the armature is sometimes provided with fan blades which promote ventilation. The most usual case is, however, the open machine without any special fan and then the armature itself

acts as a ventilator, thus keeping the surrounding air in motion and increasing its ability to carry off heat. The dissipation takes place by radiation, conduction and convection, all from the external surface, but the heat which is generated internally must flow through the body to its external surface before any cooling action can take place. The internal flow again depends on the heat conductivity. This is much greater in the iron core of the armature than in the winding of a coil. In the core itself there is a great difference of conductivity according to the direction of flow. In the direction of the plates the conductivity is from 80 to 100 times greater than across them. In a coil wound with rectangular wire it is greater than in a coil wound with round wire, and so on. The outer covering of the finished coil has also some influence. The conditions of heat dissipation are so complicated that no exact predetermination of temperature rise on purely theoretical grounds is possible and we must be satisfied with some approximate rules taken from experience. A convenient basis for any formula giving the temperature rise is the conception of the "specific cooling surface," that is, the ratio between the surface in sq. cm. and the heat energy per second expressed in watts which must be dissipated from that surface. The temperature rise we define as the difference of temperature between the interior of the body (not its surface temperature) and the temperature of the ambient, that is, air in the case of a dynamo. The temperature gradient from the interior to the surface can be estimated from experimental data, but as this book is not intended for the specialist who has to design machinery down to the minutest detail it will be sufficient to treat this subject in an approximate way*.

First we may mention some rough workshop rules. In one the electric loading of field coils is limited to 1 ampere-turn for each sq. mm. of total cross sectional area of the coil. This rule is obviously quite unscientific, since it takes no account of the winding depth. Another rule is limited to very thin coils of a winding depth not much exceeding one inch. Reckoning the whole of the surface, that is, not only that exposed to air, but also that lying against the iron, the rule is that the temperature rise will be limited to 40° C. if the specific cooling surface is 18 sq. cm. per watt to be

* For an exhaustive discussion of the problem the reader is referred to specialists' books on dynamos, such as Professor Miles Walker's excellent treatise "Specification and design of dynamo electric machinery."

dissipated. The defect of this rule is that it takes no account of any fanning action due to the armature. As regards armatures there is the rule of thumb according to which the specific cooling surface should be 7 sq. cm. per watt for moderate speeds and 3.3 for very high speeds as in turbo-dynamos. All these are merely rough workshop rules. There are also more scientific methods in use and these are based upon an equation for temperature rise of the general form*

$$T = \frac{c}{\sigma(1 + \alpha v)}$$

where c is a constant depending on the kind of coil (armature or field), σ is the specific cooling surface expressed in sq. cm. per watt, v is the circumferential speed of the armature expressed in metres p.s. and α is a constant. If $\sigma = \frac{\text{external cylindrical surface}}{\text{watts to be dissipated}}$

we may take for two pole armatures without any internal ventilation the value of c as about 600 and that of α as 0.1. With multipolar armatures having a fairly open interior and ventilating ducts c may be as low as 300, but as a fair average we may write for the temperature rise of the armature

$$T = \frac{400}{\sigma(1 + 0.1v)} \quad \dots \quad (28)$$

the specific cooling surface referring to the external cylindrical surface of the armature including the heads, but exclusive of internal ventilating spaces. A similar formula is used for the field coils, but as these are not in motion themselves, but only receive some of the air thrown out by the armature, the value of c is higher. If the cooling surface is reckoned as that of the external cylindrical part of the coils actually exposed to the air we may write

$$T = \frac{800}{\sigma(1 + 0.1v)} \quad \dots \quad (29)$$

* It should be noted that in this formula σ being the ratio of surface to power lost does itself depend to some extent on temperature rise since with increasing temperature the resistance also increases. For a coil in quiescent air we have $TS = cP$ and $P = R(1 + \alpha T) I^2$ where R is the resistance of the coil at 15°C. and α the temperature coefficient which may be taken as 4×10^{-3} . If α were zero the temperature rise T_0 would be smaller, corresponding to $T_0 S = RI^2$. If thus the current is the determining factor we have $T = \frac{1000 T_0}{1000 - 4T_0}$. If on the other hand the impressed voltage is the determining factor

$$P = \frac{E^2}{R(1 + \alpha T)} \quad \text{and} \quad T_0 = T + 0.4 \left(\frac{T}{10} \right)^2$$

It will thus be seen that the final temperature is greater in a series coil and smaller in a shunt coil than that which would be reached if the temperature coefficient were zero. Hence the danger of overheating is greater in a coil in series than in shunt connection.

Heat is generated on the surface of the commutator partly by brush friction and partly by contact resistance. Reckoning the cylindrical wearing surface and the power wasted from these two causes we have

$$T = \frac{120}{\sigma(1 + 0.1v)} \cdot \cdot \cdot \cdot \cdot \cdot (30)$$

v being in this case the linear speed of the wearing surface in m./s. These formulae refer to the ordinary type of open machine without any special provision for ventilation. If an air blast is directed against the cooling surface its efficacy is increased. Prof. Miles Walker gives the following formula for the temperature rise of coil over the surface of which a stream of air passes with a velocity of v metres per sec.

$$T = \frac{900}{\sigma(1 + 0.54v^2)} \cdot \cdot \cdot \cdot \cdot \cdot (31)$$

T being the difference between the temperature of the incoming air and that of the surface.

In a forced draught machine the ventilating air is supplied by an external fan and sent through the closed external casing of the machine. The speed at which it passes the different surfaces can be approximately calculated from the area of the passages and the volume of air passing per second. Then the temperature rise may be calculated from (31). To find the quantity of air required to carry away the heat corresponding to a given loss we reason as follows: The specific heat of air is 0.238 and since one cub. metre of air weighs about 1 kg. we find that one cub. metre of air heated by t° C. carries away $0.238t$ kilogram-calories or approximately $t \times 1000$ joules. One cub. metre supplied per second can therefore deal with a loss of 1 kw. for each degree difference of temperature between the incoming and outgoing air. Referring the temperature rise of the body to the mean temperature of the air during its flow past the body we can use formula (31) without correction, but if we refer it to the temperature of the incoming air we must lower T by $t/2$. Thus if we allow the air on its passage to be heated up by 20° we must provide $1000/20 = 50$ litres per second for each kw. that has to be dissipated and the temperature rise over the incoming air will be 10° greater than given by (31). If we allow the air to be heated up by 30° we may reduce the quantity supplied per second to 30 litres for each kw., but then we must add 15° to the value of T calculated from (31) with reference to the temperature of the

incoming air. These figures, namely 30 to 50 litres per second (60 to 100 cub. ft. per minute) for each kw. wasted in heat, may be taken as usual values for forced draught machines.

A simple empirical formula for the temperature rise of totally enclosed forced draught railway motors has been given by Mr Carter. In it not the surface, but the weight of the machine is introduced. This is obviously permissible, since the surface varies as the square and the weight as the third power of the linear dimensions. Hence the cooling surface of the machine taken as a whole must vary as the $\frac{2}{3}$ power of the weight. Let P be the loss in watts and W the weight in kg., then the temperature rise of the motor over the temperature of the air which is blown into the casing is by Carter's formula

$$T = \frac{4P}{\sqrt[3]{W^2}} \dots \dots \dots (32)$$

The temperature actually reached in any internal spot of a machine can be accurately measured by thermo couple (Vol. I, p.12), but as a general rule the measurement is made either by thermometer applied externally or by the resistance method. The temperature thus measured is called the "*observable temperature*"; and its permissible maximum as defined by the Eng. Standards Committee is for field coils 95° C. and for armatures and commutators 90° C. The ambient temperature is reckoned at 40° C., so that the limit of observable temperature rise is 55° for field coils and 50° for armatures and commutators. The temperature of armatures may be taken by thermometer, but can also be taken by a resistance method. The temperature of the field coils is always to be taken by resistance. Where the resistance is low, as in an armature or series field coil, it is advisable to measure it by potentiometer as explained in Vol. I, Chapter III. The resistance of shunt exciting coils can be measured with sufficient accuracy by the observation of voltage and current. An amperemeter is permanently inserted into the shunt exciting circuit and a voltmeter is placed across the terminals of the exciting circuit. Whilst the machine is at work, the readings of these instruments are recorded at stated intervals of, say, half an hour. The gradual rise of resistance as the coils heat up can then be followed without interfering with the working of the machine, and by plotting the corresponding values of temperature the final temperature rise can be estimated from the trend of the temperature curve. To take the temperature of the armature, whether by thermometer

or by resistance, it is, of course, necessary to stop the machine. It is generally found that after six hours run the temperature has become stationary even in large machines; in small machines the final temperature is reached somewhat sooner.

Commutation losses. These are of two kinds; one purely mechanical caused by the friction between brush and commutator and the other electrical caused by contact resistance. Both are influenced by the material, the speed, the current density, the pressure on the contact surface and also to a large extent by the care devoted to keep the commutator in good order and clean. If the brushes are badly set the surface of the commutator may become pitted and wear away quicker than the mica insulation which then projects slightly and prevents good contact. This wear is not a mechanical abrasion, but an electrical effect due to excessive local currents and consequent eating away of metal by minute arcs. Again if the commutator is not kept clean, carbon dust may accumulate on its surface and bridging over the segments may cause flashing all round, but with reasonable care such abnormal conditions can be avoided and then the losses are caused simply by voltage drop and friction. It will be shown in the next section that a certain voltage drop is conducive to good, that is sparkless, commutation and a large variety of brushes are made to suit different cases. It would be outside the scope of this book to enter into minute details of different makes of brushes; for these the reader must be referred to the publications of the manufacturers, such as the Morganite Company or the Le Carbone Company; here we can only summarise and give average figures. Copper, whether strip or gauze, is now hardly ever used for brushes. The material generally used is some form of carbon, more or less graphitic. The more graphitic the material the softer is the brush and the lower the contact resistance. Graphite has the further advantage that it forms a lubricant and gives the commutator a highly polished surface. These so-called soft brushes are used on machines of low voltage and large current. For high voltage machines a harder type of brush is used and where a very small voltage drop is desirable and sparking is excluded by the nature of the service (for instance in slip rings) so-called "copper-carbon brushes" may be used. In these brushes the loss of pressure by contact resistance at 10 amperes per cm.² is only of the order of half a volt, but they require very careful attendance

if pitting is to be avoided, so that most engineers prefer even for slip rings a purely graphitic brush although the voltage drop over both brushes is then of the order of 1.5 volt.

The voltage drop in all types of brushes does not follow Ohm's law, but is more of the nature of a true resistance drop augmented by a polarisation effect. Moreover the drop is not independent of the direction of the current. It is from 5 to 10 per cent. greater with the current flowing from metal to carbon than for a current flowing from carbon to metal. In a generator the positive brush has therefore a slightly greater drop than the negative*.

The drop is not influenced by the surface speed of the commutator and only slightly by the specific pressure. Up to about 70 gr. per cm.² it decreases sensibly with an increase of pressure, but beyond this point the decrease is not very great and since the specific pressure is generally from 100 to 150 gr. per cm.² (and about double these figures in rail and tram motors where on account of vibration some extra pressure is required to keep the brushes on) the drop is only very slightly influenced by altering the contact pressure. It is influenced by the current density, but not in proportion. Thus in a very soft brush the contact drop over both brushes is 1.3 volt with a current density of 5 amperes per cm.² If the current density is raised to 12, the drop is only increased to 1.6 volt.

The coefficient of friction increases slightly with the specific pressure and decreases slightly with an increase of speed. Here also the deviations are not very great so that we may take an average figure for calculating the loss occasioned by friction. The following table contains for three representative types of brushes the maximum current density which may be allowed and the corresponding drop; also the specific resistance of the brush material and the coefficient of friction.

Type of brush	Amperes per cm. ²	Microhms per cm. ²	Coefficient of friction	Drop in volts over both brushes
Soft	12	1800	0.20	1.6
Medium	10	2400	0.24	1.9
Hard	6	3800	0.28	2.2

Let v be the surface speed of the commutator in m. per second and assume in all cases the spring of the brush tightened so as to

* For an experimental investigation of brush drop see Arnold, *Die Gleichstrommaschine*, vol. 1, p. 362.

produce a specific pressure of 150 grams per cm.², then by using the figures of this table and making a simple calculation we find that the total commutating loss for the maximum permissible current density is given by the following formulae, I being the total current passing through the armature:

Power lost in commutation with

$$\left. \begin{array}{ll} \text{Soft brushes} & I (1.6 + 0.048v) \text{ watts} \\ \text{Medium brushes} & I (1.9 + 0.070v) \text{ watts} \\ \text{Hard brushes} & I (2.2 + 0.14v) \text{ watts} \end{array} \right\} \quad . \quad . \quad (33)$$

These formulae take no account of the loss due to the resistance of the brush material itself in a longitudinal direction, nor of the resistance of the connections from brush to terminals, the so-called "pig tails." The longitudinal resistance of the brush itself may be made much smaller than its transverse resistance by building the brush up of flaky graphite giving good conductivity axially and bad conductivity transversely, which latter is an advantage for commutation. Brushes are also made with material softer at the heel than at the toe, thus providing a sufficiently high contact resistance at the place where it is most beneficial without unduly increasing the average drop.

The current passing through the brush must be taken through the brush holder to the terminal of the machine. In some cases the holder is a lever pivoted on the brush spindle, in others it is a box in which the brush can slide. Neither the hinge contact where a brush spindle is used nor the rubbing contact between brush and its holder is good enough for passing the current. Hence the necessity for pig tails which can be firmly attached to the brush spindle or box holder by screw terminals or any equivalent device. The drop due to longitudinal resistance of brush and connection to machine terminals is additional to that given in the above table, but as it can be made exceedingly small it is generally neglected.

Commutation*. The current in any armature conductor must change sign whilst passing from one side of the brush to the other. Whatever the type of winding, the reversal of the current from its value $+i$ to its value $-i$ must take place in every conductor within a distance equal to the width of the brush reduced to the air gap radius. This process of current reversal is called commutation; it is

* For a comprehensive treatment of this subject see B. G. Lamme, "Physical Limitations in D.C. Commutating Machinery," *J.I.E.E.*, 1915, p. 1559.

diagrammatically shown in Fig. 20. Since the conductor has inductance, the reversal can only take place in virtue of the application of an e.m.f. and if this is of such a value that the current is exactly reversed during the time that the segment passes under the brush, then there will be no e.m.f. between the toe of the brush and the segment at the moment when the latter comes out of contact with it and we have perfect commutation. This is shown in the middle sketch *A*. Here the current in the conductor is represented as a function of the distance travelled by the conductor, but as the speed is constant, the diagram can also be regarded as a current-time diagram. The sloping line joining $+i$ and $-i$ may be considered a half wave of an alternating current of periodic time T and frequency f . As an approximation we may consider the sloping line

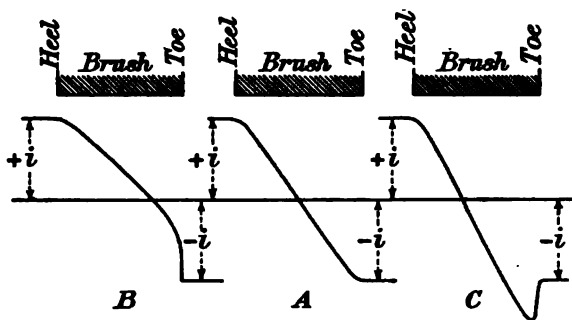


FIG. 20.

sinusoidal and then the e.m.f. required to produce reversal and no more in the time $T/2$ can be calculated. For this purpose we must know the inductance of the coil short circuited by the brush.

Now suppose that the e.m.f. is not sufficient to change the current in time $T/2$ from $+i$ to the full value $-i$, but to some lesser value, as represented by *B*. At the moment when the segment leaves the toe of the brush the current has not yet attained its correct value, but a moment later it must assume the full value $-i$. The current must change more or less suddenly and the consequence is a momentary rise of e.m.f. between brush and segment causing a spark. Such a condition is called "under-commutation." On the other hand it is conceivable that the e.m.f. acting on the coil under short circuit is too large and produces a negative value of i greater than that which must obtain after the segment has left the brush. This process, illustrated at *C*, is called "over-commutation."

Fig. 20 refers to the working current in one armature conductor, but considering it as a space diagram it can also be used to represent the current density at the brush contact surface in so far as this depends on the working current. The current density at any point of the brush contact surface is obviously proportional to the slope of the line joining $+i$ with $-i$. Constant current density over the whole contact surface can only obtain if the joining line is a straight line. The current density is then the ratio of current per brush in amperes divided by the contact area in sq. cm. If the joining line is sinusoidal the current density will be a maximum in the middle of the brush, namely $\pi/2$ of its average value. With under-commutation the maximum will be considerably in excess of the mean and shift to the toe of the brush; with over-commutation the maximum shifts to the heel and there may at the toe be a negative value of current density, that is to say, the direction of flow may be reversed. The figures of current density given on page 62 are average values and may be used for the determination of brush contact area, but only on the supposition that the commutation is fairly good, that is to say that there is neither under- nor over-commutation, and that there is no appreciable short circuit current between neighbouring segments which would cause a local change in current density. Such a short circuit current may be generated by the conductors cutting through the field which the armature produces in the brush axis. If we analyse this action we find that this short circuit current decreases the current density at the heel and increases it at the toe of the brush. The lower the armature resistance the greater is the excess of current density at the toe. Under normal conditions the exciting force of the armature current is small and the magnetic reluctance in the brush axis is great, hence the field produced by the normal armature current is not very large and the e.m.f. acting in the short circuited coil is small. This may nevertheless cause a sensible short circuit current if the armature resistance is very low. To seek a remedy in the increase of armature resistance would be wasteful, but the same effect may be obtained at very little loss of power by increasing the resistance of the connectors between the winding and the commutator segments, an expedient sometimes employed to improve the working of the brushes.

A local increase of current density under the brush, although it need not necessarily produce violent sparking, may nevertheless be harmful. With an excessive current density from copper to carbon

some action comparable to a kind of dry electrolysis may be set up transferring metal from the segment to the brush and gradually eating away the surface of the metal and producing the condition technically termed, "high mica." Some engineers hold that this condition is due to the copper wearing away faster than the mica, but it is difficult to understand how a true grinding action can result in any other but a smooth surface. If then we find that the commutator develops high mica, which means "low copper," we must conclude that this is due not to mechanical, but to a kind of electrical abrasion. The evil is cumulative, for once the copper gets low the brush can no longer make intimate contact with the metal and minute arcs must be set up, thus further roughening and pitting the surface. The mica is of course ground away by the brush so that the arcs remain very minute, but nevertheless the life of the commutator is thereby shortened. Hence the condition of good commutation is not only a commutating e.m.f. sufficient to balance the e.m.f. of self induction (the so-called reactance voltage), but also the avoidance of large short circuit currents in the coils under the brush.

The commutation e.m.f. may be obtained by induction from the fringe of the main pole, or by the employment of a separate so-called commutating field produced by series wound commutating poles (also called interpoles because they are placed between the main poles), or by the effect of the changing resistance between segment and brush due to the change in contact area as the segment passes from one side of the brush to the other. The latter effect is always present and therefore supplementary to either "fringe" or "interpole" commutation. Fig. 21 illustrates fringe commutation in a series machine. For simplicity of drawing, the armature is supposed to be straightened out. The motion in a generator is from right to left and if we retain the direction of the currents as indicated by the dots and crosses, the motion in a motor is from left to right. The little circles represent the position of the conductors in the slots. There are four conductors in the slot and those which are not marked with dots or crosses are supposed to undergo commutation. By shifting the brush axis forward in the direction in which the machine is being driven when working as a generator, the conductors connected to the brush come under the influence of the fringe. By applying the right hand rule it will be found that the e.m.f. induced in these conductors is upwards, that is, in the same

direction as the current has after reversal. Thus the coil is prepared by the fringe for the passage of the reversed current after it leaves the brush and there is no spark. By shifting the brush more or less it is possible to find the correct place for perfect commutation and having once found the amount of lead for one load it is hardly necessary to alter it for a different load since in a series machine the main field and therefore also its fringe increases with the current, whilst the e.m.f. required to overcome the inductance of the short circuited coil also increases with the current. The armature field is more or less suppressed by the fringe so that no harmful short circuit currents can develop. The balance between commutating e.m.f. and reactance voltage is thus automatically maintained within fairly wide limits of load. If the machine works as a motor

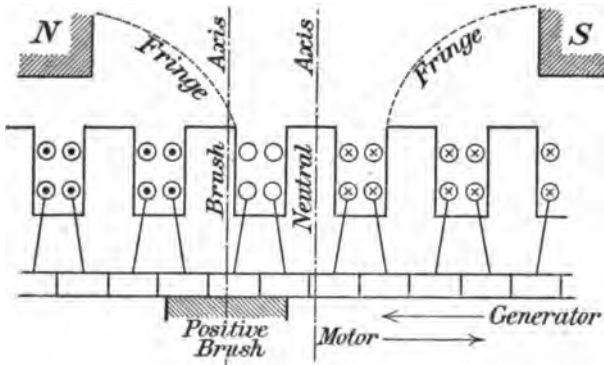


FIG. 21.

the direction of motion is reversed and if the brush is given a backward lead it will occupy the same position as shown in Fig. 21. In the conductor undergoing commutation there will be induced a downward e.m.f. that is again in the same direction as the current has after the conductor leaves the brush. Also in this case we can find by trial how to set the brushes for sparkless commutation. This setting will be correct for all loads within reasonable limits.

Whether the machine works as a generator or motor the commutation is produced by a fringe of north polarity. We can obviously obtain the same effect by leaving the brush in the neutral axis and placing there a small north pole as an equivalent to the fringe which in this case we do not use. If this auxiliary pole is produced by a series winding we have the same balance between reactance and commutating voltage. This is commutation by interpole.

Fringe commutation is not very satisfactory in shunt machines, because the commutating field decreases with the load due to distortion, whilst the reactance voltage increases with the load. Perfect balance can therefore only be obtained at one particular load and this means that to get satisfactory commutation it is necessary to adjust the brush position when the load varies. For this purpose shunt machines are provided with brush shifting gear, generally by means of worm and wheel so that the whole set of brushes can be shifted simultaneously. Where interpoles are used this is, of course, not necessary and there is the further advantage that the machine may be used as generator or motor without any attention to the setting of the brushes.

As already mentioned, brush contact resistance always helps commutation. Also the resistance of the connection between winding and segments is instrumental in making the current die out more quickly and in reducing the short circuit current due to the armature field. For this reason such resistances are sometimes employed to reduce sparking. The loss of power due to such connector resistance is simply the product of current and voltage drop over them and need therefore not be excessive, but even if we were prepared to sacrifice a sensible amount of power the connector resistance, although it may bring the $+i$ current quickly to zero, cannot start the $-i$ current; this can only be done by the e.m.f. due to a commutating field, or a changing resistance such as that between segment and brush. This process is illustrated in Fig. 22. For ease of illustration the armature winding is shown as a ring winding, but the principle is the same for a drum winding. The four coils a , b , c , d are shown in five positions. The current is supposed to leave at the brush so that $+i$ flows from right to left through the coils and $-i$ from left to right. In I a and b carry the full current $+i$. The next coil c carries certainly much less than b because the segment cb , making full contact with the brush, carries off a large amount of current. On the left d carries the full $-i$ because one of its segments has left the brush, the motion being from right to left. In II the current from a can escape to the brush, thus reducing the current in b . At the same time the contact area of the segment connected to d has been reduced, and due to the increase of resistance produced by this reduction of area some current is forced into c and b , thus opposing the current originally flowing through b . When position III is reached the whole current

flows through *c* and some of it is forced into *b*. The reversal in *b* begins. By the time position IV is reached the resistance of the segment *cb* is increased and only a part of the current in *c* can be discharged into the brush, the remainder flowing through *b* and the segment *ba* to the brush, since the resistance of this segment has become a minimum. The resistance of *cb* to brush increases still more and reaches infinity in position V by which time *b* carries the full current in the right direction. The reversal of current in *b* is therefore due to the gradual increase of contact resistance of

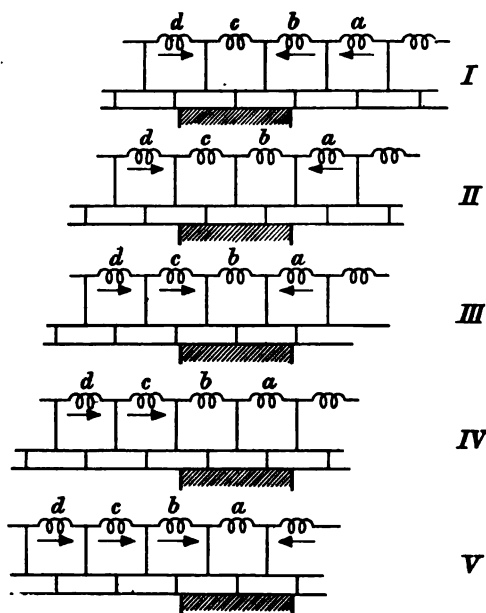


FIG. 22.

the segment *cb*, and the gradual decrease of the contact resistance of segment *ba*. This process takes place in every coil and provided the contact resistance is sufficiently high, the resulting e.m.f. acting in the direction of the reversed current may be sufficient to balance the reactance voltage. This is the reason why hard brushes having a greater contact resistance and consequently a greater voltage drop give better commutation than soft brushes having a small voltage drop. It should further be noted that with resistance commutation there is also some approach to automatic balance between reactance and commutating voltage. The brush drop and

therefore the commutating e.m.f. increases with the load and so does the reactance voltage. The balance is not so perfect as with interpole commutation because the voltage drop is not strictly proportional to the current, whilst the reactance voltage is, but on the other hand there is no possibility of over-commutation and provided we use brushes hard enough for the full load, the fact that they are unnecessarily hard for light load can do no harm.

Reactance Voltage. The reactance voltage can be calculated approximately from the known dimensions of the slot and the particulars of the armature winding by the application of the laws of the magnetic circuit. With diametral winding the current in the top and bottom conductors of a slot commute simultaneously. The change of linkage in the slot is twice that obtaining with chord winding, where the commutation of the top conductors occurs before or after that of the bottom conductors. With commutation by brush resistance only, chord winding is on this account advantageous, but the advantage vanishes if the commutation is by fringe or interpole, for in these cases the commutating field only influences one half of the coils at a time and the commutating e.m.f. is only half of what it would be with diametral winding. On the other hand the reactance voltage is a little more than half of what it would be with diametral winding because the self-induction of the armature head is the same whether the coil spans a full polar pitch or less. The problem of commutation has been treated by many authors with high mathematical skill, but the practical value of these theories is questionable, not only because any rigorous treatment leads to very complicated equations, but also because there are some uncertain factors, such as quality of brush, temperature and condition of the commutator surface, which may considerably modify any theoretical assumptions. For this reason a more simple, even if only approximate, treatment is generally preferred. Let L be the inductance of all the turns included between two adjacent commutator segments and i the current in each turn, then on the assumption of the current changing according to a sine function the crest value of the reactance voltage (RV) is

$$RV = \omega Li$$

where $\omega = \frac{2\pi}{T}$ and $\frac{T}{2}$ is the time available for commutation.

Experience shows that if RV is under 3 volts, resistance commutation is satisfactory. If it is more, then resistance commutation must be assisted by fringe or interpole commutation. To find L we map out the field produced by 1 ampere both in the slot and the head of the armature. By definition of inductance L is the linkage produced by one ampere, or the product of the self-induced flux and the number of turns. Since the self-induced flux is proportional to the number of turns the inductance is proportional to the square of the number of turns, and any expression for reactance voltage must have the form

$$RV = W^2 \cdot$$

$$RV \equiv W^2 i\omega$$

where W is the number of wires in the coil.

The problem of finding the reactance voltage is thus reduced to the determination of the self-induced field linked with all the turns in a coil and the time of commutation. We shall treat the problem first on the assumption that there are only two coil sides each of W wires in a slot and then see what reduction in the RV results from splitting up the W wires into m groups of w wires, whilst at the same time the number of commutator segments is made m times as great.

In either case the slot leakage is produced by $2W_i$ ampere-wires and the head leakage by W_i ampere-wires. It is convenient to determine the leakage flux per cm. of slot or cm. of head connection.

Fig. 23 shows a slot and roughly the different leakage fields. Φ_1 is shown as lying entirely in air, which would be the case where no interpole is used. If an interpole is used Φ_1 is the air gap field between top of teeth and interpole and can be calculated by the well-known rule for a flux across an air gap.

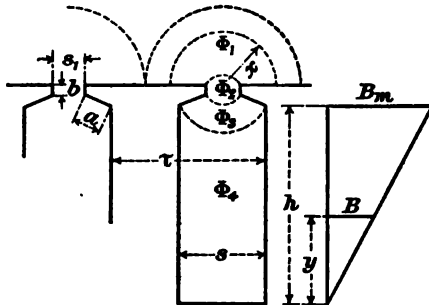


FIG. 23.

Φ_2 is the field between the lips of the slot opening, and to allow for the bulging of the lines an addition of 50 per cent. may be made to the thickness of the lip b . Φ_3 is the shelf leakage and this may be calculated on the same basis as the external air leakage, namely on the assumption that the lines are semicircles. With open slots there is no shelf leakage and the lip leakage is that between the

top parts of the slot where there are no wires. The three fields here considered are all interlinked with the whole of the wires in the slot, but this is not the case with the leakage field Φ_4 passing across the sides of the slot. The lowest wire is interlinked with the whole of this field, but the highest wire is not interlinked at all. To find the effect of Φ_4 on the two coil sides as a whole we must therefore take account of the fact that the induction across the slot increases linearly from the bottom upwards whilst the number of wires coming under the influence of the flux decreases from the bottom upwards. Using the notation as given in the figure we determine the different leakage fields as follows:

Put $c = 0.4\pi 2Wi$

$$\Phi_1 = \int_{\frac{s_1}{2}}^{\frac{\tau}{2}} \frac{c}{\pi x} dx = \frac{c}{\pi} \log_e \frac{\tau}{s_1} = c \frac{2.3}{\pi} \log_{10} \frac{\tau}{s_1}$$

$$\Phi_2 = c \frac{1.5b}{s_1}$$

$$\Phi_3 = c \frac{2.3}{\pi} \log_{10} \frac{s}{s_1}$$

$$B_m = \frac{c}{s} \quad B = B_m \frac{y}{h}$$

Flux interlinked with elementary layer at distance y from bottom is

$$\Phi = \frac{B_m + B}{2} (h - y) = \frac{B_m}{2h} (h^2 - y^2)$$

$$de = \omega \Phi \frac{2W}{h} dy = \omega \frac{B_m}{2h^2} 2W (h^2 dy - y^2 dy)$$

$$e = \int_0^h de = \omega \frac{B_m}{2h^2} 2W \left(h^3 - \frac{h^3}{3} \right)$$

$$e = \omega \frac{B_m}{3} h 2W = \omega c \frac{h}{3s} 2W$$

We also have $e = \omega \Phi_4 2W$ and therefore $\Phi_4 = c \frac{h}{3s}$.

The total slot leakage per cm. is therefore in c.g.s. lines

$$\Phi_s = c \left(\frac{2.3}{\pi} \lg \frac{\tau}{s_1} + \frac{1.5b}{s_1} + \frac{2.3}{\pi} \lg \frac{s}{s_1} + \frac{1}{3} \frac{h}{s} \right) \quad (34)$$

where $c = 0.4\pi 2Wi$.

To find the head leakage Φ_h we may as a rough approximation suppose that the two diamond heads are replaced by two semi-

circles with a diameter equal to the pole pitch τ_p . The two heads may now be considered as a toroid and applying to this the formula (121) of Vol. I, we find with a slight transformation the inductance in henrys

$$\pi\tau_p W^2 \left(0.46 \lg \frac{\tau_p}{d} + 0.064 \right) 10^{-8}$$

where d is the diameter of the bundle of W wires passing across the end face of the armature. To get the inductance per cm. of head connection we divide by $\pi\tau_p$ and the corresponding value of the flux is

$$\Phi_h = Wi \left(0.46 \lg \frac{\tau_p}{d} + 0.064 \right) \dots \dots (35)$$

The constant term 0.064 is due to the field within the bundle of wires; it is unimportant as compared to the term containing pole pitch and thickness of bundle of wires. The thickness is at most half the slot pitch and may be as little as a quarter the slot pitch. In modern machines there are at least 10 slots per pole so that the fraction $\frac{\tau_p}{d}$ will certainly not be less than 20 and may be as much

as 60. The corresponding logs are 1.3 and 1.78. We thus find that the numerical value of the bracket lies between 0.6 and 0.82. A well known rule due to Mr Hobart is that 0.8 lines should be taken as the self-induced flux per ampere-wire per cm. in the head connection. This is generally looked upon as a mere empirical rule, but the above calculation shows that there is a good scientific reason for Mr Hobart's rule. He has also given the rule that for the slot leakage one might take four lines per ampere-wire per cm. and on testing specific cases by formula (34) it is generally found that also this rule comes fairly close to the value calculated. For this reason and also on account of the great simplicity of Hobart's rules these are in extended use. Let l_s and l_h represent length of slot and head connection respectively, then the flux interlinked with a coil of W turns is

$$\Phi = 2l_s\Phi_s + 2l_h\Phi_h \text{ with full pitch winding,}$$

$$\Phi = l_s\Phi_s + 2l_h\Phi_h \text{ with chord winding.}$$

In these equations Φ_s and Φ_h are to be determined by (34) and (35). If Hobart's rule is used, we have

$$\Phi = Wi (16l_s + 1.6l_h) \text{ with full pitch winding} \dots (36)$$

$$\Phi = Wi (8l_s + 1.6l_h) \text{ with chord winding} \dots (37)$$

The reactance voltage is in all cases

$$RV = \omega\Phi W 10^{-8} \dots \dots (38)$$

In order to calculate it we must know the frequency of commutation. The case of one top and one bottom coil side per slot hitherto considered is represented in Fig. 24. The circles represent the

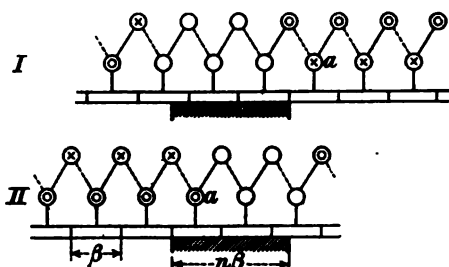


FIG. 24.

a pole pitch away. The head connections in front are shown in full lines, those at the back in dotted lines. The position I refers to the moment when the group of conductors marked *a* begins to commutate, whilst in II the commutation is just completed. The distance travelled by the armature is equal to the width of the brush reduced to the air gap radius.

Let the commutator contain *S* segments of width β and let the brush cover *n* segments, then at the speed of *u* r.p.s. the linear speed is $Su\beta = \frac{2n\beta}{T}$ or $Su = 2nf$ where *f* is the frequency of an alternating current containing the half wave from + *i* to - *i*. We have also $Su\pi = 2\pi fn$ and therefore

$$\omega = \frac{\pi u S}{n} \quad \dots \dots \dots (39)$$

This expression is only valid if there are as many segments as slots; if, however, we put *m* coil sides into the top layer and *m* into the bottom layer of each slot, then we shall want *m* times as many segments as there are slots and the value of ω will be altered. In Fig. 25 commutation begins in the group of coils *a*, *b*, *c* in the position I and is finished in position II. Taking the three coils as one group it will be seen that the slot has travelled over a distance $\beta(2 + n)$ during the process of commutation, or generally with *m* coil sides in each layer and *m* segments per slot the distance travelled is

$$v \frac{T}{2} = \beta(m + n - 1)$$

and

$$\omega = \frac{\pi u S}{m + n - 1} \quad \dots \dots \dots (40)$$

S having in this case m times the previous value; but with the same width of brush n has also m times the previous value, so that by subdividing the W wires into m groups of w wires we reduce ω and with it the reactance voltage between $m + 1$ segments. If this voltage were evenly divided over the m intervening segments we would have the reactance voltage between segments

$$RV = \frac{1}{m} \frac{\pi u S}{m + n - 1} \Phi m w 10^{-8}$$

$$RV = \frac{\pi u S}{m + n - 1} \Phi w 10^{-8} \quad \dots \quad (41)$$

An exactly even division of the reactance voltage of the mw wires as a whole between the m intervening commutator segments cannot be expected since the mutual induction of conductors in the same

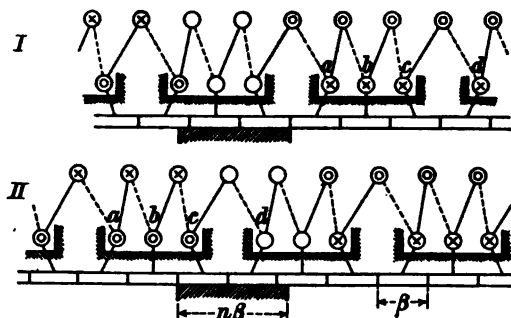


FIG. 25.

slot facilitates a displacement of the whole current volume towards the conductors which commute last. In consequence the RV between segments a, b will be a little less than between b, c , and this again a little less than that between c, d . If there is any tendency to sparking this will first show itself on segments c, d . This is a common experience in the working of dynamos if commutation is by brush resistance. With interpole commutation each wire is forced to commute at the right moment and there can be no piling up of reactance voltage towards the last wire in a slot. The same applies to fringe commutation.

I am indebted to the General Electric Company of Witton, Birmingham for the following formula for reactance voltage and for their permission to publish it. In this formula no distinction is made as to the number of coil sides housed in a slot, but as was shown such distinction is not required in commutation by interpole

and as this is generally used in modern machines the formula may be considered as generally applicable,

$$RV = 8 \left(\mu_1 + \mu_2 + 0.2 \mu_1 \frac{l - l_0}{l_0} \right) \Delta n v l 10^{-6} \quad (42)$$

Δ = linear current density per cm. air gap circumference.

v = circumferential speed of armature in metres per second.

l = gross length of armature core in cm.

l_0 = net iron length of armature core in cm.

w = number of wires or bars in a coil side.

μ_1 and μ_2 are coefficients to be taken from the curves of Fig. 26

where abscissae $x = n + 1 - \frac{a}{p}$.

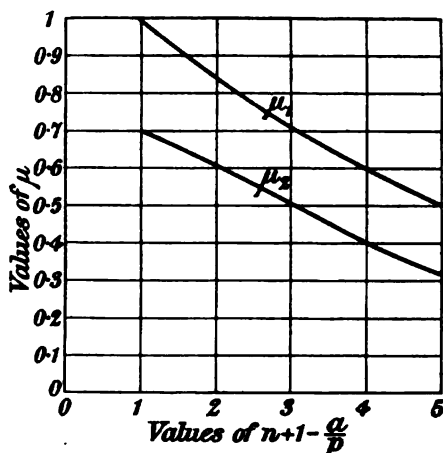


FIG. 26.

n = number of segments covered by the brush.

a = number of pairs of parallel circuits.

p = number of pairs of poles.

Exciting Force required for Interpoles. In order that the reactance voltage may be balanced by the e.m.f. induced by the interpoles the induction B produced by it must satisfy the equation

$$RV = Bl_0 2cv 10^{-8}$$

It has been shown in connection with Fig. 25 that the space within which the commutation takes place is $m\beta + \beta(n - 1)$.

Since $m\beta$ is the slot pitch and n is generally larger than 1 the width of the commutating field will generally be a little greater than the slot pitch and as this includes the fringes we may take the width of the interpole as about equal to a slot pitch. Since the fringe on either side is comparatively wide we may take the shape of this field to be approximately sinusoidal so that the total flux carried by it is

$$\frac{2}{\pi} l_p B\beta (m + n - 1) = \Phi$$

and combining this with the above expression for RV we get

$$RV = \omega w \Phi 10^{-8}$$

which simply shows that the flux which the interpole must inject must be equal to the self-induced flux, a conclusion which might have been deduced from first principles without making any calculation. The exciting force required for the interpole may then be determined in the usual way from the air gap, pole and tooth surface. This is, however, not the whole of the exciting force required. It has been shown in the section on armature reaction that the armature current produces in the brush axis an exciting force of $\frac{ZI}{4pa} = \Delta\tau$. This exciting force must also be balanced.

Hence the exciting force applied to the interpole must be correspondingly increased, that is to say, $\frac{ZI}{4ap}$ ampere-turns must be added to the ampere-turns required to produce the induction B across one air gap. In practice a somewhat larger exciting force is provided so as to allow for the margin always advisable in a calculation which can only be approximate; and the series coil on the interpole is shunted by a resistance which is experimentally adjusted to the correct amount.

Where series or series-parallel winding is used on the armature it may happen that the total number of coil sides is not an exact multiple of the number of slots. In such a case one slot may contain less active wires than the rest and then the space is filled with so-called dummy bars. This particular slot has the same magnetic reluctance as all the other slots, but a smaller self-induced field because the total volume of current in it is smaller. If the commutating field provided has the correct value for all the other slots we shall have over-commutation in the wires of this particular slot and probably sparking. The evil may be cured by decreasing

the magnetic reluctance of the particular slot so that round it there shall be the same self-induced field as round all the others. This may be done by replacing the copper dummies by iron strips.

Advantages of Interpole Commutation. If the polar fringe of the main field is used to produce the commutating e.m.f. the main flux is not fully utilised for the production of the working e.m.f. and in addition to this reduction we have a reduction in total exciting force due to the back turns corresponding to the angular displacement of the brushes. Moreover the e.m.f. injected into any one armature conductor during the time it forms part of a short-circuited coil is not constant, but increases more or less in conformity with the slope of the fringe from heel to toe of the brush. Hence the current density under the brush even with sparkless commutation increases towards the toe and to minimise this effect, which may cause overheating of the brush and high mica, care should be taken to make the slope of the fringe not too steep. This may be done by chamfering the pole shoes as shown in Fig. 13. The air space is then greater at the edge of the pole shoe than farther towards the middle, thus causing a gradual increase of induction so that the difference in the strength of the commutating field within the space between heel and toe is lessened. Another method sometimes employed is to slightly skew the polar edges out of parallel with the shaft. If that is done the fringe at any point may be fairly steep, but as the conductor does not enter the whole of the fringe simultaneously the effect is the same as that of a chamfered pole shoe.

With interpole commutation none of these drawbacks obtain. The whole of the main field is utilised for the production of the working e.m.f., there are no back turns and the e.m.f. injected for the purpose of reversing the current is constant or nearly so over the distance between heel and toe of brush. Hence a more even distribution of current over the brush contact surface and less tendency to undue local heating and high mica. This means an increased output with a given size of machine and this advantage is so great that practically all modern machines of moderate and large size are provided with interpoles. By the use of interpoles an increased reactance voltage may be permitted. Theoretically there should be no limit to the reactance voltage since the commu-

tating field is produced by the same current which produces the armature field and the reactance voltage; practically there is however a limit, because the balance cannot be perfect. At high load the magnetic leakage from the interpole weakens its effect and there is therefore a danger to sparking if the commutating pole has been designed for perfect commutation at moderate load. If, on the other hand, the design is correct for high load involving consequently considerable leakage from the commutating pole, then at low load, when the leakage is small there will be a rather stronger commutating field than required and we shall have over-commutation. Thus a perfect balance at all loads cannot be obtained, but this is not necessary, since brush contact resistance can deal with any want

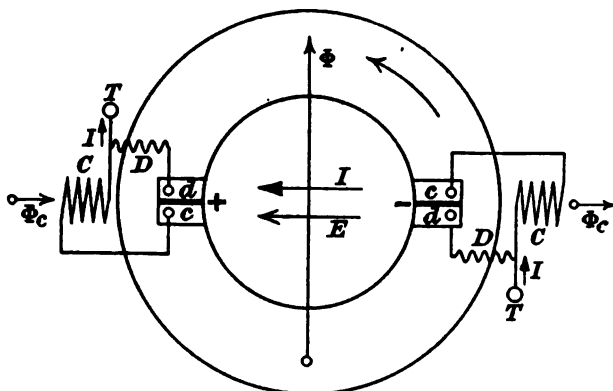


FIG. 27.

of balance, say up to 2 volts or even more, so that if the reactance voltage calculated by formula (38), (41) or (42) comes out at, say, 7 or 8 volts, this value is admissible. An excess or deficiency of as much as 25 per cent. in the injected voltage means about 2 volts want of balance and brush resistance can deal with this.

A very ingenious method for automatically varying the excitation of interpoles and thus compensating for the disturbing effect of magnetic leakage has been devised by Professor Miles Walker and practically demonstrated at the 1915 British Association meeting in Manchester*. The arrangement in a generator is diagrammatically shown in Fig. 27. The outer circle represents the armature winding and the inner the commutator. Each

* *The Electrician*, 1915, No. 1948, p. 872.

brush consists of two halves slightly insulated from each other, but both carrying working current. With the flux Φ as shown by the vertical arrow and counter-clockwise rotation the brush on the left is positive and the current passes through the armature from right to left as shown by the current arrow I . T , T are the terminals of the armature circuit. The field magnets are not shown. The working e.m.f. generated in the armature is represented by the arrow E . The coils marked C , C excite the interpoles and produce the commutating fields Φ_c . Only part of the total working current passes through the coils C , the rest being diverted through a resistance D in parallel. Such a resistance is used generally as already pointed out on page 77 for the purpose of experimentally adjusting the excitation of the interpole to the right amount.

It is convenient to make a slight digression at this place with regard to the action of such a shunt or diverter to the exciting coil when the load is liable to sudden and great fluctuation as may occur in the generator in a tramway power house or in a rolling mill motor. Since the coil on the interpole has not only resistance, but also a certain inductance, any sudden increase in current will be accompanied by a more than proportional increase in the e.m.f. over its terminals and consequently by a larger percentage of the total current being forced through the diverter. Hence the commutating flux will be smaller than it ought to be and the result will be under-commutation during the short period that the current is on the increase. For the same reason we shall have over-commutation if the load suddenly decreases. To avoid these evils it is necessary to give the diverter not only resistance, but also inductance, so that its time constant shall be the same as that of the interpole coil. Only under this condition can there at all times be true proportionality between the exciting force on the interpole and the total current to be commutated. The use of inductance in the diverter does, however, not overcome the difficulty due to the effect of saturation in the interpole. Since the interpole is placed into the neutral space its sides are fairly near to the edges of the main poles and there is necessarily a good deal of magnetic leakage. The commutating flux injected into the armature can therefore not increase in strict proportionality with the working current, unless some provision is made to automatically increase the exciting force on the interpole at a greater rate than corresponds to an increase in the strength

of the working current, and it is this automatic adjustment of inter-pole excitation which in the Miles Walker arrangement is brought about by paralleling the diverter to a split brush. The action will be understood from Fig. 28. On the left is shown the condition of under-commutation and on the right that of over-commutation, or rather the conditions which would obtain if the split brush device did not automatically correct them. On the top are shown the split brushes. That marked *c* is connected to the exciting coil and that marked *d* to the diverter. The lower part of the figure shows the current density under the brush, the ordinates of these curves being proportional to the slope of the line representing the current

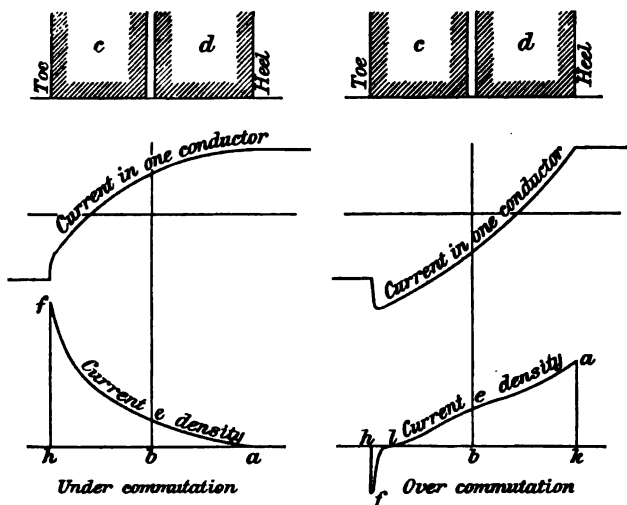


FIG. 28.

in one conductor whilst passing from heel to toe of brush. Since current density multiplied with contact area is total current, the area between the density curve and the horizontal represents the current carried by each of the two halves of the brush. Thus with under-commutation the area *aeb* represents the current passing through the diverter and the area *befh* that passing through the exciting coil. The latter area being considerably the greater it will be seen that far more than half the total current is used to magnetise the interpole. On the other hand if there is a tendency to over-commutation the current going through the diverter is proportional to the area *aebk*, and that magnetising the interpole is proportional to the area *elb* less the negative part *lh*. The exciting force on the

interpole is now considerably reduced. Herein lies the automatic adjustment; any tendency to under-commutation, which on account of magnetic leakage always arises at heavy load, is corrected by the increase of that part of the total current which is forced through the magnetising coil on the interpole and similarly at light load, when the leakage is less pronounced and the commutating field would with the ordinary arrangement of diverter be too strong, the excitation of the interpole is brought down to the correct value by the fact that now a greater part of the total current is forced through the diverter. The practical value of this automatic adjustment is that a machine may be overloaded by 100 per cent. or more without sparking, and where the overload is only intermittent and not of sufficient duration to increase the heating materially, the split brush and diverter arrangement makes it possible to use a smaller machine than would otherwise be necessary.

Compensating Windings. In 1884 Mr Menges suggested and in 1892 Profs. Ryan and Thomson practically applied the principle

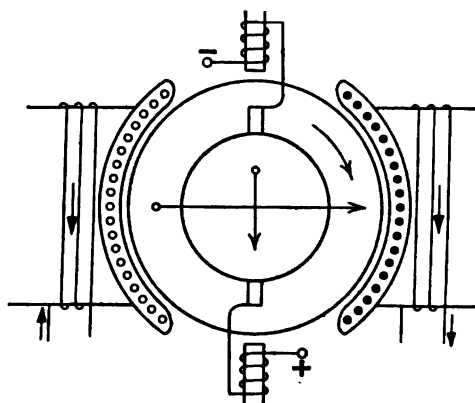


FIG. 29.

of placing a winding into slots in the pole shoes for the purpose of counterbalancing the magnetising force of the armature conductors. Such a winding is called a *compensating winding* or a *neutralising winding*. The latter term is more usual in America. The principle is shown in Fig. 29 as applied to a two pole machine for simplicity of illustration, though the application to

a multipolar machine is not only easier because of shorter end connections, but much more important since it is especially in large machines that armature cross turns become troublesome. The little circles represent the conductors of the compensating winding which are housed in slots. The black circles represent downward, the others upward, currents. If this fixed winding passing through the pole shoes is so arranged as to form an exact counterpart to the

armature winding there will be no cross magnetisation and no distortion of the main field; the compensation will be perfect. If the winding is restricted to the polar faces as shown in Fig. 29 but so arranged as to produce the same ampere-turns as the armature, the compensation will be very nearly perfect. In such a case interpoles may be added to provide a commutating field as shown in the figure. In the construction devised by Profs. Ryan and Thomson the stator was provided with teeth and slots all round, but in every neutral gap there was one larger tooth wound with a series coil which acted as interpole. It will be obvious that the excitation to be applied to the interpole need in this case only suffice to produce the induction required for commutation, but nothing to oppose the field corresponding to the armature ampere-turns since that is compensated by the distributed winding. As a result the effect of a variable leakage from the interpole is so far reduced that a correct balance between reactance voltage and commutating e.m.f. can be maintained over a wide range of load.

Messrs Parsons and Stoney have gone a step further in the development of the principle of compensating the armature cross

turns by arranging the compensating winding all round the armature and giving it considerably more exciting force than that exerted by the armature. This arrangement is shown in Fig. 30. The black dots represent conductors in which the current flows downwards, the clear dots conductors

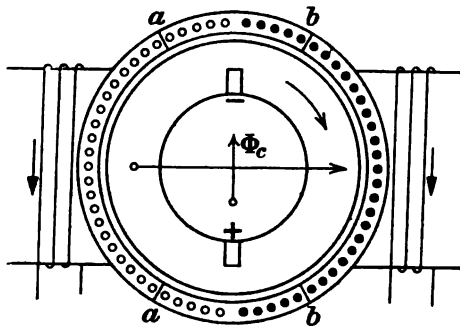


FIG. 30.

in which the current flows upwards. The current in the fixed winding has therefore the opposite direction to that flowing in the movable armature conductors, but the total ampere-turns in the compensating winding being considerably in excess of the armature ampere-turns the result is a field Φ_c in the brush axis acting in such a direction as to induce the required commutating e.m.f. The pole shoes *aa* and *bb* are of the usual extent, but the gap *ab* on either side is bridged by gun metal pieces which merely serve as a mechanical support to the conductors within the neutral space.

Since air is a non-magnetic material of constant permeability the commutating flux remains strictly proportional to the currents producing it and thus the balance between reactance voltage and commutating e.m.f. is right for one load is right for any other load. Fig. 31 shows the distribution of induction around the armature. The excess of compensating ampere-turns over armature cross turns is represented by the dotted line, the main field on open circuit by the thin full line and the resultant field by the thick line, the scale being assumed such that in air ampere-turns and induction may be represented by the same ordinate. Thus under the brush the induction is represented by the ordinate of the thick line and this is obviously independent of the induction under the pole. The

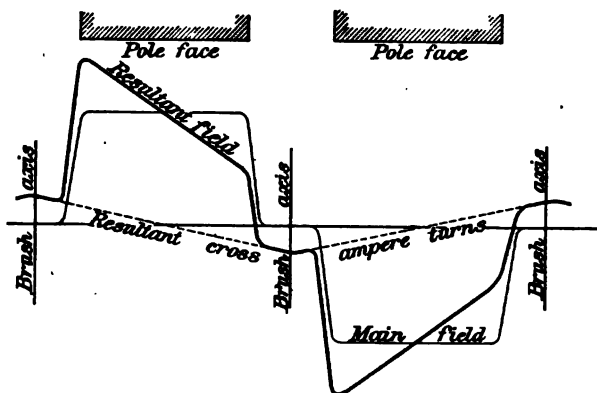


FIG. 31.

commutation will therefore be equally good whether the main field is strongly or weakly excited. Thanks to the proportionality between the induction in the brush axis and the working current the commutation is equally good at light load and at full load, and as a matter of practical experience machines fitted with this type of compensating winding can support momentary overloads of 100 or even 200 per cent. without sparking.

Some special Types of Dynamo. For certain services, such as train lighting or motor car lighting, it is desirable to have a machine the output of which is not overmuch affected by speed variation within reasonable limits. For searchlight work the machine although running at a given speed should produce approximately the same current whether the carbons are in contact or

opened out to produce the arc. This ensures a more steady light and moreover a light which is available at any instant by simply separating carbons, an important matter in naval or military service. A machine for this purpose must therefore give approximately the same current with a reasonable variation of resistance in the external circuit. Train lighting machines must further satisfy the requirement that the direction of the current shall not change with a reversal of rotation, because the machine being necessarily used in conjunction with a storage battery must always preserve the polarity of the battery. The conditions of approximately constant current at variable speed and constant polarity are fulfilled in the *Rosenberg Dynamo**. In this machine the field which provides the

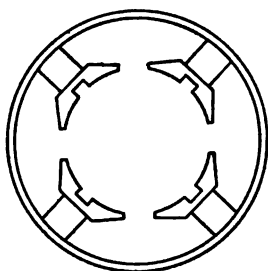


FIG. 32.

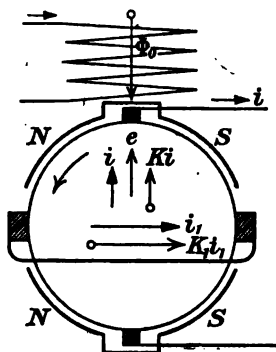


FIG. 33.

working e.m.f. and current is produced by the armature itself in the axis of a pair of short circuited brushes. The short circuit current is produced by a field in quadrature which is impressed by a system of physical magnets excited in the usual way. The field system of a four pole machine is shown in Fig. 32. The commutation of the working or external current takes place in the conductors, whilst these pass under the middle of the pole shoes and to avoid sparking the pole shoes are grooved out in those places. The short circuited brushes which have to carry a much larger current are so placed that the commutation of the short circuit current takes place in the conductors whilst they are passing through the neutral spaces. Fig. 33 illustrates the action of the machine, but for the sake of simplicity for a two pole arrangement. The coil at the top provides the flux through the physical magnet system. With

* *Elektrotechnische Zeitschrift*, 1905, p. 393.

counter-clockwise rotation and the axis of the short circuited brushes horizontal, a current i_1 passes through the armature producing a flux from left to right in proportion to i_1 . Let this flux be $K_1 i_1$. This flux produces an e.m.f. e directed upwards and therefore also a working current i directed upwards. If this current were acting alone it would produce an upward flux in proportion to i ; let this be Ki . This flux therefore opposes the impressed flux Φ_0 and the flux actually existing in the vertical axis and producing the short circuit current i_1 is

$$\Phi = \Phi_0 - Ki$$

If R is the resistance in the working circuit and ρ that in the short circuit, u the speed in revs./sec. and ϵ a constant, we have

$$i_1 = \frac{\epsilon u \Phi}{\rho} \text{ and } i = \frac{\epsilon u i_1 K_1}{R}$$

$$i = \frac{\epsilon^2 u^2}{\rho R} K_1 \Phi = \frac{\epsilon^2 u^2}{\rho R} K_1 (\Phi_0 - Ki)$$

Solving this equation we find

$$i = \frac{\Phi_0}{\frac{R\rho}{K_1 \epsilon^2 u^2} + K}$$

The condition of constant current will be more nearly approached the smaller the first term in the denominator is in comparison with the second term. This means that the armature resistance must be small, the value of ϵ large and the speed not too small. A large value of ϵ and a small value of ρ implies a more heavily wound armature than would be used in an ordinary machine. Such an armature will for any given current also have a larger self induced flux, so that the second term K will be comparatively large. If a properly designed machine of this type is used for a searchlight and the speed is reasonably high it is quite easy to keep the increase of current with the carbons in contact within about 6 per cent. of the normal. If the machine is to be used in conjunction with a battery for train lighting an aluminium-iron cell is inserted between machine and battery, the lamp circuit being connected not to the machine, but to the battery terminals. In this case the variable is the speed. From a certain speed upwards (about 20 m.p.h.) the battery is charged with an approximately constant current, at lower speed or when standing there is no charging current and the battery gives current to the lamps. In this case the aluminium-iron cell

prevents the current flowing through the dynamo circuit the reverse way. Such a cell permits current to flow through the electrolyte from iron to aluminium, but not in the reverse direction. Since the terminal voltage of the battery and therefore also the voltage in the circuit must vary somewhat the individual lamps are protected by ballast resistances (iron wire in an atmosphere of hydrogen, as explained in Vol. I, p. 4). The above equations for i show that the charging current may be adjusted to approximately a constant value by adjusting the strength of the impressed flux Φ_0 , that is to say, by adjusting the exciting current in the field system.

In dynamos for motor car lighting no provision need be made for reversal of rotation, but the machine should give approximately the same output at greatly varying speed. It is also desirable that machine and battery should be strong enough to start the engine. Of the many types available only a few can be mentioned*. The Lucas dynamo has three field windings as shown in

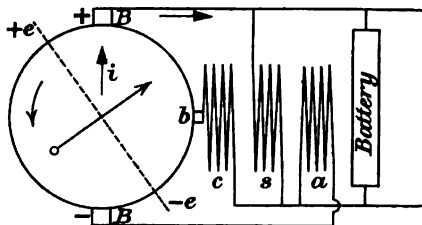


FIG. 34.

Fig. 34. The winding s is excited from the battery terminals, the winding a is in series between negative brush and negative battery terminal and the winding c is connected to the negative battery terminal and a third brush b . The shunt winding s acts always in the same direction. At low speed when the dynamo current is weak and the battery discharges a acts also in that direction and strengthens the field. At low speeds all the three windings act the same way, but as the armature winding is "strong" the current i produces an upward field, so that the flux actually passing through the armature is inclined as shown by the flux arrow. The e.m.f. diameter is therefore not vertical from the $-B$ brush to the $+B$ brush, but as shown by the inclined line $-e + e$. In this condition the brush b is still slightly positive

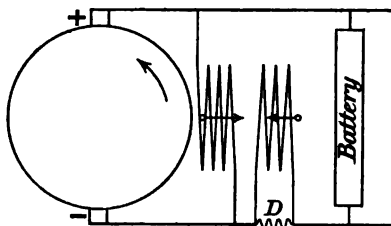


FIG. 35.

* For a fuller account see J. D. Morgan, *Journal I.E.E.*, 1912, p. 749, and 1913, p. 106.

as compared to $-B$ and the c coil acts in the same direction as the other coils. If the speed increases and the current becomes stronger

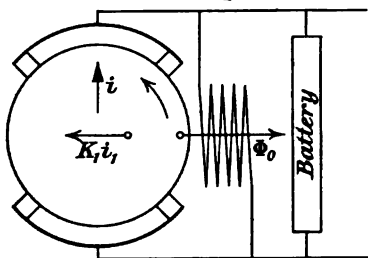


FIG. 36.

the e.m.f. axis is swivelled farther round making b negative as compared with $-B$ and the c winding begins to demagnetise, while at the same time robbing the a coil of some of its current and finally reversing it, thus weakening the field at high speed.

The Rushmore dynamo is a compound machine with the main acting in opposition to the shunt. As shown in Fig. 35, the current for the reversed series winding is taken from the terminals of a resistance D technically termed a "diverter." An increase of current results in a decrease of flux and thus a partial compensation for speed variation is obtained.

In the Brolt dynamo use is made of armature reaction in a similar way as in the Rosenberg dynamo, but instead of producing the

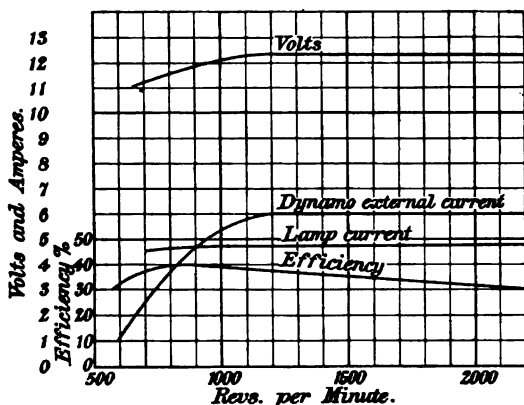


FIG. 37.

short circuit over a diameter it is produced over two segments of the armature as shown in Fig. 36. The effect is a self-induced flux which opposes the flux produced by the shunt winding on the field magnets. In Fig. 36 the same notation is used as in Fig. 33 and the theory of the machine is similar. Fig. 37 is a graph of its performance which has been published by Mr Morgan in his paper already cited. It will be noticed that the regulation shown is extremely good.

CHAPTER IV

WORKING CONDITIONS OF DYNAMOS

Graphic representation of working conditions—External characteristic of separately excited generator—External characteristic of series generator—External characteristic of a shunt generator—External characteristic of compound generator—Current-speed-torque characteristics—Speed of a shunt motor as influenced by the supply voltage—Motor starting and speed regulating switches—Energy loss in starting any type of motor—Series-parallel control—The Ward-Leonard system of speed regulation—Speed regulation by booster—Electric drive for reversible rolling mill—Coupled dynamos—Transmission of power—The Thury system—Reversal of polarity.

Graphic Representation of Working Conditions. Any two related quantities in the working of a dynamo may be represented by a graph. We may for instance plot exciting force in ampere-turns on the horizontal and the corresponding flux on the vertical; or if the speed is constant plot exciting current on the horizontal and induced e.m.f. on the vertical. We may also represent the relation between current and torque, or between external resistance and current, or current and speed in a motor by a curve. All these are characteristic curves of the machine, but the term characteristic is generally confined to the relation between ampere-turns and flux, or to that between ampere-turns and induced e.m.f., or exciting current and induced e.m.f. If the e.m.f. plotted is that appearing on the terminals we speak of an external characteristic. If the e.m.f. plotted is the induced e.m.f. (in the case of a motor the counter e.m.f.) we speak of an internal characteristic.

External Characteristic of Separately Excited Generator. In order to obtain this curve we proceed as follows. Let in Fig. 38 OB represent the magnetisation characteristic and with an appropriate change of scale also the internal e.m.f. characteristic at constant speed. Then with an excitation of $X = OA$ the open circuit voltage at the terminals will be OE . E is a point on the desired characteristic, namely that corresponding to zero current. If there were no armature reaction the terminal e.m.f. at any other value of the current could be found by deducting from OE the

ohmic loss ρI and the external characteristic would simply be a sloping straight line starting from E . But there is armature reaction; we have to reckon with a certain amount of back turns as given by (22) and there may also be some weakening of the field due to cross turns though this is generally small. Both causes may with a fixed brush position be considered as producing a demagnetising effect, that is back ampere-turns proportional to the current. Let X_b represent these back ampere-turns, then we may write

$$X_b = bI$$

where by (22)

$$bI = 2\kappa \left(\frac{zI}{4ap} \right)$$

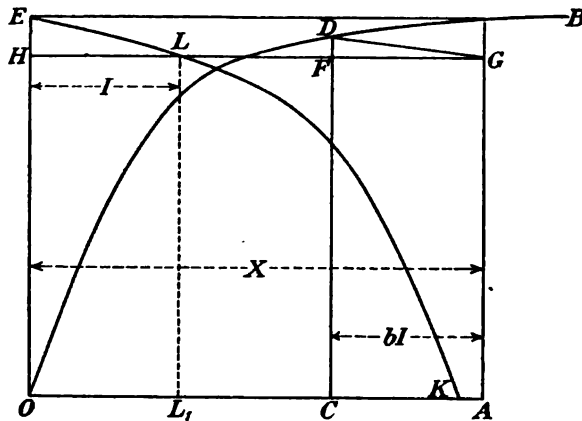


FIG. 38. External I.E. characteristic of separately excited generator.

and this value of bI may be slightly augmented if the effects of the cross turns should be important. It is not necessary to enter into details, since this subject has already been discussed in connection with Fig. 17. All that concerns us at present is that the back turns may be represented by the product bI . Let in Fig. 38 the length AC scaled off to the left from A represent the back or demagnetising ampere-turns, then the induced e.m.f. will be reduced to the value corresponding to the flux CD . The terminal voltage will be lower than this value by the ohmic loss ρI , where ρ is the true ohmic resistance of armature and brushes. Let $DF = \rho I$ measured to the volt scale on the ordinate, then CF is the terminal voltage if the current has the value I , which produces the back turns $GF = AC$. Since in the rectangular triangle DFG the two cathetes are in the

ratio of ρ to b the inclination of the hypotenuse DG is the same for any current and we obtain thus the very simple construction for finding the external characteristic of a separately excited machine running at constant speed shown in the figure. Use an ampere-scale b times smaller than the ampere-turn scale, mark this off from O to the right giving the point L_1 and also from A to the left giving the point C . The terminal voltage is found by drawing from D a straight line under an angle to the horizontal whose tangent is $\frac{\rho}{b}$. Mark the point G where this line cuts the ordinate corresponding to the fixed excitation OA . Project to the left and thus obtain the point L on the ordinate through L_1 . L is a point on the external ampere-volt characteristic and by repeating the construction for different values of the current the complete external characteristic ELK can be drawn.

If the machine has interpoles there are no back turns since κ is zero and if it is worked well above the knee the effect of the cross turns is negligible. In this case the first part of the characteristic up to the point corresponding to full load is a straight line forming an angle with the horizontal whose tangent is ρ . The equation of the external characteristic is then simply that of a straight line, namely

$$E_t = E - \rho I$$

where E is the voltage on open circuit.

External characteristic of Series Generator. In a series generator with fringe commutation (that is with brushes shifted forward) there must be back turns weakening the field in proportion to the current and the above reasoning applies. But in this case the external characteristic must start from zero and have an upward sweep, not a downward sweep as in a separately excited machine.

The construction of the external ampere-volt characteristic of a series generator is shown in Fig. 39. Here again we use two scales, both for the ordinates and the abscissae. The outer curve passing through D is the magnetisation characteristic giving the relation between X and Φ in one scale and the relation between X and induced e.m.f. in the other scale. Similarly OA in one scale represents exciting force $X = nI$, where n is the number of turns in one magnetic circuit ($\frac{n}{2}$ turns per pole), and in the other scale

it represents I . The two scales are related as 1 to n . Draw the line OC_1 under such an angle β that $\tan \beta = \frac{b}{n}$. Then to the current I represented by OA corresponds the back exciting force $AC_1 = AC$ and OC is the exciting force actually present and producing the induced voltage CD . If the armature had no resistance the projection of D on the ordinate through A would give a point B on the external characteristic; but as there is resistance causing a drop ρI the point lies lower by this amount. Draw OG under such an angle γ that $\tan \gamma = \rho$, then $\rho I = AG$. Make $BL = GA$, then L is the true point on the external characteristic. Repeating the construction for other points we find the complete curve. If the machine

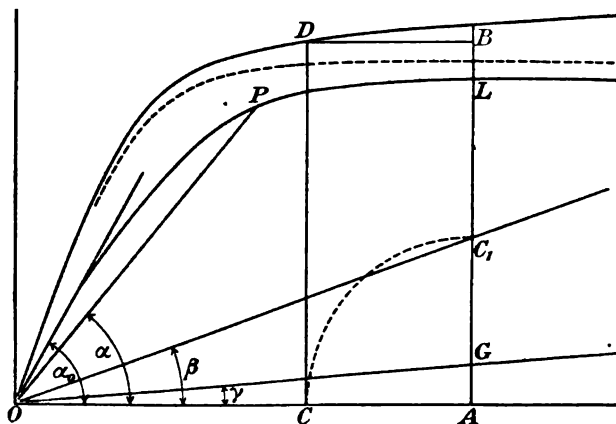


FIG. 39. External I.E. characteristic of series excited generator.

has interpoles, then there are no back turns and the external characteristic is found by simply deducting the ordinates of the resistance line OG from those of the internal characteristic. This gives the dotted curve.

The working point with a given resistance R in the external circuit can now be found by using the same reasoning as in the last chapter in connection with Fig. 19, the only difference being that we use the external instead of the internal characteristic. Draw in Fig. 39 OP under such an angle α that $\tan \alpha = R$, then the point where this P line cuts the curve is the working point. The critical resistance R_0 at which the machine is liable to cease working is $R_0 = \tan \alpha_0$. This is smaller than in the case of the machine provided with interpoles.

External Characteristic of a Shunt Generator. In Fig. 40 dND is the magnetisation curve. It is shown for a machine having some residual magnetism and does therefore not go through the origin, but cuts the e.m.f. axis some distance above O . The straight line $OgMG$ is the shunt resistance line drawn under such an angle α to the horizontal that

$$\tan \alpha = \frac{R}{n}$$

where R is the shunt resistance and n the number of shunt turns on a pair of poles. To an armature current I correspond bI back ampere-turns and an ohmic drop of ρI . This relation may be represented by a triangle GFD in the same way as in Fig. 38. The angle

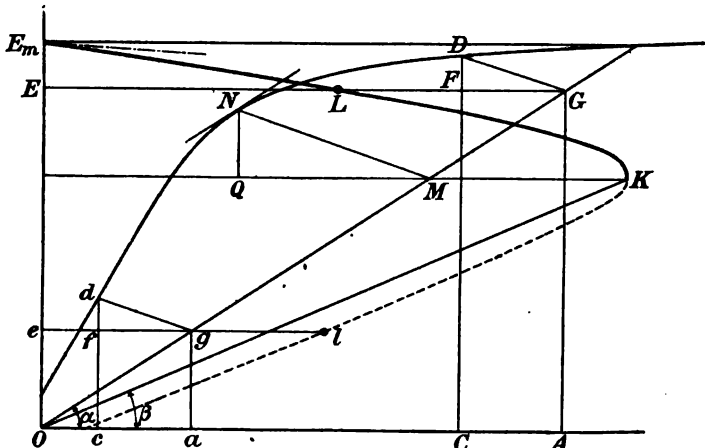


FIG. 40. External I.E. characteristic of shunt excited generator.

at G is constant and the line DG has the same inclination wherever the point G may be chosen on the resistance line. The ordinate AG represents the terminal e.m.f. Draw through G a line under the slope ρ/b and mark the point D where the magnetisation curve is cut. The induced e.m.f. is then DC and DF is the ohmic drop. Any side of the triangle may be used as a measure for the current. Taking for instance the horizontal we have

$$I = \frac{FG}{b}$$

and the current may be plotted to any convenient scale on the horizontal through G . This gives the point L on the desired characteristic. It should be noted that I is the armature current and

contains therefore the small current absorbed in excitation. To get the current in the external circuit the shunt current has to be deducted from EL , but the correction which can easily be made is not shown in the diagram to avoid complication. The correction is moreover so small that it is generally neglected. By repeating the construction here explained we get the full curve as shown. The lower part of the curve is dotted, since it represents an unstable condition. Where two quantities such as e.m.f. and current are interdependent the condition of stability may be expressed thus: An increase of current must produce a reduction of e.m.f., for if it produced an increase of e.m.f. this would again increase the current and we should have a cumulative process. The condition of stability is fulfilled for the point G on the resistance line, for an increase of current means that G shifts to a position lower down on the resistance line, that is to say, an increase of current (brought about by lowering the resistance in the external circuit) results in a decrease of e.m.f. For a point g on the resistance line an increase of current that is a lengthening of the line fg can only be obtained by shifting g to a higher position and this means more e.m.f. Maximum current is obtained for a particular resistance in the external circuit. To find the value draw a tangent to the characteristic of magnetisation parallel to the resistance line and where this touches the curve at N draw the inclined line NM parallel to DG . The ordinate of the point M is the external e.m.f. at maximum current and the current itself is given by the length $\frac{QM}{n}$. Let this be the abscissae of the point K . Draw the line KO , then the tangent of the angle of this line to the horizontal taken in the appropriate scale of current and e.m.f. represents the lowest resistance in the external circuit at which the machine can work. It need hardly be mentioned that the maximum current is with modern machines far beyond the safe limit for heating and sparking so that this question has only so far a practical interest as it shows that a shunt machine connected to an external circuit of resistance

$$r < \tan \beta$$

cannot work at all.

On open circuit where $\tan \beta$ is infinite the terminal voltage is E_m . At load it is smaller, say E , but the difference is less than shown in the diagram, especially if the machine is worked at high saturation. If the machine has interpoles the back turns vanish and the effect

of the cross turns being small, the length GF is much reduced and $1/b$ much increased so that the external characteristic takes the position shown by the chain dotted line. In this case (the machine has interpoles and is worked at high saturation) the drop is practically only that due to ohmic resistance in the armature circuit.

External Characteristic of Compound Generator. Since shunt excitation gives a drooping and series excitation a rising characteristic it is possible by a combination of both methods of excitation to obtain a fairly constant terminal voltage throughout the load range of the machine. The terminal voltage at different loads may be found by the construction shown in Fig. 41. The process is necessarily one of "trial and error" since we must start

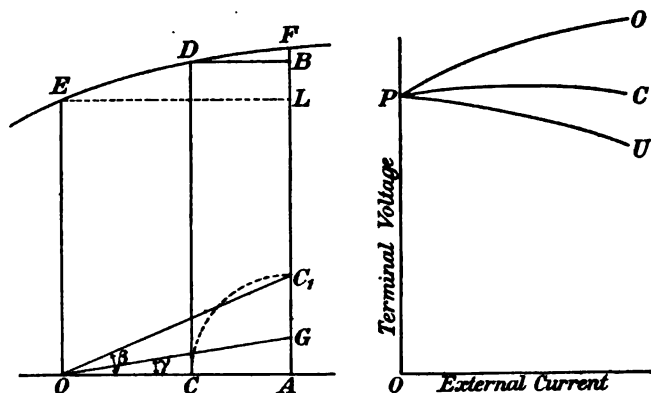


FIG. 41. External I.E. characteristic of compound excited generator.

the construction on a certain point, say E , of the internal characteristic EDF and arrive at a point on the external characteristic which has the same ordinate. (We neglect here the small correction for ohmic drop in the case of a short shunt and the exciting force of the shunt current passing through the series coils in the case of a long shunt, but if desired these corrections can easily be made.) Assuming then that the excitation due to the shunt alone produces the internal e.m.f. OE and that OA represents the excitation in ampere-turns due to the main current I , then OA may with an appropriate change of scale also represent the main current itself. The lettering of this figure corresponds as far as the main current is concerned to Fig. 39. $AC_1 = AC = bI$ represents armature back turns, CD represents e.m.f. actually induced and AG represents

ohmic drop. This deducted from CD gives the terminal voltage AL . This value should come out equal to OE ; if there is a difference, select a different point E and repeat the construction until L and E are on a level. L is a point on the desired characteristic and corresponds to the current OA . By repeating this trial and error method for different values of external current we get the external characteristic. The diagram on the right shows three such curves, all starting from the same point P which refers to open circuit working and is therefore independent of the action of the series coils. The middle curve PC is that of a correctly compounded generator in which the number of series turns is so chosen that at no load and full load the terminal voltage is the same. At an intermediate load it is slightly greater and this is due to the fact that the magnetisation characteristic is not quite a straight line, even if the working region be chosen above the knee. A very close approach to a straight line PC may be obtained by giving the series coils a few more turns than necessary and shunting their terminals by an iron resistance. Since iron has a high temperature coefficient the percentage of current shunted through such an iron "diverter" becomes less as the external current increases and hence the exciting force of the series coils increases rather faster than the current in the external circuit. In this way the middle of the curve PC is depressed and the end C raised.

By reversing the series coils we get a drooping characteristic such as PU . This is called "under-compounding." By giving the series coils more turns and not reversing we get a rising characteristic such as PO which is called "over-compounding." Over-compounding to the extent of about 10 per cent. is sometimes used in generators for tramways, so as to make up for the extra drop in feeders at times of heavy load. A slight amount of over-compounding may also be employed to correct drop in speed of the prime mover when the load comes on. There is no governor so perfect as to keep the speed absolutely constant at all loads and a drop of a few per cent. in speed is unavoidable. By over-compounding the generator to this extent the voltage can be kept constant at all loads.

It is interesting to enquire how a compound wound generator will act if used as a motor on a constant voltage circuit. The two conditions are shown in Fig. 42. It will be seen that when used as a motor the field is weakened by the main current. At the same time the ohmic loss in the armature circuit is increased, that is to

say, the e.m.f. impressed on the armature and therefore also the counter e.m.f. produced in the armature is reduced. Since the speed is proportional to the quotient of e.m.f. divided by flux it will be constant if both increase or decrease together. This is the case since the series coils act in opposition to the shunt coils. We thus find that a generator correctly compounded for constant voltage will, if used as a motor, run at constant speed under variable load. To start such a motor it is advisable to short circuit the series winding, so that the starting torque due to shunt excitation may not be weakened by the demagnetising action of the series coils. The action of a compound generator may be represented in a simple manner mathematically if we admit the following approximations, namely the magnetisation curve is a straight and slightly sloping

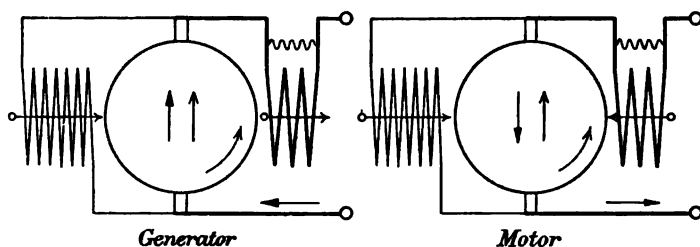


FIG. 42.

line above the knee and armature reaction may be represented by a certain increase of resistance as explained on p. 52 (Chapter III). Let Φ_1 be the flux due to the shunt which with correct compounding is constant and mI the additional flux due to the series coils, then the terminal e.m.f. is

$$E_t = u\epsilon\Phi_1 + u\epsilon mI - \rho I$$

where ρ is the resistance including the increase to represent the effect of armature reaction. The condition of correct compounding is therefore

$$u\epsilon m = \rho$$

If the same machine is worked as a motor the impressed terminal voltage must be larger than the induced voltage by the amount ρI , so that

$$E_t = \rho I + \epsilon u (\Phi_1 - mI) = \epsilon u \Phi_1 + I(\rho - \epsilon um)$$

Since by the condition of correct compounding as generator

$$\rho - \epsilon um = 0$$

the expression for the speed is obtained from

$$\epsilon\Phi_1 u = E_t - \rho I + \epsilon u m I$$

$$u = \frac{E_t}{\epsilon\Phi_1}$$

Φ_1 being constant and the impressed voltage also being constant, it follows that the speed will be constant for all loads. If for such a motor we draw a graph connecting torque as abscissae and speed as ordinates, the so-called speed-torque characteristic, we find simply a straight horizontal line.

Current-Speed-Torque characteristics. Let, in a shunt motor working on a constant voltage circuit, Φ_0 be the flux when running idle at speed u_0 and let Φ be the flux when the armature carries the current I . If the motor has a compensating winding then $\Phi = \Phi_0$, but if the cross turns are not compensated then Φ will be a little smaller than Φ_0 . If ρ is the ohmic resistance in the armature circuit, ρI is the ohmic drop and with an impressed voltage of E the speed u is determined by the relation

$$E - \epsilon u \Phi = \rho I$$

where $E = \epsilon u_0 \Phi_0$. We thus obtain

$$u = u_0 \frac{\Phi_0}{\Phi} \left(1 - \frac{\rho I}{E} \right) \dots \dots \dots (43)$$

The second term in the bracket is the ratio of the ohmic drop to the supply voltage, a matter of a few per cent. If then the motor is compensated the speed at load will drop by this percentage, but

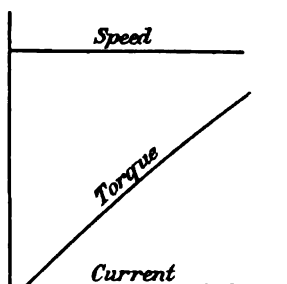


FIG. 43. Current-speed-torque lines of shunt motor.

if there is no compensating winding $\Phi < \Phi_0$ and then the speed at load may actually be a little higher than when the motor is running idle. The same effect might be obtained by over-compounding and in a more satisfactory manner since there would be no difficulty with the commutation. Cases where an increase of speed at load is required are, however, quite exceptional; what is generally desired is a constant speed under variable load and the above investigation shows that the ordinary shunt motor is eminently suited for this purpose. Fig. 43 shows the current-speed-torque lines of a shunt motor. The speed line is practically a horizontal; the torque being

proportional to the product of flux and current is also a straight line for a compensated motor where the flux is constant, but when there is weakening of the field by armature reaction the line is slightly curved. Theoretically it should pass through the origin, but in reality it is slightly lower, due to friction and hysteric losses.

In a series motor the flux increases with the current and since the e.m.f. to be induced by rotation decreases with increasing flux the speed line must have a downward slope. The heavier the load the smaller is the speed with a constant supply voltage. The torque, being proportional to the product of flux and current, increases at

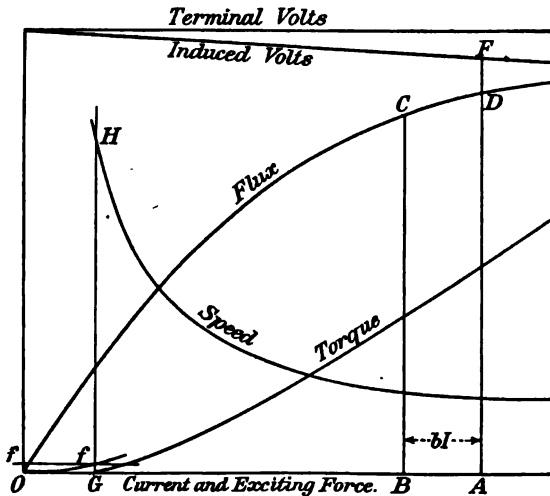


FIG. 44. Current-speed-torque lines of series motor.

rather a faster rate than the current. Fig. 44 shows these curves. For the abscissae two scales are used, namely current and exciting force. To the exciting force OA corresponds the flux AD , but to the current OA corresponds the somewhat smaller flux BC , since the back turns $bI = AB$ reduce the actually effective exciting force to OB . Let the effective flux be Φ , the terminal voltage E and the true ohmic resistance in the armature circuit ρ , then the speed in r.p.s. with a current I is

$$u = \frac{E - \rho I}{\epsilon \Phi} \dots \dots \dots (44)$$

The torque is

$$T = \tau \Phi I$$

The numerator in (44) is graphically represented by the ordinate AF corresponding to the current OA . The torque equation refers to the torque exerted by the armature, not that actually available on the shaft. To get the latter a deduction for friction and hysteric losses must be made. Both may be taken as approximately constant and their sum is represented by the ordinate of the line ff . Where this line cuts the theoretical torque curve we have the condition of idle running, that is to say, zero torque on the shaft. OG represents the current when the motor runs idly and GH the corresponding speed. This condition is not definitely fixed since a small change in the torque loss makes a big change in no load current and no load speed. The latter is in any case several times the normal load speed and is generally termed the "run-away speed." As it may be dangerously high it is advisable to fit the motor starting gear with some appliance which will trip the main switch in the event of the current strength falling below a certain minimum, a so-called "no load release."

Speed of a Shunt Motor as Influenced by the Supply Voltage. It will be obvious that if the working point is very high on the magnetisation curve a change of supply voltage will result

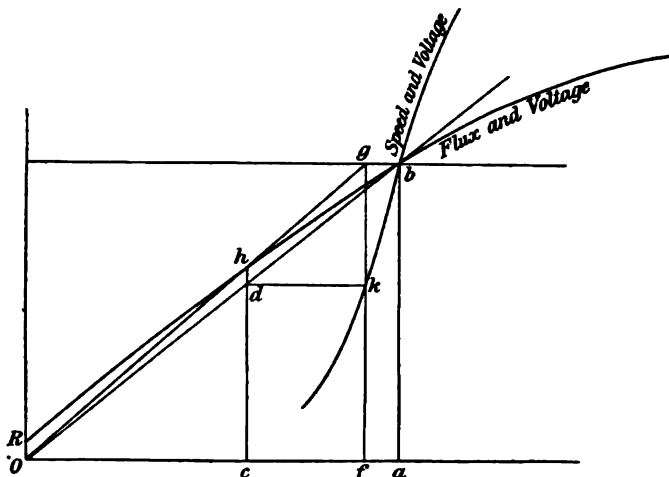


FIG. 45. Speed curve of idle running shunt motor under variable voltage.

in an almost equal change of speed, for at high degrees of magnetisation a moderate change of exciting force can only produce a very small change in flux, whilst it produces a proportional change in

induced voltage, thus causing an almost proportional change in speed. If, however, the field is far from saturation, flux and induced e.m.f. will vary more or less together and thus the speed will not be influenced so much as with a highly saturated field. We shall now investigate this case on the supposition that the motor is running idly. In Fig. 45 Rhb is the magnetisation curve and for a constant speed also the e.m.f. curve. For the abscissae we use two scales, namely exciting force and speed. With the exciting force Oa corresponding to the normal voltage E the flux is ab and with an appropriate change of scale the e.m.f. is also ab . Now let the supply e.m.f. be reduced to $E_1 = cd$, producing the excitation Oc and the flux $\Phi_1 = ch$. The two working conditions are represented by

$$E = \epsilon \Phi u \quad \text{and} \quad E_1 = \epsilon \Phi_1 u_1$$

u_1 being the speed to be determined and to be plotted on the axis of abscissae to the same scale that represents $Oa = u$. Combining the above equations we get

$$\frac{u_1}{u} = \frac{E_1}{E} \times \frac{\Phi}{\Phi_1}$$

and since
$$\frac{E_1}{E} = \frac{Oc}{Oa} \quad \text{and} \quad \frac{\Phi}{\Phi_1} = \frac{fg}{ch} = \frac{Of}{Oc}$$

we find
$$\frac{u_1}{u} = \frac{Oc}{Oa} \times \frac{Of}{Oc} \quad \text{or} \quad u_1 = \frac{u}{Oa} \times Of$$

If we measure the distance Of with the speed scale corresponding to $Oa = u$ we get

$$u_1 = Of$$

Since ab represents the original voltage E , cd represents the reduced voltage E_1 . Draw the horizontal through d and find the point k where it cuts the ordinate fg , then k is a point on the volt-speed curve, speed being abscissae and e.m.f. ordinates. The rule for finding this curve is therefore: Draw the characteristic of magnetisation and through the working point at normal voltage b draw a horizontal. Through O and b draw a straight line and mark off on it points such as d , the ordinates of which give the reduced voltage. Through d draw a vertical and determine the point of intersection h with the magnetisation curve. Join O and h and prolong to the point of intersection g with the E line. Drop a vertical from g and draw a horizontal through d and where these lines meet is a point on the desired volt-speed curve. The same construction is of course also applicable if there is not a reduction but an increase of voltage. It will be noted that on either side of the normal working point b ,

a fairly large change in the voltage produces only a small change in speed and this effect is the more marked the more the characteristic approaches a straight line. In the figure it has been assumed that there is residual magnetism in the field system, giving an initial flux OR . This is the reason for the lower part of the curve bending to the left. If residual magnetism can be nearly avoided (as may be done by using sheet steel for the field system) and the machine be worked at very low flux density, the curve would become considerably steeper than shown in the figure with the result that the machine would run practically at constant speed although the supply voltage might vary up or down by say as much as 10 per cent.

Motor Starting and Speed Regulating Switches. The usual type of starting switch for a shunt motor is diagrammatically shown in Fig. 46. Such a switch has to fulfil the following requirements. It must limit the starting current to a predetermined amount, it must sever the motor from the supply mains in the event of the supply failing even momentarily or the shunt circuit being broken and it must also interrupt the connections in case the motor is overloaded beyond a predetermined limit. The reason why the motor should automatically be switched off if the supply fails is that on re-establishing the supply the armature would form almost a dead short circuit and become damaged by the excess of current which passes it whilst at rest. Also the fuses in the supply circuit (not shown in the diagram) would blow and would have to be renewed. L, L are the supply leads coming from the fuses, D is a double pole main switch and F is the contact lever of the starting switch. The whole of the resistance R is put into the armature circuit as soon as the starting lever touches the first contact. As the motor gathers speed the lever is pushed farther to the right until it reaches the last contact when the resistance R is completely cut out. The shunt current passes through a little electromagnet a which holds an armature attached to the starting lever in place. As long as the shunt current flows, the connection between the mains and the brushes is kept up, but should the shunt circuit be broken or the supply be cut off either accidentally or by the main switch D being opened, the electromagnet can no longer hold the lever in its last position and then the spiral spring l throws the starting lever back into the "off" position. M, M are the terminals for the main or armature current and S is a shunt terminal, the other terminal of

the shunt winding being permanently joined to one brush. The electromagnet *b* inserted into the armature circuit protects the motor against an overload. If the current increases beyond a safe value this magnet attracts its armature and thus closes the contact *c* and short circuits the magnet *a*. This releases the starting lever and allows it to fly into the "off" position. Sometimes a press key (shown at *K*) is added for stopping the motor by hand. This key needs only thin leads and may therefore be put into any convenient place even at a considerable distance from the starting switch.

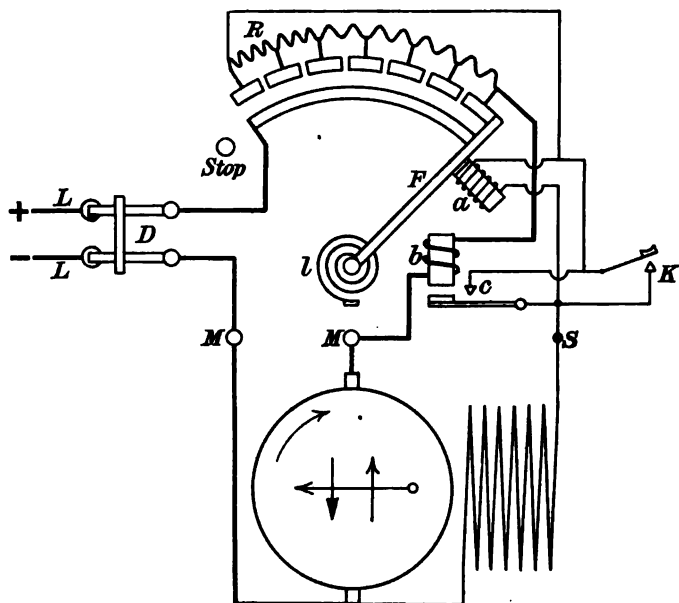


FIG. 46. Starter for shunt motor.

The apparatus shown merely diagrammatically in Fig. 46 is technically known as a starter with "no volts and overload release." It will be seen that a similar arrangement can be used for series motors if the magnet *a* is energised by the main current.

The type of starter shown in Fig. 46 becomes impracticable when applied to large motors of some hundreds of h.p. The difficulty is not only in the amount of resistance material which would be required to take up a large amount of energy, but also in the contacts which would have to carry enormous currents. In such cases a liquid starter is used. It consists of an iron tank into which can be gradually lowered an iron plate, the tank being filled with an

alkaline solution. The deeper the plate is dipped the less is the resistance and by suitably shaping the contour of the plate it is possible to ensure a gradual start without any excess of current during the process. A supplementary knife edge contact is generally provided to short circuit tank and plate when the latter is in the lowest position.

The speed of any motor may be regulated by the insertion of resistance (whether solid or liquid) into the armature circuit, but

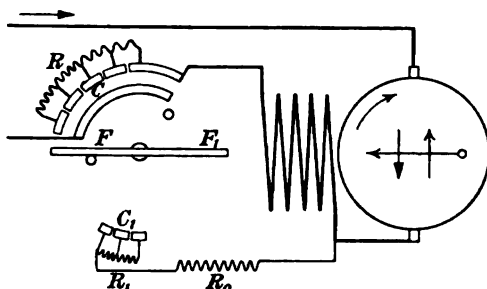


FIG. 47. Starter and speed regulator for series motor.

this method is wasteful. One method to avoid this waste is to use an armature with two equal windings and two commutators. By joining the two armature circuits in parallel we get full speed and by joining them in series

about half speed. Apart from the drawback of complication this method has the disadvantage that no adjustment for any intermediate speed is possible. The method generally adopted is to adjust for speed by varying the excitation.

Fig. 47 shows the principle as applied to a series motor. F is the switch lever passing over the contacts C of the starting resistance R and the other end of this lever F_1 passes over the contacts C_1 of a resistance by which the field winding is shunted. When the lever

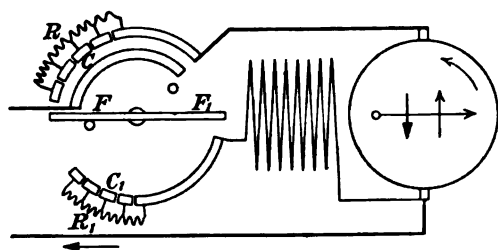


FIG. 48. Starter and speed regulator for shunt motor.

has been turned so far that the whole of R is cut out, but before F_1 has come into contact the motor has a full field and its speed is normal. If the lever be turned farther the field winding is shunted by the resistances R_1 and R_0 . The field is thereby weakened and the speed is increased. Maximum speed is obtained on the last contact when the whole of R_1 is cut out and only R_0 remains in circuit.

The same principle may be applied to a shunt motor as shown in Fig. 48. Here the weakening of the field is obtained by the

insertion of the resistance R_1 in series with the shunt exciting coils.

Energy Loss in Starting any Type of Motor. During the starting of a motor energy must be wasted in the starting resistance. The quicker the start the smaller is the time integral of power, that is the total energy input. The most favourable condition is therefore one in which the starting rheostat is so manipulated as to keep the current constantly at its permissible maximum value, or as near this as the steps in the rheostat will permit. This means a constant starting torque and with a constant resisting torque it also means constant acceleration. A graph of the starting period will then have the character shown in Fig. 49. Oa is the starting time t , ab the supply voltage E , bc the ohmic drop which is a certain fraction αE of the supply voltage, ai the current and at the torque. The speed is given by the sloping line Oc .

The diagram shows ideal conditions not realisable in practice since whether the starting resistance be solid or liquid it is impossible to adjust it so accurately at any moment as to keep torque and current absolutely constant. Any departure from this condition must lengthen the starting time and increase the waste of energy. If then we determine the energy

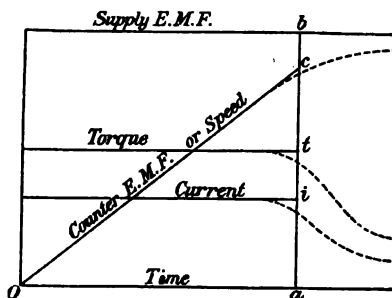


FIG. 49. Graph of start.

efficiency on the basis of this diagram we shall slightly overestimate it. The input of energy is obviously $W_i = EIt$, graphically represented by the area of the full rectangle. The energy output W_o is represented by the area of the triangle Oac , or in figures

$$W_o = EIt \left(\frac{1 - \alpha}{2} \right)$$

The energy efficiency during the starting period is therefore

$$\eta = \frac{1 - \alpha}{2}$$

Thus if at normal load the motor has an ohmic drop of 5 per cent. it may have 8 per cent. drop with the increased current permissible during the start and then the energy efficiency would be 46 per cent.,

in reality a little less since the current cannot be kept absolutely constant and some deduction from W_0 must be made for frictional losses. With an ideal motor having no losses at all under normal working conditions the starting efficiency cannot exceed 50 per cent., with a practically possible motor the starting efficiency cannot much exceed 40 per cent.

Series-Parallel Control. This matter is of considerable importance in electric traction, especially on tram lines and suburban railways, where high acceleration and frequent stops are conditions of the service. A large portion of the resisting torque is due to the requirement of quick acceleration and the rest to tractive resistance. At full speed the torque required for acceleration is zero and the current is correspondingly reduced, so that in the diagram for a traction motor the speed line after passing point *c* bends over and becomes horizontal at a height corresponding to the torque required for overcoming tractive resistance only. This condition is shown by the dotted lines, but as it does not materially modify the general

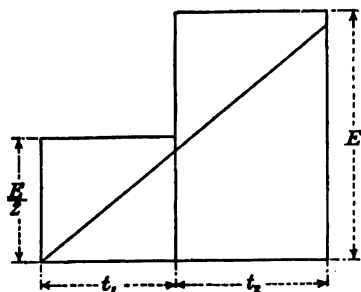


FIG. 50.

problem we can say that also in railway and tramway motors more than half the energy supplied during the starting period is wasted. This waste may be lessened by the so-called "series-parallel control." The vehicle is provided with two motors which up to about half speed are coupled in series and beyond half speed in parallel. The graph of a start with series-parallel control is shown in Fig. 50. The energy input is $W_i = IE(t_1 + 2t_2)$ and the energy output is $W_o = IE(t_1 + t_2)(1 - \alpha)$. The energy efficiency during the starting period is therefore

$$\frac{(t_1 + t_2)(1 - \alpha)}{t_1 + 2t_2}$$

Since t_1 and t_2 are not greatly different from $\frac{t_1 + t_2}{2}$ we find in close approximation

$$\eta = \frac{2}{3}(1 - \alpha).$$

Comparing the two equations for efficiency for single and series-parallel working we find their ratio to be 3 to 4. This means that

the energy input during the starting period is with series-parallel control only 75 per cent. of what would have to be provided in order to reach the same speed in the same time with the two motors permanently worked in parallel. By using series-parallel control we therefore save 25 per cent. of the starting energy, economically an important point, since in urban and suburban traction most of the energy is required for bringing the cars up to speed, a good part of the run being performed under so-called "coasting conditions" that is letting the train run on by its momentum until the brakes have to be applied. Fig. 51 is a running diagram of a suburban train and that of a tramcar having to make frequent stops is not very different. Time is plotted on the horizontal, speed and current on the vertical. The current is that for both motors and since at half speed they are switched into parallel there is a sudden rise in the current line. The motors are started in series, the starting rheostat being so worked by the driver as to keep the current at its permissible maximum until the time that half speed is reached. Then the motors are switched into parallel again with a certain amount of resistance in the armature circuit which is gradually reduced until at time b each motor gets the full e.m.f. As the speed must still rise (since the torque exerted exceeds that due to tractive resistance) the counter e.m.f. rises also and reduces the current. At the time c the current has fallen to the amount corresponding to tractive resistance and there is no more acceleration. On a long run this steady condition would be maintained, but the diagram is drawn under the supposition that the distance between stations is so short that the rest of the journey can be made by coasting.

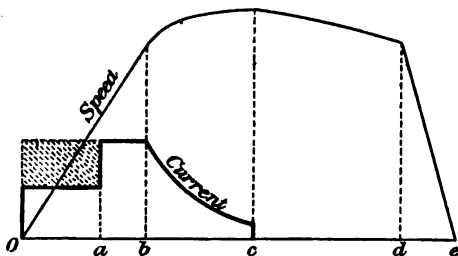


FIG. 51. Series-parallel control.

The current is therefore switched off at time c and the train runs on till at d the brakes must be applied to bring it to a standstill at time e . The energy used is proportional to the area of the current line; if the motors were worked in parallel from the start double current would be required not at a , but already at O , thus increasing the area by the shaded part. This then represents the saving of energy obtained by series-parallel control.

In the case here considered the torque required on account of acceleration is considerably greater than that necessary for traction. It can be calculated from the known weight of the train and the desired acceleration (about 0.5 to 0.7 m. per second per second). Since the train must not only be accelerated linearly, but there is also a rotary acceleration of armatures, gear and wheels a suitable addition of say 10 per cent. must be made to the total weight when calculating the torque required.

Apart from the saving in starting energy the series-parallel system of control has also other advantages. It implies a reduction in the total amount of material required for the rheostats, since that of each motor has only to take up at the start half the e.m.f. and there is the further advantage that on heavy gradients current may be saved by working in series, whilst generally the speed regulation is better.

For heavy railway work the system has been extended to four motors. At the start all these are in series, then they are connected in two parallels of two in series and finally for top speed all four motors work in parallel. The insertion and withdrawal of resistance and the change in the coupling is done by means of a so-called "controller."

The Ward-Leonard System of Speed Regulation. In this system the field of the motor is kept constant, its exciting coils being permanently connected to a source of constant e.m.f. and the e.m.f. supplied to the armature is raised or lowered accordingly as a higher or lower speed is required. The supply voltage may also be reversed, in which case the motor may be made to run either way at any desired speed. This system of control had originally been devised for passenger and goods lifts, but it has also found many other applications. It is diagrammatically represented in Fig. 52 in which however all unessential details such as switches are omitted. M_1 is a shunt motor connected to the supply mains and running at constant speed. It drives a separately excited generator G the field of which can be adjusted by the rheostat R and M is the motor driving any kind of machine T (the drum of a hoist, or a printing press or any other appliance which must be able to work at an adjustable speed). If reversal of motion is required a reversing switch S is added. By means of the rheostat R the voltage generated in G can be varied from almost nothing to its

full value and thus the motor M may be made to run at any speed from a mere creeping round to normal speed and even more if a

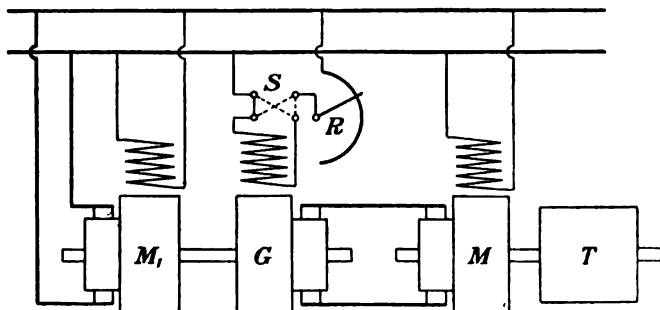


FIG. 52. Ward-Leonard system of speed regulation.

rheostat for weakening its field be added. It will be seen that the advantage of speed variation from zero upwards and ability to reverse is obtained at a rather large provision of machinery. In addition to the motor two other machines must be provided, G a little larger than M and M_1 a little larger than G , since not only the whole of the power absorbed by T , but also the power lost in the intermediate machines must be given to M_1 . Thus if the motor M is required to have at normal speed an output of 20 kw., the generator G must give an electrical output of at least 23 kw. and M_1 a mechanical output of about 27 kw., making in all 70 kw. of machinery to work a tool requiring 20 kw.

Speed Regulation by Booster. If reversal of motion in the tool is not required and if only a moderate range of speed need be

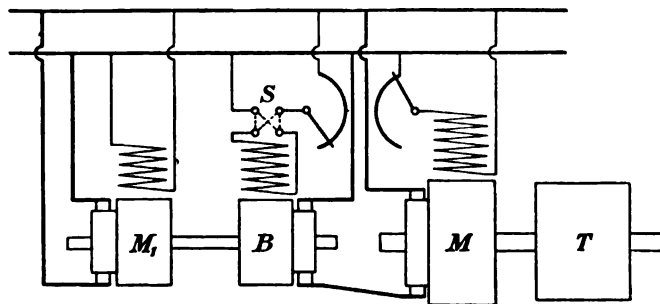


FIG. 53. Speed regulation by booster.

provided, then the system can be made more economical by arranging the intermediate machine not as a generator for the full

power, but merely as a reversible booster to add or subtract e.m.f. This system is shown in Fig. 53. B is the reversible booster giving when fully excited the e.m.f. E_b . With a supply voltage E in the mains the voltage supplied to the generator can then be varied between $E - E_b$ and $E + E_b$. If the field of the motor M were kept constant the ratio of its extreme speeds would be $\frac{E + E_b}{E - E_b}$, but by adding a field rheostat as shown it is possible to extend this ratio to

$$n = r \frac{E + E_b}{E - E_b}$$

where r is the ratio between the strongest and weakest field. The voltage of the booster is then

$$E_b = E \frac{n - r}{n + r} \quad . \quad . \quad . \quad . \quad . \quad (45)$$

For $r = 1.67$ the weakest field will be 60 per cent. of the strongest field and a speed range of 1 to 5 may be obtained with a booster half the size of G in Fig. 52 and a primary motor also half the size of that required in the original Ward-Leonard system. The total amount of machinery now comes to only 45 instead of 70 kw.

Electric Drive for Reversible Rolling Mill. For certain services such as winding engines and rolling mills provision must be made for a short time torque largely in excess of the normal. In a winding engine this large torque is required for acceleration and in a rolling mill for the pass of the billet. Were the motor fed directly from the mains these and the generating station would have to be much larger than corresponds to the average power required and there would be voltage fluctuations disturbing other motors on the system. For these reasons a store of energy in the shape of a flywheel is used to supply the peak load for the short time in question, but a flywheel in order to give out energy must vary its speed and it is therefore necessary to add to the Ward-Leonard system not only a flywheel but also some means by which the speed of the primary motor is reduced during the time that the peak load lasts and increased again when the load is light. Fig. 54 shows the system employed for reversible rolling mills. The field rheostat R_1 of the primary motor is controlled by a centrifugal regulator which reduces resistance as the speed diminishes and cuts it in again as the speed rises. The same object may however be

attained in a simpler way by compound excitation of the primary motor as shown. As the load increases the current also increases and strengthens the field, causing the motor to run more slowly and thus enable the flywheel to give out energy. In the figure G is the generator which supplies the whole of the power electrically to the main motor M . T is the rolling mill. The brushes of generator and motor are permanently connected and no switch is used in this circuit. The field of the motor is also permanently connected to

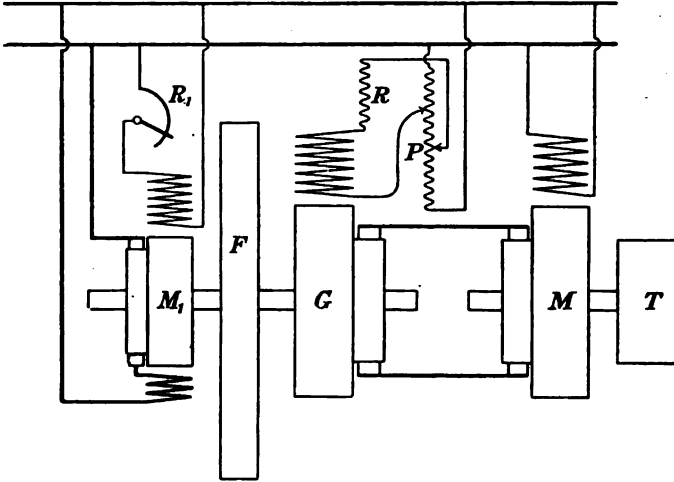


FIG. 54. Electric drive for reversible rolling mill.

the supply mains and the reversal of motion is obtained by reversing the field of G . For this purpose a potential slide P is used. If the movable contact is at one end the motor runs one way; if it is pushed to the other end the motion is reversed. As it is necessary for economical working to reverse quickly an inductionless resistance R is inserted into the exciting circuit of the generator. This reduces the time constant (see Vol. I, p. 223) and makes it possible to reverse very quickly, say 5 to 10 seconds in a 2000 h.p. mill.

Coupled Dynamos. Provided the prime movers are powerful enough to keep running in the right direction whatever the load, two series machines can be coupled in series to supply the external circuit at a voltage which is the sum of the voltages of the two dynamos. If the machines are equal in all respects we are thus able to double the voltage supplied to the circuit, the current being

that which can be produced in one machine. If it is a question of doubling the current at the voltage of one machine, the two generators must be coupled in parallel and then some special arrangement in connection is necessary to ensure stability. On the left of Fig. 55 two series machines are shown coupled in parallel without any alteration in their internal connections. We cannot expect the internal e.m.f. of the two machines to be absolutely the same; if then by way of example *B* has an e.m.f. a little less than *A*, then the current through *B* will be a little smaller than that through *A* and the field of *B* will be weaker than the field of *A*. The original difference in the induced e.m.f. may be due to some slight difference in the characteristics, say, from the air gaps not being absolutely the same or from a spongy casting, or it may be due to a slightly lower speed of the prime mover of *B*. Whatever the cause, the inevitable result is that the original deficiency of e.m.f. in *B* produces

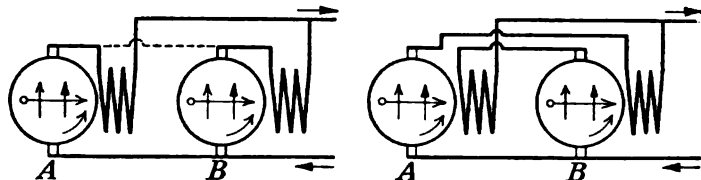


FIG. 55. Series generators in parallel.

a weaker field in *B* and thus a still lower e.m.f. The process is cumulative with the result that the current through *B* is quickly reversed and each machine forms a dead short circuit to the other. As a remedy it has been suggested to join the two positive brushes by a connection shown dotted in the figure, but this remedy is only reliable if the two field windings have the same resistance and this is large as compared to the resistance of the balancing wire. A much better way in which two series generators can be coupled in parallel is by "cross excitation" as shown on the right of Fig. 55. If we again assume *B* to be the weaker machine it will give less current to the field system of *A* and thus *A* will be weakened and this will restore the balance. To put it another way, if *A* should try to give more than its fair share of the current, it will strengthen the field of *B*, causing this machine to take more load and thus correct the initial difference to a great extent.

There is no difficulty in coupling two equal shunt generators driven at the same speed in parallel. The field is necessarily the

same in both and if there should be a tendency in the prime mover of one generator to slacken speed, the result would be a reduction of current and torque of this generator and an increase of load on the other prime mover. This means that the stronger set would reduce and the weaker increase speed, thus again correcting to some extent the original inequality. Even if the supply of power to the one machine were cut off completely, this would not stop its running. It would continue to run as a motor in the original direction. It is therefore perfectly safe to couple shunt machines in parallel.

It is also safe to couple them in series in the sense that (unlike series machines) they cannot burn each other up, but it is not a very stable arrangement. In Fig. 56 on the left are shown two shunt machines coupled in series. Let for some reason (drop in speed of prime mover, or increase of shunt resistance) the induced e.m.f. in one machine diminish. If the machine were working alone

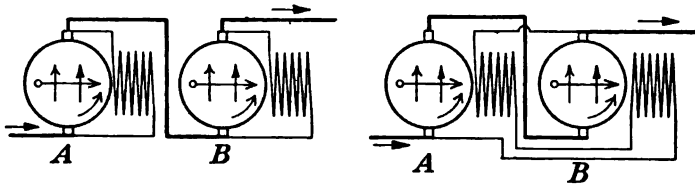


Fig. 56. Shunt generators in series.

this would imply a lowering of the current and the machine would settle down in a new working condition at a lower output, but as the machine must take the larger current supplied to it by the stronger machine the brush voltage will again be lowered and the working point will be on a lower part of the characteristic, possibly so low as to bring the machine within the critical condition (p. 53) when it will fail to be self-exciting. The impressed current still passing from the lower to the upper brush will make the former positive and thus force current through the shunt in the reverse way. The machine will then build up its field again but in the reverse sense thus putting itself into parallel with the stronger machine. Whether this happens or not will depend on how near the machine is under normal working condition to the critical condition. To make the working stable under all conditions the two shunt circuits should be coupled in series as shown on the right of Fig. 56. Then even if *B* should slacken in speed, its field must be the same as that of *A* and reversal of polarity becomes impossible.

Under-compounded generators, having a drooping characteristic, can be run in parallel without any alteration in their connections.

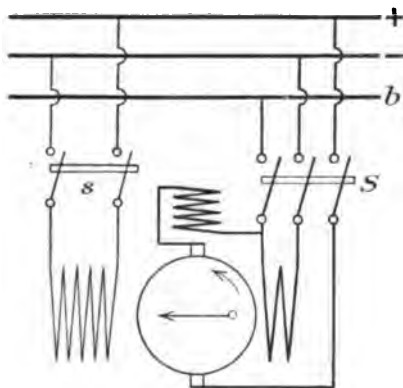


FIG. 57. Compound machines in parallel.

Such machines are, however, exceptional. The general case is that of level or over-compounded machines and to run these in parallel it is necessary to use a balancing (or equalising) connection as shown in Fig. 57. Only one machine is shown working on to the bus bars marked + and -. The bus bar to which the equalising connections are coupled is marked *b*. The exciting current is made by closing switch *s*. When the

machine has been run up to speed and the voltage adjusted to that of the bus bars, the main switches may be closed. The switch in the equalising circuit should be closed either before the others, or simultaneously. To ensure stability it is essential that all main fields should have the same resistance and the equalising circuit a very low resistance especially if the machines are over-compounded. If the machine has interpoles their coils must be treated as part of the armature circuit as regards the balancing wire. This is shown in Fig. 57. Compound machines of unequal size can be run in parallel if the resistances of the series field circuit are so adjusted as to give equal drop.

Transmission of Power. If two machines are used for transmission of power, stable working is always possible if the generator is either a shunt machine or a level compounded machine, because its field cannot be reversed however the main current may vary. With a series generator and a shunt motor stable working is impossible. The reason is this: In the event of the mechanical load on the motor being suddenly reduced, there must be a reduction of current and therefore also a reduction of e.m.f. in the generator. The back e.m.f. of the motor cannot be suddenly reduced, since the motor by its inertia will run on for a short time at its previous or only a slightly reduced speed. The e.m.f. generated in the motor armature cannot fall as quickly as that of the generator because the

inductance of the motor field is necessarily great. Hence whilst the motor slows down and eventually may stop, the current through the generator is reversed and its field builds up in the reverse sense. By the time the generator has built up its field, the motor may have stopped or may be running very slowly, thus forming almost a short circuit to the generator.

A sudden increase of mechanical load produces an equally sudden increase, not only of current but also of e.m.f. since the series generator has a rising characteristic. This excess e.m.f. again calls forth a greater current and the motor is accelerated and finally reaches a speed and counter e.m.f. which reduces the current and the torque on its shaft. The speed drops in consequence and it will thus be seen that any sudden alteration of the load must cause considerable variation in speed and may even produce violent sparking and overheating in both machines.

Stable working of a power transmission plant with a series machine as generator is, however, possible if the motor is also a series machine. In this case a reduction of mechanical load, whether sudden or gradual, produces not only a reduction in current, but also a reduction in the e.m.f. of the generator, whilst the motor, even if it runs on by inertia, can never act as a generator and send current the reverse way through the primary machine. This can, therefore, never be reversed. The only question is whether, if the generator is driven at a constant speed, the motor will run at a constant speed under variable load. This requirement can be fulfilled by so designing the two machines that their characteristics have a certain relation to each other. Let R be the resistance of the line and ρ the effective resistance of the machine, that is its ohmic value augmented by an appropriate amount to take into account armature reaction, then the e.m.f. equation is

$$\epsilon_1 \Phi_1 u_1 = \epsilon_2 \Phi_2 u_2 + I(\rho_1 + \rho_2 + R)$$

where the indices 1 and 2 refer to the primary (generator) and secondary (motor) machine. If the primary machine is driven at constant speed and it is required that the secondary machine shall also run at all loads at a constant (though possibly different) speed, it must be so designed that its magnetisation characteristic fulfils the equation

$$\Phi_2 = \Phi_1 \frac{\epsilon_1 u_1}{\epsilon_2 u_2} - \frac{\rho_1 + \rho_2 + R}{\epsilon_2 u_2} I$$

This may also be written $\Phi_2 = \alpha \Phi_1 - bI$

where a and b are constants. In the special case that the two armatures are identical we have $\epsilon_1 = \epsilon_2 = \epsilon$ and

$$\Phi_2 = \Phi_1 \frac{u_1}{u_2} - \frac{\rho_1 + \rho_2 + R}{\epsilon u_2} I$$

Let $u_2 = au_1$ where a is the constant ratio of secondary to primary speed, then

$$\Phi_2 = \Phi_1 \frac{1}{a} - \frac{\rho_1 + \rho_2 + R}{\epsilon u_1 a} I$$

The numerator of the second term is the fraction pE of the primary e.m.f. lost by armature reaction and resistance in the whole circuit; the denominator is aE/Φ_1 . We thus obtain

$$\Phi_2 = \frac{\Phi_1}{a} (1 - p)$$

By selecting the speed of the motor so that it is $(1 - p)$ that of the generator we get

$$\Phi_2 = \Phi_1$$

that is to say, not only the armatures, but also the fields of the two machines may be identical and with a voltage loss of p per cent. the motor will run at a constant speed, which is p per cent. less than the constant speed of the generator.

Motors taking current from the same circuit are obviously electrically coupled. The question arises, what will be their working condition if they are also mechanically coupled? If the motors are of different type the working need not necessarily be impossible, but such a case has not much practical interest, since one would naturally use motors of the same type and size if mechanically coupled. If the coupling is by belt to a common counter shaft the motor with the smaller pulley must run faster. If the machines are series motors and series coupled that with the smaller pulley will take more than its fair share of the load, but unless the difference of speed is excessive no harm will be done. With shunt motors in series and the shunts coupled as on the right of Fig. 56 the case is the same, but with shunt motors coupled in parallel, that having the larger pulley will take more than its fair share of the load and if the ratio of difference of speed to average speed exceeds the ratio of drop to e.m.f. impressed, it will take the whole or even more than the whole load, thus making the working impossible. For safe operation of mechanically coupled motors it is therefore necessary that they should be not only electrically, but also mechanically in the same condition.

This point is of some importance in traction work. D.C. traction motors are nearly always series machines. Speed regulation, whether by resistance in shunt to the field circuit or resistance in series with the armature circuit should therefore always be so arranged as to affect all motors equally, but even if this is done there still remains the question of equal speed of all motors. They are mechanically coupled by the rails and the requirement of equal speed is naturally fulfilled if the diameter of the driving wheels is the same. Let us now see what happens if the diameter of the wheels driven by one motor is a little smaller. If the motors are coupled in series the torque of both is the same, but being exerted on a smaller diameter the tractive force is a little greater, thus increasing the attrition of the smaller wheels. This means that in series connection any initial inequality in the diameter of the driving wheels will increase. The opposite is the case when the motors work in parallel connection. The force between wheel and rail is proportional to torque divided by the diameter of the wheel. If the torque remained the same there would be a slightly greater wear on the small wheel, but the torque is even for a small increase of speed considerably reduced. It is proportional to the product of current and flux, and since the flux is a function of the current, the product of the two changes rather faster than the current. The current is due to the difference between impressed and induced voltage and as this difference is small in comparison with the induced voltage a slight change in the latter results in an appreciable change in the current. We thus find that a very slight increase of speed due to the reduced diameter of the wheel will result in an appreciable reduction of current and a still more marked reduction in the torque. The torque exerted by the motor on the small wheel is reduced to a much greater extent than the diameter so that the force between wheel and rail is sensibly lessened. The reverse is the case for the other pair of wheels. The motor runs slower, allows more current to pass and exerts appreciably more torque. This means that the larger wheels exert more tractive force and suffer more attrition. There is thus a natural tendency to reduce any initial inequality in the diameter of the driving wheels when the motors are in parallel. As shown above the tendency is the other way when the motors are in series, but as most of the running is in parallel connection the general tendency is for the wheels to wear equally.

The Thury System of Power Transmission. Transmission

of power over long distances necessitates high voltage. This may easily be obtained with an alternating current plant and the usual method is to work all units both on the primary and secondary side in parallel, the voltage being constant, whilst the current varies in accordance with the load on the secondary units. Such a system is impracticable with continuous current since at the present state of the art 3 to 4000 volts may be considered the limit of e.m.f. with which a commutator can deal. Parallel distribution only becomes commercially practicable at much higher voltage (20,000 to 100,000 according to distance and size of plant) so that if D.C. is to be used, generators and motors must be put in series. We have

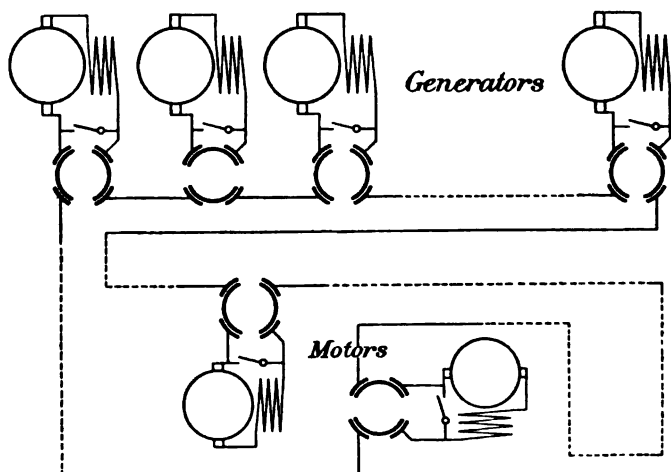


FIG. 58. The Thury system of power transmission.

then power transmission under constant current, but variable voltage. This is the basic idea of the Thury system* of which on the continent about a score of examples exist with voltages ranging up to 57,000 (Moutier-Lyons), whilst in this country there has been in operation since 1911 a plant designed by Mr Highfield. It belongs to the Metropolitan Electric Supply Company and is intended to serve an area of 300 square miles, the most remote point being 28 miles from the Generating Station at Willesden. The plant has been designed for an ultimate capacity of 10,000 kw. and 100,000 volts maximum working pressure.

* For details of the series system of power transmission see Highfield, *Journal I.E.E.*, No. 183, p. 471.

At the generating station all dynamos are placed on insulating foundations and the connection between dynamo and prime mover, generally a water turbine, is made by insulating couplings. Each generator is provided with a switch by which the machine is automatically short-circuited on itself in the event of any internal connection being broken. The object of this switch is to preserve the continuity of the circuit. There is no danger in short-circuiting a series generator on which is impressed a definite driving torque, because the current can never exceed that value which corresponds to the torque. All that happens is that the short circuited machine runs dead slow. The voltage is adjusted by putting a greater or lesser number of generators into circuit and by shifting the brushes under the action of a solenoid and servo motor relay, the solenoid being in circuit with the line current. The regulating gear thus automatically keeps the machines at that voltage which will give constant current. Fig. 58 shows diagrammatically generators, line and motors. The speed of the motors is regulated by shifting the brushes, but in this case the regulating gear is under the control of a centrifugal governor. To stop a motor the line current is by-passed round it by a four pole switch of similar pattern to that used to put a generator into or withdraw it from the circuit. In Fig. 58 the principle is shown diagrammatically. The circuit is taken all round the district to be served and wherever required a motor is inserted. This may be used to drive any kind of generator, generally a three phase alternator for supplying current to the surrounding neighbourhood. Of the four constant current generators shown in the sketch Nos. 1, 3 and the last in the series are in circuit, No. 2 is out of circuit as seen by the position of the four way switch. The two motors shown are both in circuit.

Reversal of Polarity. How the reversal of polarity may be brought about in a series generator if used to supply current to a shunt motor has already been explained. It may, however, also happen that a shunt generator reverses polarity without interference by any motor. Such a reversal may have serious consequences if the generator is used in an electrolytic plant for the manufacture of hydrogen and oxygen, and the author knows of one case where an explosion of the oxygen container was the result of a reversal of polarity in the dynamo which was not noticed at the time. Oxygen was therefore collected and by mistake put under pressure

into the hydrogen retainer. If a shunt machine is accidentally short circuited and if there is any delay in the circuit breaker acting the shunt excitation is momentarily overpowered by the back ampere-turns of the armature, which may for a moment amount to ten or twenty times their normal value. Thus the field is reversed and although the short circuit may be cleared very soon after, the machine is left with residual magnetism of reverse polarity, so that at the next start it will build up its field in the reverse sense, making what was previously a negative brush into a positive brush.

CHAPTER V

THE DYNAMO IN CONNECTION WITH A STORAGE BATTERY

The storage cell—Advantages of using a battery—Willans' law—Arrangement of dynamo and battery—Central station battery—Buffer batteries for electric traction—Boosted buffer battery—The Lancashire booster—The Highfield booster—The Entz booster.

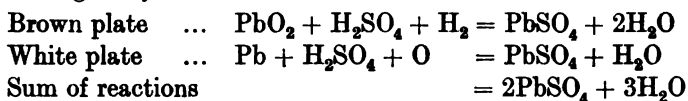
The Storage Cell. In 1859 Planté published his discovery that electricity can be stored in a galvanic cell having lead for both electrodes and dilute sulphuric acid for the electrolyte. On sending a current through such a cell the plate at which the current enters becomes coated with peroxide and that at which it leaves remains metallic and bright. If both plates are originally covered with a layer of any oxide of lead, one becomes more oxidised and the other partly reduced to the metallic condition. The former is generally called the positive and the latter the negative plate, but to avoid any misunderstanding, which might arise if the direction of the current were taken as determining the polarity, we will call the former the brown and the latter the white plate. If after passing the current for some time in at the brown and out at the white plate the cell is detached from the circuit and connected to an incandescent lamp requiring less than two volts, it is found that the lamp lights up and current keeps flowing until a very large proportion of the quantity previously sent through the cell has flown back again in the opposite direction. The first process is called charging the cell, the second discharging it. The discharge current in passing through the electrolyte from the white to the brown plate carries hydrogen to the latter and oxygen to the former. The new compounds produced must therefore be derived from the combinations:

Active material at brown plate plus sulphuric acid plus hydrogen.

Active material at white plate plus sulphuric acid plus oxygen.

The active material at the brown plate may be taken to consist mainly of peroxide, though other oxides may be also present. At the white plate we have mainly metallic lead, though also here other compounds may be present. For the purpose of representing the reactions in a simple chemical formula we need only take account of PbO_2 and Pb , but it should be noted that these materials are

in an allotropic state and have a molecular structure far more complex than the simple chemical symbols indicate*. The reaction on discharge may then be written as follows:



Thus for each molecule of water decomposed by the current there enter into the electrolyte three molecules of water. The solution becomes less dense. If the discharge is not carried too far so that a certain quantity of the active materials originally present still remains on the plates, the conductivity of the active material is only slightly lowered by the presence of the sulphate and this is probably the reason why the terminal e.m.f. falls off with advancing discharge to only a moderate extent at first; but if the discharge be carried too far or if the cell be left standing after having been discharged, a process called "sulphating" sets in accompanied by a considerable increase in electrical resistance due to an excessive amount of sulphate, also increase of bulk and bursting off of the active material and buckling of the plates. Batteries should therefore never be left standing idle after having been discharged and should not be discharged beyond a certain point which is indicated either by the density of the electrolyte, or more precisely by the terminal voltage.

To overcharge a battery with a strong current is also harmful because active material is torn off the surface of the plates. An overcharge with a weak current is generally not very harmful. The reaction consists in the decomposition of the sulphate, the higher oxidation of the brown and the reduction of the white plate. If the charge be continued with a weak current after the white plate has been completely reduced, hydrogen will be liberated. At the brown plate an excess of oxygen after the surface has been converted into peroxide cannot do much harm as any further oxidation can only proceed very slowly into the lower parts of the plate, so that the principle objection to overcharging is waste of energy. Yet an occasional and moderate overcharge is advisable as thereby any tendency to sulphating is prevented. The electrolyte is made by diluting concentrated sulphuric acid (density 1.84) with about $3\frac{1}{2}$ times its volume of pure distilled water. The density of

* For a detailed investigation of the chemical process see E. J. Wade, "Storage Battery Problems," *Journal I.E.E.*, No. 145, p. 460.

the electrolyte when the cell is fully charged and fully discharged varies somewhat with the different makes of plates, but the following figures may be taken as fair averages:

When fully charged the density is 1.215

When fully discharged the density is 1.180

The terminal e.m.f. both on charge and discharge alters as the

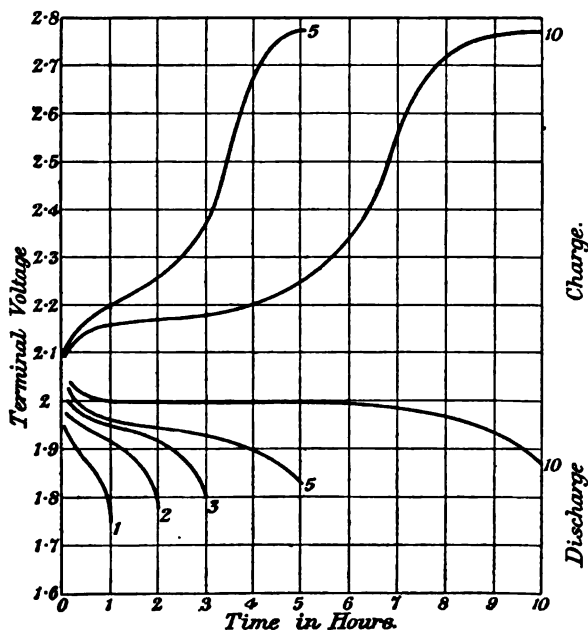


FIG. 59.

process goes on; it also alters with the rate at which electricity is sent into or taken out of the cell. When a cell is fully charged and the brown plate can no longer take up oxygen and the white plate can no longer take up hydrogen, these gases are given off in bubbles—a condition technically termed “gassing.” The bubbles burst at the surface of the liquid and carry a little spray of acid with them. To prevent the latter diffusing into the atmosphere the cell is loosely covered with a glass plate to condense the spray and let it drip back into the cell. Gassing begins at about 2.5 volts terminal pressure and becomes vigorous at 2.7. By continuing charging the e.m.f. rises still a little, but approaches a limit somewhere between 2.7 and 2.8 volts.

The rate of charge and discharge is specified by the number of hours required to complete the process. The capacity of a cell or battery is stated in ampere-hours obtainable on discharge at a given rate. The shorter the time, that is the heavier the discharge current, the lower is the capacity. Fig. 59 shows terminal voltages of a cell at various rates as a function of time. For the charge two curves are shown representing a 5 hour and a 10 hour rate; for the discharge five curves are shown representing rates of 1, 2, 3, 5 and 10 hours. The exact shape of these curves may differ slightly in different makes of plates, but the general character is that shown in the figure. After discharge at the 10 hour rate the terminal voltage should be 1.85. To test whether both plates are in good condition a thin electrode made of cadmium is dipped into the acid and the voltage taken both to the brown and the white plate. As cadmium is negative to both, the difference of the two readings should give 1.85 if both plates are in good condition. The figures are

Brown plate to cadmium	2.03
White plate to cadmium	0.18
Difference representing terminal e.m.f.	1.85

The relation between capacity and discharge rate and current may be approximately represented by Peukert's equations

$$C = C_0 \left(\frac{t}{t_0} \right)^\alpha \quad \text{and} \quad i = i_0 \left(\frac{t_0}{t} \right)^{1-\alpha}$$

where C is capacity in ampere hours, t time of discharge and i discharge current. If the capacities are known for two discharge rates we can find the value of the exponent α from

$$\alpha = \log_{10} \left(\frac{C_0}{C} \right) / \log_{10} \left(\frac{t}{t_0} \right)$$

and the capacity at any other rate can be calculated. The value of α is slightly different in different makes, but as a fair average may be taken at 0.3. This means that a battery discharged at a one hour rate has half the capacity available at the ten hour rate.

Temperature has also a slight influence on the capacity. Within such limits of temperature as occur in practical use an increase or decrease of capacity of $1\frac{1}{4}$ per cent. for every degree C. rise or fall of temperature respectively may be taken as representing this influence. The efficiency of a battery may be expressed either by the ratio between quantity of electricity discharged and charged,

or by the ratio between energy obtained on discharge to energy required for the charge. The former is called the ampere-hour-efficiency and the latter the watt-hour-efficiency. With a battery in good condition the efficiencies obtainable in practice are about 0.92 as regards quantity and 0.73 as regards energy. To obtain reliable figures in an efficiency and also in a capacity test it is advisable to carry the battery through several complete cycles of charge and discharge, taking care to have the same density in the electrolyte at the start and finish.

Various makes of storage cells differ greatly in the manner of arranging and supporting the active material, but in all cases the object is to obtain good conductivity, that is intimate cohesion of the active material with the plate itself and at the same time a large contact surface with the electrolyte. For this purpose the plate may be provided with fine and deep ribs, or pockets in the plate may be filled in with plugs of active material, or the plate may be in the form of a grid with so-called rosettes of rolled up corrugated lead strip pressed into them. The actual contact surface with the liquid is by these means made many times that of the external surface of the plate, but as it is impossible to measure the actual contact surface, the carrying capacity of a plate is generally referred to the external surface, counting of course both sides. On this basis we can speak of the current density per sq. dm. of plate and the following values are average figures of modern practice:

At the 10 hour rate	...	0.5	ampere	per	sq.	decimetre.
At the 3 hour rate	...	1.3	"	"	"	"
At the 1 hour rate	...	2.5	"	"	"	"

As regards weight in relation to output no very definite figures can be given, not only because of the difference in weight per unit surface of plate due to difference in construction, but also because a considerable part of the weight of a cell in working condition is due to the box and the liquid. The latter varies with the spacing of the plates and the former with the material of the box. Glass boxes are lighter than wooden boxes lead lined. The weight per kw. or kwh. naturally also depends on the rate of discharge and on the size of the cell. On a 3 hour rate the weight per kw. may be assumed to lie between the following limits:

50 to 100 amperes	...	500 to 700	kg.	per	kw.
300 to 500 amperes	...	400 to 600	kg.	per	kw.

These figures refer to stationary batteries. Storage cells intended as a source of energy for automobiles are of special construction and have a much higher weight efficiency. At the 5 hour rate (this rate is adopted because an electric vehicle may be considered to be actually running for 5 hours daily) a kw. output is obtained with about 250 kg. of battery, or a kwh. with 50 kg. of battery. At the one hour rate a kw. is obtained with 85 kg. of battery.

In the Edison storage cell these figures are still further reduced to 170 and 50 kg. respectively. This is a nickel iron cell the electrolyte being a strong solution of potassium hydrate with a small percentage of lithia. The "positive" active material is nickel hydroxide with fine nickel flakes to give it conductivity and the negative is oxide of iron with a trace of mercury for the same purpose. The electrolyte merely acts as a conductor and does not enter into any chemical union with the electrodes; hence the cell is not damaged by over-discharge or by standing idle. The normal discharge rate is 5 hours and the average terminal pressure 1.2 volts and the final 1.1 volts. The normal charging rate is 7 hours, when the terminal pressure rises from 1.55 to 1.84 volts. The ampere-hour efficiency is given by the maker as from 75 to 80 per cent. and the watt-hour efficiency as from 55 to 60 per cent.

The total floor space required for lead cells in the battery room may be roughly estimated from the output at a given rating. A convenient arrangement is to place the cells in four rows, one at each side wall and a double row along the middle, leaving a free space at either end for access to the passages between the rows. These passages are generally provided with a floor of wood grating to insulate the attendant from earth. The cells are placed on glass or porcelain insulators and these are placed on wooden stands. Small cells may be arranged in two tiers, but a one-tier arrangement (in any case necessary with large cells on account of weight) is preferable as it facilitates inspection from above. On a three hour rating a 5 kw. battery requires if arranged in double tier about 15 sq. m. floor space and a 10 kw. battery about 23 sq. m. Larger batteries erected in single tier require a floor space per kw. on the three hour rating as hereunder:

Power in kw.	20	50	100	150	250
sq. m. per kw.	2.5	1.9	1.3	1.0	0.8

These figures are merely approximate and considerable variations

may be caused by the necessity to use rooms of given shape and by a more or less liberal provision of gangways and free spaces to facilitate inspection or removal of cells.

Advantages of Using a Battery. The chief reasons for the use of batteries are first that the mechanical plant may be shut down altogether for prolonged periods daily without interrupting the supply and secondly that the power which has to be provided mechanically to meet a given peak load may be reduced by the amount the battery can take on. There is the further advantage that the mechanical plant is more evenly loaded and therefore works with greater efficiency. These advantages will be the more felt the greater the difference between maximum and average power and the longer the periods of minimum power. In a small electricity works for the supply of a district demanding mostly light and comparatively little mechanical power, the peak time of demand lasts only a few hours in the late afternoon. At night hardly any current is required for private lighting and none for motors, so that the current for public lighting constitutes the whole of the load. During the day the chief demand is for power current, but as this is small it will hardly suffice to load up even one engine at the central station. As lighting current begins to be required in the afternoon in winter, whilst there is no slackening off in the demand for power current, the total demand for current rapidly rises and attains a maximum some time between 5 and 8 p.m. This is called "peak time" and the maximum demand for electrical power sent out from the station is called the "peak load." The ratio between the average daily load during the whole year and the heaviest peak load observed during the year is called the annual load factor and this in a small station providing chiefly lighting current is of the order of 12 per cent. and seldom exceeds 15 per cent. A station having a peak load of 1500 kw. would send out during the year only about 1.8 million kwh. which represents a continuous load of only 200 kw. If no battery were used either three sets of 750 kw. or four sets of 500 kw. each would have to be installed, since even at peak time one set must be spare. Adopting four sets of 500 kw., one of these would be running 24 hours a day, summer and winter. In winter another set would be running about seven hours daily and the third at peak time for three hours, making 34 engine-hours daily. In summer the second set would have to run for four hours and

the third set would not be required, since the peak is much lower. We thus get 28 engine-hours in summer and the average throughout the year would be about 31 engine-hours daily.

If a 500 kw. battery is installed the peak may be taken by it so that only two engines need be running at any time. The battery can also take the very light load from 10 p.m. to 6 a.m. so that for eight hours out of the 24 the mechanical plant may be out of action. Thus the engine-hours will be reduced to little more than half, whilst all the time the engines work with a fair load and not greatly under-loaded as in the previous case. Fig. 60 shows a winter load curve of a small station supplying current mainly for street and

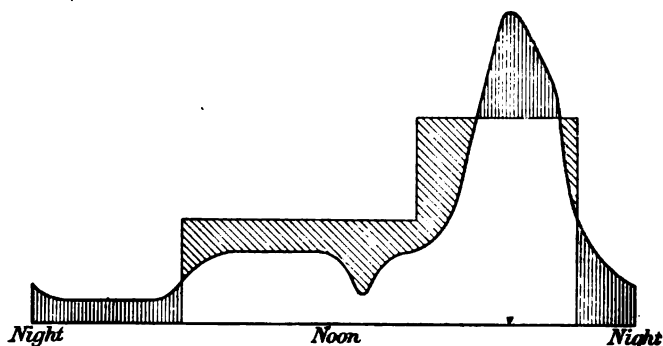


FIG. 60. Winter load curve of small central station.

private lighting and only little for power. The indentation at noon is due to the dinner hour when motors are switched off and what little light may be required during the day is reduced. The areas shaded by vertical lines represent energy given by the battery, those shaded by inclined lines available energy from the engines for charging. The two areas must be in the relation indicated by the efficiency of the battery, say 3 to 4. Of the total energy output of 1.8 million kwh. the engines would deliver directly about 1.2 million and the battery 0.6 million. With an efficiency of 70 per cent. the engines will have to supply to the battery 0.85 million so that the total output demanded from the engines comes to 250,000 kwh. units more than in the case of all the energy being supplied direct from the engines. This is 14 per cent. on 1.8 million. The cost of a battery on the three hour rating is about the same per kw. as that of the mechanical plant from the coal bunker to the dynamo terminals, so that whether a battery is used or not there will be no

material difference in capital outlay. The annual charge for upkeep, and amortisation is for a battery about 5 per cent. greater than for the mechanical plant, so that on account of this item the battery station has slightly higher fixed charges. Against this and the 14 per cent. extra energy required to be produced by the mechanical plant must be set the large saving in steam per kwh. due to the fact that no engine is worked far below its normal load; and also the saving in labour cost due to the reduction of engine hours by about half. On the whole there is a sensible balance in working expenses in favour of using a battery. This balance becomes less and less and finally vanishes or is reversed with an increase in the size of the station, because in a large station the load factor is much higher and the minimum load is large enough to enable at least one engine to work with a fair load even at slack time.

Willans' Law. The question as to how the mechanical plant should be subdivided so as to ensure economical working with a given load curve and how to determine whether a battery would be advisable can only be decided if the amount of working agent (whether steam or gas) required by the prime mover at different loads is known. The late Mr Willans has formulated a law according to which the total consumption as a function of the load may be approximately expressed by the linear equation

$$K = A + bP \quad \dots \dots \dots (46)$$

where K is the quantity of working agent (kg. of steam, kg. of coke with producer gas or cub. m. of town gas) used per hour and P is the power output of the dynamo. A and b are constants depending on the type of prime mover and its normal rating.

STEAM CONSUMPTION OF TURBINES

Rating in kw.	Revs. per min.	Full Load kg. per kw.	Half Load kg. per kw.	Constant A	Constant b
100	3,000	10.75	13.12	237	8.38
250	3,000	8.80	10.45	420	7.13
500	3,000	7.57	8.85	650	6.26
1,000	3,000	6.81	7.59	820	6.00
2,500	3,000	6.31	7.04	1,820	5.60
5,000	3,000	6.00	6.67	3,400	5.32
10,000	1,500	5.86	6.45	5,900	5.27
15,000	1,500	5.81	6.35	8,160	5.27
20,000	1,500	5.77	6.27	10,000	5.27
25,000	750	5.72	6.17	11,350	5.27

130 PRINCIPLES OF ELECTRICAL ENGINEERING

I am obliged to Mr Gerald Stoney, F.R.S. for the preceding table of steam consumption of turbines. The figures relate to a gauge pressure of 12 atm., a steam temperature of 250° C., 760 mm. barom. and 95 per cent. vacuum. With different steam conditions the consumption will of course be different, but can be determined by reference to Mr K. Baumann's paper "Recent Steam Turbine Practice" in Vol. 48 of the *Journal Inst. El. Eng.*, p. 768.

Reciprocating engines are rather more economical than turbines in small sizes, but less so in large sizes. This will be seen from the following table for *A* and *b* supplied to me by leading makers of this type of engine.

STEAM CONSUMPTION OF RECIPROCATING ENGINES
(Vacuum 87 per cent.)

Rating in kw.	Revs. p.m.	Type	Gauge, atm.	Steam Tem- perature. Degrees C.	Constant <i>A</i>	Constant <i>b</i>
50	575	Compound	11	300	35	8.20
100	525	"	11	300	68.5	7.95
250	375	"	11	300	168	7.72
500	350	"	11	300	332	7.50
500	350	Triple	12.5	300	307	6.65
750	300	"	12.5	300	454	6.55
1000	250	"	12.5	300	600	6.50
1500	200	"	12.5	300	890	6.35

In cases where non-condensing engines are installed the total power available at the works (such for instance as a colliery) may be sensibly augmented without additional expenditure of fuel by the use of a generator driven by an *exhaust steam turbine*. As in all turbines a high vacuum is also in this case essential. With 95 per cent. vacuum and an absolute steam pressure of 1.125 atm. (*i.e.* 0.125 above atmosphere) the following figures represent modern English practice.

STEAM CONSUMPTION OF EXHAUST TURBINES
(Absolute pressure 1.125 atm. Vacuum 95 per cent.)

Rating in kw.	Revs. p.m.	Constant <i>A</i>	Constant <i>b</i>
500	3000	1660	10.85
1000	3000	2720	10.5
1500	3000	3810	10.3

Arrangement of Dynamo and Battery. A great variety in the way the generators and storage cells are combined is found in practice, but whatever arrangement is adopted, it must be possible to obtain the increased voltage required for charging without at the same time raising the supply voltage above its normal and constant value. A cell requires about 2.7 volts towards the end of the charge, whilst at the end of its discharge it gives only 1.8 volts, so that between these extremes there is a difference of nearly a volt per cell. The total number of cells in a battery is found by dividing the normal voltage by 1.8 and if the battery is so arranged that the number of cells in circuit cannot be reduced the maximum voltage required towards the end of the charge is found by multiplying the number of cells by 2.7. If the number of cells can be changed by a charging and discharging switch, then the maximum voltage is less, because the end cells, being only brought into service towards the end of the discharge, are less discharged than the cells in the main body of the battery. They can therefore on charge be switched off long before the main battery is fully charged. Thus the case never arises in practice when the whole number of cells in a battery opposes a back e.m.f. of 2.7 volts per cell to the charging current simultaneously. Whilst the end cells have risen to this e.m.f. those in the main body of the battery are yet a long way short of it and only attain this back e.m.f. after the end cells have been cut out. The result is that the provision of a maximum charging voltage of 2.3 to 2.4 volts per cell is sufficient. This means that the maximum charging voltage need only exceed the normal supply voltage by about one-third, whereas if the battery is treated as one block the excess must be 50 per cent. Where charging and discharging switches are provided the end cells under their control are technically termed "milking cells" and their number is about one-quarter of the total number of cells in the battery.

Fig. 61 shows an arrangement suitable for a small country house installation. *G* is the generator driven by a gas or oil engine or any other constant speed prime mover. *D* is the discharging and *C* the charging switch and *S* is a switch by which the dynamo can be connected either to battery or to line. With the connections shown it is possible to charge the battery at the same time that it supplies current to the mains; it is also possible to supply the mains from the dynamo direct, the battery being out of circuit, or to supply the mains direct from the battery, the dynamo being out of circuit;

and finally dynamo and battery may be put into parallel at peak time. In the latter case care must be taken to put *D* and *C* on to

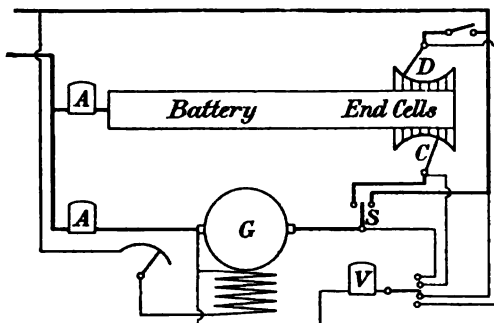


FIG. 61. Dynamo and battery for country house lighting.

the same number of cells. Unless a spare generating set is provided the battery must be large enough for the whole service, the determining factor being the week end demand. The necessity of providing ample storage in a plant of this type results in a battery of

rather more power than the dynamo and then the extra cost of making the dynamo capable of giving the increased voltage required for charging is not a very important item in comparison with the advantage of a very simple plant. The increase of cost does not extend to the prime mover because at the higher voltage towards the end of the charge the current may be smaller than its normal value and at the beginning of the charge, when the current is larger, the voltage is smaller.

Central Station Battery. The case is different in a central station, where the power output of the battery need only be about

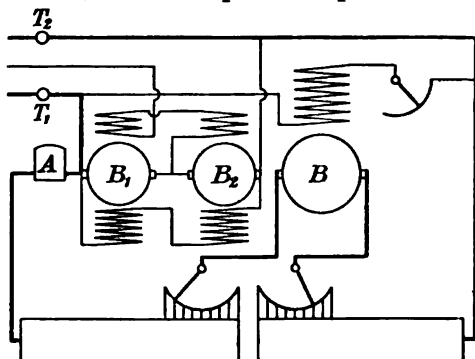


FIG. 62. Balancer, charging booster, and batteries in a three-wire central station.

equal to that of one dynamo and since three or four dynamos are required it would obviously be too expensive to make them all larger so that each may be able to produce the full charging voltage. The plan usually adopted is to instal dynamos which can give the voltage required when working direct on to the feeders and to provide

the additional voltage required for charging by a special charging dynamo or booster put in series with the battery. Fig. 62 shows

diagrammatically this system as applied in a three wire station where the same home voltage may be kept on all feeders and the division of voltage between the positive and negative sides of the system is made by a balancing set serving for all feeders (compare Vol. I, p. 77). In the Fig. B_1 and B_2 represent the balancer and B the charging booster. The battery is arranged in two parts with the milking cells in the middle. This has some advantage over a battery in one block with all the milking cells and the booster at one end. The milking cells require more attention than the cells in the main body of the battery and it is therefore convenient to place them where the potential to earth is small; also the handling of the booster is thereby rendered quite safe. The chief advantage is, however, that at times of light load when the machinery is not running, the battery may be used for balancing the two sides of

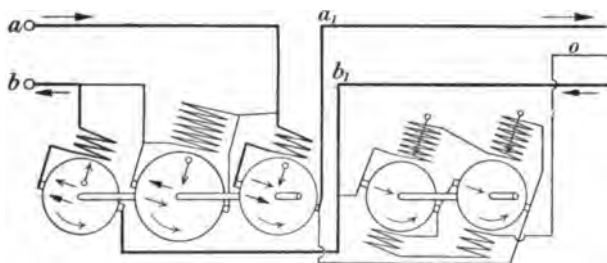


FIG. 63. Automatic booster and balancer.

the system. In order not to complicate the diagram the connections necessary for this purpose are not shown, but it will be readily understood that by disconnecting or stopping the booster and connecting the two battery charging levers to the zero wire, the battery, whilst giving what little current is required during the night to the outers, also acts as a balancer.

If the feeders have in service all approximately the same drop, then no other provision need be made than a charging booster and a balancer as shown in Fig. 62, but it may happen that the drop in some important feeders is not at all times equal to that occurring in the other feeders and then it becomes necessary to regulate the home voltage of particular feeders independently and possibly also to provide independent balancers. Fig. 63 shows how this may be done. For the sake of simplicity switches and instruments are omitted, the illustration merely showing the principle. a and b are connected to the bus bars, a_1 and b_1 represent feeder terminals;

a is boosted up to a_1 and b is boosted down to b_1 . Where the particular feeder supplies a common network of mains the potential at all the feeding boxes should be the same and this makes it necessary to boost both outers. The boosting set shown on the left consists of a motor driving two boosters, which are series machines with the fields so little saturated as to make the boost about proportional to the current. On the right is shown the balancer arranged and connected as already explained in Vol. I, p. 77. The diagram is self-explanatory so that no further description is needed.

Buffer Batteries for Electric Traction. The demand for current in small stations for electric traction is liable to very large and rapid fluctuation, the peaks, which may last seconds or minutes only, exceeding the valleys in the load curve many times. The engines, if not assisted by a battery, would work under very unfavourable conditions, much under-loaded at one moment and over-loaded at the next. By putting a battery parallel with the dynamos the load on the mechanical plant can to a large extent be smoothed out and the advantage already mentioned in connection with central stations can also be obtained in a traction power house. There is, however, this difference, that whilst in a central station the fluctuation of load takes place slowly and in a manner which can be more or less foretold, in a traction power house it is extremely rapid and irregular, so that it is quite impossible to regulate the action of the battery by hand as is done in a lighting station. A cell may either take or give current according to the terminal voltage impressed; at one particular voltage, depending on the state of charge, no current flows either way, but a slight increase produces a large charging current, and a slight decrease a large discharging current. If a battery is in parallel with generator and line the terminal voltage is the same for generator and battery. If the line demands a large current, the terminal voltage drops below the value at which the battery can just balance it and the battery discharges, thus assisting the engine; if the demand of the line falls below the balancing point, the battery takes current and thus helps to load up the engine. The battery acts as an automatic buffer. The balancing point is called the "floating voltage." It varies somewhat with the state of charge, but under practical conditions which imply the battery retaining all the time a sufficient charge, it may be taken to lie between 2.07 and 2.08 volts per cell. In order to render the study

of the working of a buffer battery simple we shall assume that the current follows Ohm's law, that is to say, that the voltage drop or rise is proportional to discharge and charge current respectively. This assumption is not correct because polarisation must have some influence, but since even at the one hour rate the total drop including it is only of the order of 5 to 10 per cent. the error is not very important and we may therefore as a rough approximation take a straight and slightly sloping line passing through the floating point as the external characteristic of the battery. The external characteristic of the shunt wound generator is also a straight line, but sloping the other way. In Fig. 64 currents are abscissae and terminal voltages ordinates. F is the floating voltage and E the

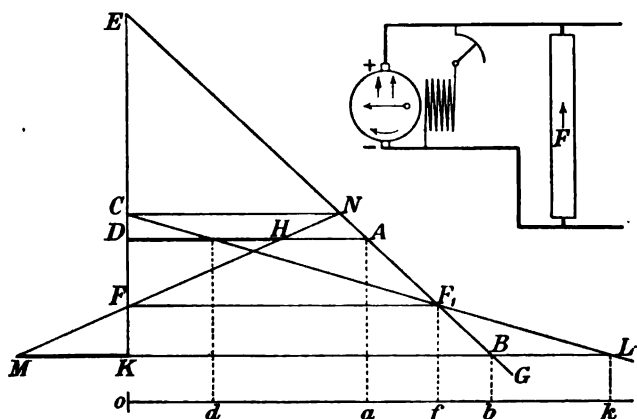


FIG. 64. Buffer battery in parallel with generator. No booster.

generator voltage on open circuit. EG is the external characteristic of the generator and MN that of the battery. If the line is switched off the whole of the generator current goes into the battery, charging it. The voltage has then its highest possible value C . If the line takes current this voltage drops and when it has come down to the floating value F no more current goes into the battery and all the generator current goes into the line. Since all ampere-volt functions are linear, two points suffice to draw the line characteristic; these are C for zero line current and F_1 for zero battery current. We thus get CL as a characteristic connecting voltage and line current. Let od represent the minimum and ok the maximum line current, to which correspond the voltages D and K ; then at minimum current the battery receives the charging current DH and at maximum

current it discharges with the current KM . If the extremes occur at regular and equal intervals the battery will not get exhausted, since in the diagram the extreme values of current have been so chosen that $MK < DH$. The generator current at minimum and maximum line current is oa and ob respectively and at the floating voltage it is of . The primary object of the buffer battery, namely the steadying of the load on the engine, is thus well attained; but the simple system here illustrated (and first used more than 20 years ago in Zurich and therefore sometimes spoken of as the "Swiss System") has two defects. In the first place the voltage variation from C down to K and even lower at exceptional peaks is rather large and secondly the assistance given by the battery to the engine is not very great. MK is small in comparison to ob . MK will automatically increase at exceptionally heavy peak loads, but then it becomes larger than DH and the battery will be gradually exhausted. This is actually what happens in practice and to make up for the exhaustion it is necessary to replenish the charge when the line is switched off during the night.

It has already been stated that the diagram, Fig. 64, is only a rough approximation of the working of this system. A battery does not behave quite in the simple manner there illustrated; its characteristic is not quite a straight line, but has a break somewhere near the floating point and the latter itself is not a fixture, but shifts about with the state of charge. Hence to correct this defect, to reduce the large voltage drop and to make the battery take up a larger share of the load without exhausting itself, some other element must be added and this is a small dynamo in series with the battery and so regulated that it assists both on charge and discharge, at the same time taking care of the battery to avoid its being exhausted. We thus get a

Boosted Buffer Battery. Of the many systems which have been devised and some of which are in practical use only a few will be here described*. Hand regulation being out of the question, the action must be automatic and as the machine must help both on charge and discharge, its polarity must be capable of reversal, hence the name "automatic reversible booster." One of the earliest forms is the simple *differential booster*, Fig. 65, where the field has two windings. One is a shunt winding S connected across the bus

* For a fuller account see R. Rankin, *Journal I.E.E.*, No. 212, p. 283.

bars and the other a main winding M traversed by a proportional part of the line current branched off from the diverter D . The direction of winding is such that the shunt coil produces a charging voltage, that is opposes the battery e.m.f., whilst the main coil produces an e.m.f. in the same direction as the battery and consequently helps the discharge. The booster is driven at constant speed and at a particular value of the line current shunt and main field are equal and no boost is produced. The battery simply floats

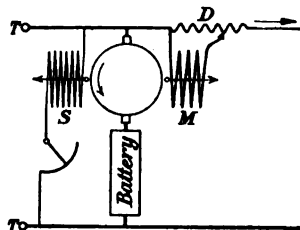


FIG. 65. Differential booster.

on the voltage of the terminals T, T' and all the machine current goes to line. An increase of line current produces a resultant field directed to the right and consequently an e.m.f. directed upwards, that is helping the discharge; a decrease of line current causes the shunt coil to overpower the main coil and gives a charging boost. If the booster is so designed that its e.m.f. equals the drop due to resistance in battery and armature either way, then with a terminal voltage equal to the floating voltage the load on the engine should be constant. In practice this is, however, not the case. The floating voltage is not constant nor is the drop proportional to the battery current for reasons already mentioned; hence the machine current does increase and decrease with the line current, though not nearly to the same extent as with a non-boosted battery. The chief defect of the simple differential booster is that it cannot adapt itself to variations in the floating voltage. This defect is overcome in

The Lancashire Booster where by the addition of a third field coil the machine voltage is rendered independent of the floating voltage of the battery. Fig. 66 illustrates this arrangement; it differs from Fig. 65 by the addition of the coil C and by the position of the diverter. The current passing through M is proportional to the machine current and not to the line current as in Fig. 65. This is an important improvement, since it is the machine current which should be regulated. The coil C is in shunt to the booster armature

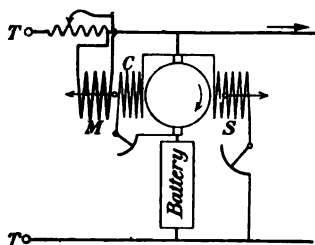


FIG. 66. Lancashire booster.

and is therefore excited by the difference between machine and battery voltage, whatever the floating voltage may be. Since the field is worked below the knee the total boost is simply proportional to the resultant ampere-turns of the three coils. Assume now that by adjustment of the diverter and the rheostat of S the e.m.f. of the booster has been brought to such a value that no current flows through it. In this case the fluxes of M and S are equal. If the floating voltage happens to be the same as the terminal voltage, C will give no flux and the booster armature will give no e.m.f. If the floating voltage is lower than the terminal voltage the main and shunt coils still balance, but the C coil produces a flux from right to left and the armature an e.m.f. directed upwards and this must in all cases be equal to the deficiency of floating voltage as compared with the terminal voltage. The battery is thus able to float in whatever state of charge it may be at the time. If now the line current increases, coil M preponderates over S and an additional upward e.m.f. is produced, causing the battery to discharge; if the line current diminishes S preponderates over M and the battery charges. It should be noted that when charging C acts with S , and when discharging it acts with M so that in both cases it assists. Since in all conditions of working two coils act against each other the amount of copper on the field is rather large in comparison with the exciting force actually required; but this is a drawback unavoidable in all differential boosters. In

The Highfield Booster only two field windings are used, but a separate little dynamo acting as an exciter to the booster is added. In Fig. 67 e is the exciter which opposes a constant e.m.f. to the e.m.f. of the battery. At a certain average line current the main field and the field produced by the C coil have such a resultant that the algebraic sum of machine voltage, boost and floating voltage is zero. Line and machine current are then equal. If the line current diminishes the diverter

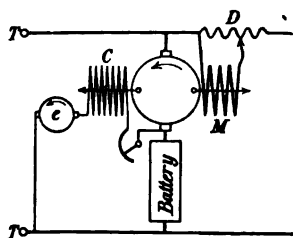


FIG. 67. Highfield booster.

field M is weakened and the field of C strengthened, producing a charging current. The opposite action takes place if the line current increases. An advantage of this system is that the line e.m.f. is kept up even if the machines are out of action. The tram service may

thus be kept up in the early morning hours and late at night when the traffic is so light that it would not pay to keep the engines running. In all types of diverter-booster the action must be somewhat sluggish though less so if the diverter is itself inductive and has the same time constant as the booster field. It is also advisable to laminate the field system and where the fluctuations of load are very great to fit a flywheel to the booster to prevent drop of speed of the motor which drives the booster. One of the difficulties to be guarded against in any type of booster is a tendency to instability, especially if the main generators are compounded or over-compounded and for this reason it is preferable to use plain shunt excited generators. In all the boosters above described the regulation of boost is brought about by the differential action of two or more exciting windings and no mechanical device is used. The absence of any mechanical gear is an advantage, but where the load is rapidly changing there is the disadvantage of sluggish action. Time is required for the change in the magnetic condition of the booster and it may happen that the regulation takes place when the need for it has passed. There are also systems in practical use in which the line or machine current acting through some kind of electro-mechanical relay produces the change in booster excitation necessary to give the required amount of positive or negative boost. As a representative example of this principle we may take

The Entz Booster. A diagrammatic sketch of this is shown in Fig. 68. The field of the booster *B* consists of a single winding which receives current from a little exciter *e*. Booster and exciter are mounted together with the motor which drives them on the same shaft and are always running at about constant speed. The field of the exciter is regulated by the intervention of a split carbon resistance, consisting of two piles of carbon blocks *C, C* through which current derived from the battery is always flowing. The field is

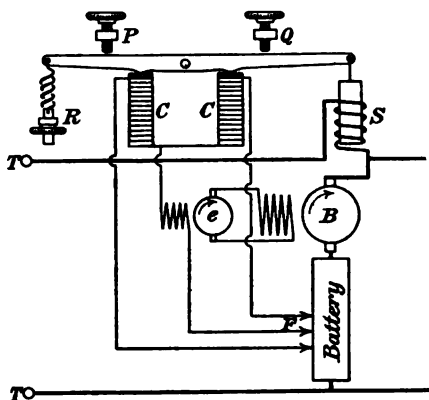


FIG. 68. Entz booster.

inserted midway between the two carbon rheostats and midway between the points of connection with the battery. These battery connections shown at F are varied from time to time, all three tapping points preserving however their mutual relation. The object of this arrangement is to avoid unequal treatment of any particular group of cells. The resistance of a pile of carbons decreases with increase of mechanical pressure. Both piles are under the action of a lever which is loaded on the right by the force exerted by the line current on the plunger of a solenoid and on the left by a spring, the tension of which is adjustable. It is thus possible to set the rheostat so that at a particular value of the line current there is balance between S and R and the resistance of the left and right carbon pile is the same. Under this condition no current flows through the field winding of the exciter and no e.m.f. is produced in the armature of the booster. Now suppose that the line current increases; part of the increase falls on the machine and consequently the downward pull of the plunger becomes greater than the amount previously balanced by the spring. The right pile becomes therefore compressed and on the left the pressure is reduced. The effect is an increase of resistance on the left and a reduction on the right, thus producing a current from left to right in the field coil of the exciter. With clockwise rotation the lower brush of e becomes positive and the corresponding current produces a flux in the field system of the booster from right to left. With clockwise rotation this means that a boost e.m.f. directed upwards is produced and thus the battery is caused to discharge. This reduces the current in S again and thus the machine is almost completely relieved of the excess current which the line demands at the time. If the line current falls off, the spring overpowers the solenoid, the carbon pile on the left is compressed and the field of e is produced from right to left with the result that the boost e.m.f. in B is now downwards, that is, assisting the machine in charging the battery. Adjustable stops P and Q are provided to protect the battery from over-discharge and over-charge respectively. They only come into action under exceptional circumstances and the general economy of working is not affected by the fact that under such circumstances the engine is heavily overloaded or only very lightly loaded; whilst the protection of the battery from damage consequent on an excessive discharge is of paramount importance. It should be noted that the quickness of action of any

booster regulated on the relay principle, as the one here described is, must be greater than that of any diverter booster, because the time constant of the exciter and booster does hardly affect the working. Even with a slight change of balance in the lever there is more boost produced than actually required; in other words there is for an instant over-regulation and this swamps the effect of the time constant, so that the balance is very quickly restored to a point where the machine current is not very different from the value to which it has been adjusted by the spring R so as to allow the battery to float. If it should be desired to charge up the battery all that is required is to turn the wheel of R so as to tighten the spring.

The various systems of booster buffer batteries here described in connection with the supply of traction current are equally applicable in other cases where the load is intermittent or fluctuating such as in rolling mills, winders, cranes, etc.

CHAPTER VI

EXPERIMENTAL WORK ON CONTINUOUS CURRENT DYNAMOS

Measurement of temperature rise—Determination of the magnetic flux and of its distribution—Separation of losses by electrical tests—Separation of losses by running down test—The determination of the moment of inertia—Other purely mechanical methods for the determination of the moment of inertia—Electrical method for determining the moment of inertia—Efficiency tests.

Measurement of Temperature Rise. Certain limits as to temperature are generally included in specifications and it is therefore necessary to make appropriate measurements. For such measurements either the thermometric or the resistance method can be used or, as an alternative for finding "hot spots," thermo-couples. The Engineering Standards Committee prescribes the two first methods; the thermometric for armatures, series main field and interpole windings and the resistance method for shunt windings. The permissible temperature rise depends on the material used for insulation. For cotton, paper and silk, all impregnated, it is 55° C. in field coils and 50° C. in armature windings*.

Whatever the method used, the temperature is first taken "cold" or at the temperature of the ambient air before the machine is set to work and at definite time intervals during working. By plotting temperatures against time we get a graph. The graph shows that after a certain time (of the order of 6 hours) the curve approaches a horizontal asymptote, the ordinate of which is the final working temperature. If the temperature is taken by thermometer this must be placed in as good contact as possible with the part to be measured and it must be shielded from external influences by surrounding the bulb with cotton wool or other thermic insulation. To measure the temperature of the armature the machine must, of course, be stopped and the temperature reading should not be taken immediately, but after such time when no further rise is observed. Whilst working, the outer surface of the armature is fanned and sensibly cooler than the interior. The internal heat requires time to filter out and hence a reading taken immediately

* See the Committee's publication, *British Standardisation Rules for Electrical Machinery*. Crosby, Lockwood and Son, London.

after stopping would be too low. When measuring the temperature of field coils the thermometer may be permanently fixed. Any thermometric measurement applied to the outside of the body can obviously only give the temperature of the outside surface and since there must be a temperature gradient to bring the heat out, the thermometer gives always too low a reading.

The average internal temperature of a winding can be more accurately deduced from a resistance test, whilst for the discovery of so called "hot spots" thermo-couples must be used, as explained in Vol. I, p. 12. When internal thermo-couples have not been provided during construction a direct hot spot measurement is impossible, but it may be assumed that any hot spot temperature will not exceed the "observable" temperature (by thermometer or resistance method) by more than 10° C.

The application of the resistance method to shunt exciting coils is very easy; all that is required is a reliable amperemeter in the exciting circuit and a reliable voltmeter connected across its terminals. A continuous record of current and potential difference can then be kept without stopping the machine. To measure the resistance of the armature the machine must be stopped. The measurement may be by potentiometer (see Vol. I, Fig. 30, where *X* represents the armature) or by sending a known current through the armature and measuring the P.D. over two commutator segments 180 electrical degrees apart. It is essential to exclude the brushes from this measurement as the uncertainty in the resistance of brush contacts would make the test very inaccurate. The brushes may, however, be used to send the known current through the armature.

The potential difference over the commutator may be measured by using metal points held on to the segments or by insulating a positive and negative brush from the others and connecting the voltmeter to these "pilot" brushes. Since only very small potential differences are to be measured the insulation of the brush from its holder can be obtained by wrapping a sheet of paper round the brush before inserting it.

If *E* is the voltmeter reading and *I* the current passing through the armature the resistance of the armature winding is

$$R = \frac{E}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Determination of the Magnetic Flux and of its Distribution. If the machine can be run idle as a generator, the total

flux may be found from observation of brush voltage, speed and excitation. The voltmeter may be connected to the brushes since there is no appreciable drop with the small current required by the voltmeter. If the machine can only be run as an idle motor the same method may be used, but as in this case some current must pass through the brushes to provide the small amount of power necessary for idle running, there is some drop at the brushes, and to eliminate the error which this would introduce the voltmeter must be connected to brushes which carry no current. Either a pair of very small separate pilot brushes may be used, or two of the machine brushes may be insulated from the others and connected to the voltmeter. Retaining the previous notation, let n be the number of exciting turns in one magnetic circuit, i the observed exciting current, and E the observed P.D. at the brushes, then we have by (2) the flux in megalines

$$\Phi = 100 \frac{a E}{p z u} \quad \dots \dots \dots (48)$$

whilst the excitation in ampere-turns is

$$X = ni$$

We thus obtain the characteristic X, Φ of magnetisation. If the winding data of the machine are not known the test gives the relation between exciting current, speed and e.m.f. To get the internal characteristic as a function of the exciting current the readings must be reduced to the same speed. The curve obtained with an increasing magnetising current lies generally slightly below that obtained with a decreasing magnetising current. This is due to hysteresis.

If it is not possible to run the machine, the characteristic of magnetisation may be obtained by flux meter as shown in Fig. 69. W represents the field magnet winding, C the commutator, P a potential slide, S a reversing switch and F the flux meter, which must be heavily shunted as explained in Vol. I, p. 261. The wires leading to the flux meter shunt are attached to two neighbouring sections of the commutator and by a preliminary trial the armature is put into the position which gives maximum deflection when the field is reversed. The whole of the armature flux then goes through the coil connected to the two commutator segments. The ampere-meter A must be a central zero instrument. The exciting current is regulated by the potential slide and the series resistance for fine

adjustment and reversed whilst a reading is taken on the flux meter. In this manner the relation between excitation and flux can be found step by step and the characteristic plotted.

The distribution of flux round the armature may be obtained by the use of two pilot brushes set a small distance apart, generally not much more than the width of a commutator segment. These pilot brushes are attached to a graduated circular scale so that the angular position for any setting may be noted. The e.m.f. between them is proportional to the air gap induction and is therefore zero in the axis of the main brushes and a maximum in any polar axis.

When the machine is rotating at normal speed the graph connecting angular position and e.m.f. observed on a voltmeter connected to the pilot brushes gives the distribution of the flux round the armature. It is advisable to shunt the voltmeter by a condenser of large capacity so as to bridge the gaps in e.m.f. supplied to the voltmeter at the time that a pilot brush is resting on the insulation separating the segments.

The pilot brush must not be wider than the insulation as otherwise, when under the pole, it would bridge two live segments and there would be danger of flashing over. For very accurate work the voltmeter may be replaced by a potentiometric arrangement.

To test flux distribution when the machine cannot be run, a flux meter may be used. A test coil is fastened to the armature and brought successively into different parts of the field and the excitation is reversed by the arrangement shown in Fig. 69. The test coil is in the form of a narrow rectangle, its length being that of the armature core and its width not less than a tooth pitch. It is held between strips of stout paper and fastened to the outside of the armature so that the longitudinal wires of the coil lie midway between two teeth. Readings are taken for successive angular positions. To determine these positions the commutator segments may be used. The area of the curve showing air gap induction as a function of angular position is obviously proportional to the total

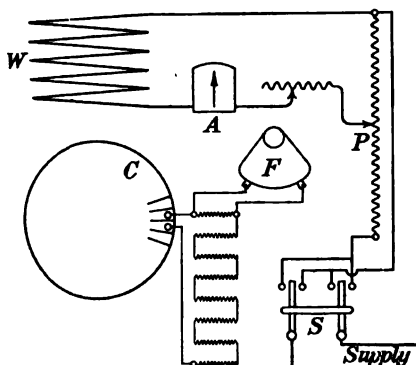


FIG. 69.

flux and if this has been found by one of the tests above explained, the scale of this curve can be determined.

Separation of Losses by Electrical Tests. The losses occurring in the working of a machine are mechanical, magnetic and electrical. The mechanical losses are friction in bearings and between brushes and commutator and ventilating or windage losses. The magnetic losses are due to hysteresis in the armature iron and in the pole shoes, and the electrical losses are due to ohmic resistance and eddy currents in the conductors, and eddy currents in the armature and pole face iron. When the machine is running idle the ohmic losses are negligible, so that we are only concerned with friction and windage as mechanical losses, hysteresis as magnetic losses and eddies as electrical losses. The latter are with a given flux obviously proportional to the square of the speed, whilst the hysteretic loss is proportional to the speed. The loss due to brush friction is proportional to the speed, whilst journal friction loss increases a little faster than proportional to speed and windage a good deal faster. The mechanical losses are, of course, independent of excitation; hysteretic and eddy current losses increase with increased excitation, the latter with the square of the total flux.

The relations are too complicated for any other than experimental determination and this is most easily made by running

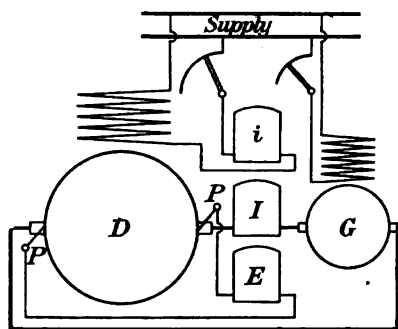


FIG. 70.

the machine light as a motor. Fig. 70 shows a convenient arrangement of apparatus for this purpose. D is the dynamo, the losses in which, when running idle, are to be analysed. G is a small generator driven by any kind of prime mover at constant speed. The voltage of G must be the full voltage of D , but the current need only be sufficient to supply the losses in

G , or say, not more than 8 per cent. of the full load current of D . P, P are the insulated pilot brushes. Let E be the induced voltage in D (reading of the voltmeter corrected for armature resistance), i the exciting current and I the main current. Then Ei is the power supplied to the armature under test. The voltage supplied to D can

be adjusted by the rheostat in the field circuit of G and the excitation of D can be adjusted by the second rheostat. The armature D may therefore be run at any desired speed and voltage. In making a test the field of D is adjusted to a certain value and kept constant, whilst the voltage supplied to D is varied. By noting speed, main current and induced voltage a graph may be drawn showing the power lost as a function of the speed for any given strength of flux. The same test is made for different excitations and thus a series of curves may be obtained. It is convenient to plot not power against speed, but energy lost per revolution against speed, because

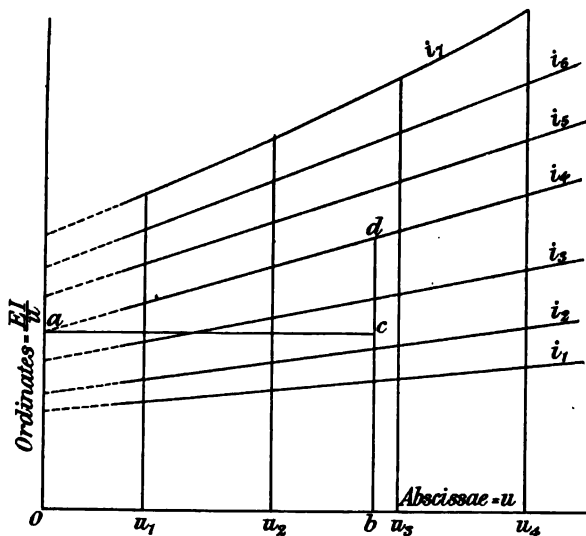


FIG. 71.

then the curves become nearly straight lines and lie clear of each other. Fig. 71 shows such curves for seven different values of exciting current marked i_1, i_2 , etc. The ordinates represent joules per revolution. The hysteretic loss of power is for any given excitation obviously proportional to the frequency and therefore to the speed. The torque to overcome hysteretic loss is therefore constant, that is, independent of speed. So would be the frictional torque if the whole of it were due to friction between solids. The greater part of it, namely that between brushes and commutator, is indeed solid friction, but the journal floating on a thin oil film may be considered to have liquid friction, the torque due to which increases slightly

with the speed and increases also at very low speed. Journal friction is, however, so small that it hardly influences the shape of the curves. The power wasted in eddy currents increases with the square of the speed and the corresponding torque is therefore proportional to the speed. Approximately the same holds good for windage torque so that for all these losses taken together the graph of joules per revolution plotted against speed is nearly a straight line. Since $2\pi uT = EI$ we have for the torque

$$T = \frac{1}{2\pi} \frac{EI}{u}$$

in electrical measure; that is to say, the ordinates of the lines in Fig. 71 measured with an appropriate scale give the torque required to overcome all the losses at the different speeds. Since at zero

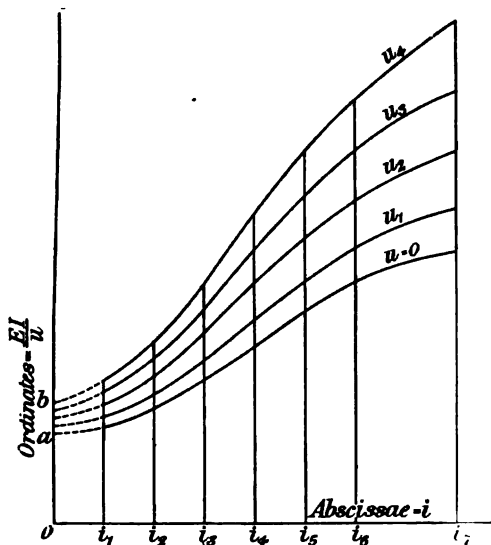


FIG. 72.

speed there can be no windage and no eddy current loss, the ordinates for zero speed give the torque for friction and hysteresis at the different excitations. These points can not be directly determined, but as the lines for moderate speeds are straight, we can find the points by prolonging the lines backwards, as shown dotted, to the axis of ordinates. Thus for excitation with i_4 amperes the power lost at speed $u = Ob$ is $u \times Oa$ on account of mechanical friction and hysteresis and $u \times cd$ on account of eddy currents and windage.

The test here described effects some separation of losses, but not quite cleanly, for with hysteresis there is still mixed up some friction and with eddies there is mixed up windage. To separate friction from hysteresis and windage from eddy current loss proceed as follows. Draw in Fig. 71 ordinates for definite speeds such as u_1, u_2 , etc., and draw as in Fig. 72 a new graph connecting exciting current and joules per revolution for each of the selected speeds. By extrapolating backwards, as shown in dotted lines, we get joules per revolution for the non-excited machine. Since in a non-excited machine there can be neither hysteretic nor eddy current losses the ordinates at the origin represent the torque due to purely mechanical losses, namely friction and windage. The influence of windage and of the liquid character of journal friction is shown by the increased torque at higher speeds. At a very small speed we have no windage and only friction and this is represented for the curve $u = 0$ by the ordinate Oa , whilst ab represents windage loss and the small increase of journal friction at full speed u_4 . From the two sets of curves it is therefore possible to determine separately all the different losses.

Separation of Losses by Running Down Test. In the test above described the readings are taken whilst power is supplied electrically so as to keep the machine running steadily at any speed. In the test about to be described the machine is run up to full speed or preferably more and then the supply of electric power is cut off and the machine is allowed to run down until it stops; readings of time and speed are taken during the period of "running down." In this case the energy to cover losses is supplied by the inertia of the rotating mass. Let Θ be the moment of inertia expressed in mass units of 9.81 kg. and metres squared, then the power given out by the retardation of $\frac{d\omega}{dt}$ at the angular speed ω is $\Theta\omega \frac{d\omega}{dt}$ in kg. m. per second. To get the power in watts we must multiply by 9.81, or

$$P = - 9.81 \Theta \omega \frac{d\omega}{dt}$$

the negative sign indicating that the angular speed diminishes as the time increases. Introducing number of revolutions per second instead of angular speed, the equation becomes

$$P = - 9.81 \Theta (2\pi)^2 u \frac{du}{dt} \quad . \quad . \quad . \quad (49)$$

This may also be written

$$P = -387 \Theta u \frac{du}{dt} \text{ watts.} \quad (50)$$

If speed is plotted as a function of time we obtain the so-called running down curve and this may be taken with the machine in different conditions; excited and unexcited, with some brushes lifted or all brushes lifted. To each condition corresponds a separate curve and if the moment of inertia of the armature is known we may from these curves determine the amount of power lost at any definite speed when the machine is in the condition represented by that particular curve. Thus hysteric loss at different excitations may be separated out from the other losses, the loss caused by brush friction may be separately determined and the loss by windage may be found by taking different points on the same curve. All this is

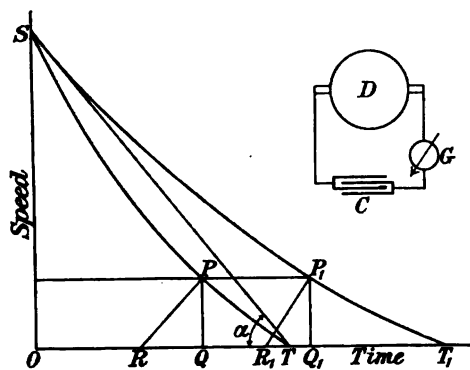


FIG. 73.

so simple that no detail description is necessary. There are, however, difficulties. With a non- (or weakly) excited machine the time of running down is generally sufficiently long for securing enough observations to plot the curve correctly, but when the machine is excited the process of running down is rather too quick for

accurate observation, hence the running down test is mainly of use in the determination of friction and windage. It may be used for the measurement of hysteric loss if the machine is fitted with a heavy flywheel so as to make the time long enough for accurate observation. Another difficulty is the evaluation of the graph in concrete figures. The line SPT in Fig. 73 is a typical running down curve. To find the power absorbed by all losses at a certain speed such as QP we must lay a tangent to the curve in P and multiply its geometrical value with the speed so as to get $u \frac{du}{dt}$ in the above formula. This is obviously the subtangent to P , that is the length QR . The power wasted is therefore represented by the subtangent corresponding to any point P on the running down curve. There is not much difficulty in

drawing a tangent to the curve correctly near either end, but at intermediate points this is not so easy and even a slight error in the inclination makes a considerable difference in the length of the subtangent. For this reason it is better not to attempt an evaluation of du/dt from the curve (except next to either end) but determine this ratio directly by the following expedient. Connect as shown in Fig. 73 a galvanometer G and condenser C to the brushes and observe the deflection as a function of the time, another observer noting speed as a function of time. For this test the machine should be weakly excited, so as to avoid its running down too quickly, but at the same time sufficiently excited to make sure that the e.m.f. at the brushes shall be accurately proportional to the speed. Since the discharging current of the condenser is proportional to de/dt and with constant excitation e is proportional to u , the discharging current is proportional to du/dt and since the deflection of the galvanometer is proportional to the current, we find that the deflection is an accurate measure of du/dt ; or in symbols

$$-\frac{du}{dt} = k\Delta$$

where k is a constant the value of which can be found by comparing the deflections Δ at time zero and time OT with the value of the tangents drawn for these points. A still more accurate method for finding the scale to represent deflection and rate of decrease of speed and also of finding the ratio of the speed at any time to the speed at which the test was started is as follows. Let in Fig. 74 the line adb represent

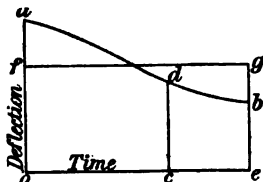


FIG. 74.

$$\Delta = f(t) = -\frac{1}{k} \frac{du}{dt}$$

then $du = -k\Delta dt$ and $\int_0^t du = -k \int_0^t \Delta dt$

Let u_0 be the observed speed at the time that the observation begins and u the speed at time $t = oc$ then

$$u_0 - u = k \times \text{area } oadc$$

and for zero speed at time oe we have

$$u_0 = k \times \text{area } oabe$$

combining we get

$$u = u_0 \frac{\text{area } cdb e}{\text{area } oabe}$$

The initial speed u_0 can be determined quite accurately and so can the time of running down, for the armature stops suddenly. By the use of a planimeter it is therefore possible to determine the time-speed curve with great accuracy from the galvanometer readings and the initial speed. To find the scale on which the ordinates of the adb curve represent deceleration we reason as follows. If the time-speed curve in Fig. 73 were a straight line ST , the time-deceleration curve in Fig. 74 would be a horizontal straight line whose ordinate is $-\tan \alpha/k$. In this case the initial speed is represented by the area of the rectangle $ofge$. If then after finding the area of the time-deceleration curve by planimeter we determine the height of of a rectangle of equal area, we get the numerical relation between deceleration and deflection

$$\frac{du}{dt} = \frac{\tan \alpha}{of} \Delta$$

The Determination of the Moment of Inertia. Since by this method both u and du/dt can be accurately found for various conditions of the machine and for any speed, it is possible to use equation (49) for the separate determination of the losses, but for this purpose the moment of inertia must be known. If this is not the case we must make two tests, one with the machine in normal condition and the other with some element added which will influence the deceleration in some well-defined and calculable manner. This added element may either be an additional flywheel of known moment of inertia or a brake having a constant resisting torque. A convenient form of flywheel is a plain disc cut out of boiler plate. If D is the diameter in m. and W the weight of such a disc in kg., the moment of inertia is

$$\Theta_1 = \frac{WD^3}{78.48}$$

On account of the smooth surface of the disc the windage is not sensibly increased, nor is the small weight sufficient to sensibly augment journal losses, so that whether the disc is fitted or not the power wasted in losses remains the same. If then we make a running down test with the disc fitted we get in Fig. 73 the curve SP_1T_1 and for any speed such as $QP = Q_1P_1$, we have the same loss, namely $C\Theta S = C(\Theta + \Theta_1)S_1$ where C is a constant which need not be known and S and S_1 are the subtangents corresponding to the points P and P_1 lying at equal height on the two curves. It

has been shown above how these may be accurately found. We thus get

$$\Theta = \Theta_1 \frac{S_1}{S - S_1} \quad \text{and} \quad \Theta + \Theta_1 = \Theta_1 \frac{S}{S - S_1}$$

Another method for finding the moment of inertia consists in applying a brake to the shaft which shall resist with a constant torque. A convenient form is a composite band laid over a small pulley and weighted on both sides. The band is a leather belt to which is joined a thin strip of metal. The strip of metal receives the larger of the two weights and the band is laid on in such way that the rotation shall tend to lift the larger weight. The higher the weight is lifted the smaller becomes the arc spanned by the leather and the larger that spanned by the metal. Since the friction of the leather belt is the greater the system has a definite point of equilibrium when the resisting torque of the brake is equal to the product of the radius of the pulley and the difference of the two weights. Expressing the radius r in metres and the difference of weights W in kg. the power absorbed in watts is $9.81 \times 2\pi ruW$. Let P represent the power lost in the machine when running down with the brake band off and Δ the deflection of the galvanometer of Fig. 73 at a definite speed u ; and let Δ_1 be the deflection at the same speed with the brake applied, then

$$9.81 \Theta (2\pi)^2 uk\Delta = P$$

$$9.81 \Theta (2\pi)^2 uk\Delta_1 = P + 9.81 \times 2\pi ruW$$

$$9.81 \Theta (2\pi)^2 uk (\Delta_1 - \Delta) = 9.81 \times 2\pi ruW$$

$$\Theta (2\pi) k (\Delta_1 - \Delta) = rW$$

$$\Theta = \frac{rW}{2\pi k (\Delta_1 - \Delta)}$$

The disadvantage of this method is that except in cases where the moment of inertia is very large, the running down time is rather too short for accurate observations; also the band brake sometimes is unsteady and it may be necessary to fit a dash pot.

Other Purely Mechanical Methods for the Determination of the Moment of Inertia. If an armature weighing W kg. is suspended in the line of its axis by at least two wires of length l and distant from the axis a ; and is set oscillating round the axis, then the periodic time is by a well-known law of mechanics

$$T = 2\pi \sqrt{\frac{\Theta l}{W a^2}}$$

where $\Theta = \Sigma mr^2$, m being, as before, expressed in mass units of 9.81 kg. and r in metres. If n is the number of complete (to and fro) oscillations per minute, the moment of inertia is

$$\Theta = W \frac{91a^2}{n^2l} \dots \dots \dots (51)$$

Where swinging in a bifilar or trifilar suspension is not practicable

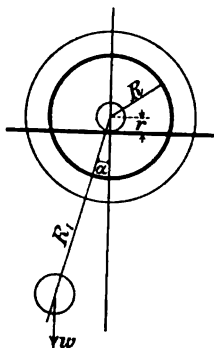


FIG. 75.

the moment of inertia may be determined by rolling the armature to and fro on two straight edges which take the place of the bearings in supporting the axle. The motion is produced by a light bar clamped to the axle and weighted at one end. The horizontal line in Fig. 75 represents the straight edges, w is a weight of w kg. attached at the end of a very light bar at a distance of R_1 metres from the axis and R is the radius of a ring in which we may imagine the whole weight of W kg. of the armature concentrated, so that its moment of inertia is

$$\Theta = \frac{W}{g} R^2$$

By deflecting w a rolling swing is produced and from the time of swing the moment of inertia may be calculated. The torque on the system exerted by gravity is $wR_1 \sin \alpha$. W has both linear and angular acceleration and w has also both, but the angular acceleration is so small as compared to the linear that we may neglect it. Let $M = W/g$ be the mass supposed concentrated in the circle shown in a thick line. If v is the linear speed and dv/dt the linear acceleration the horizontal force is $M dv/dt$ and the corresponding torque is $rM dv/dt$, r being in metres. Let ω be the angular speed at the moment, or $\omega = d\alpha/dt$, then $v = r d\alpha/dt$ and $dv/dt = r \frac{d^2\alpha}{dt^2}$. The torque on account of linear acceleration of mass M is therefore

$$r^2 M \frac{d^2\alpha}{dt^2}$$

The peripheral speed due to rotation is ωR and the circumferential force due to the acceleration $R d\omega/dt$ is $MR \frac{d^2\alpha}{dt^2}$, so that the corresponding torque is $R^2 M \frac{d^2\alpha}{dt^2}$. The small weight w moves with the

linear velocity $\omega (R_1 - r)$ m./sec., and the torque is $m (R_1 - r)^2 \frac{d^2\alpha}{dt^2}$, where $m = w/g$.

Since the angle α is very small we may write α instead of $\sin \alpha$. The differential equation of the rolling motion is then

$$mgaR_1 + \frac{d^2\alpha}{dt^2} \{M(R^2 + r^2) + m(R_1 - r)^2\} = 0$$

$$ga + \frac{d^2\alpha}{dt^2} \left(\frac{M(R^2 + r^2) + m(R_1 - r)^2}{mR_1} \right) = 0$$

The expression within the bracket has the dimension of a length. Calling this l we may write

$$ga + l \frac{d^2\alpha}{dt^2} = 0$$

This is the well known differential equation of the motion of a pendulum of length l whose angular deflection is so small that the sine may be assumed to have the same value as the angle itself. Since by the pendulum equation the periodic time is

$$T = 2\pi \sqrt{\frac{l}{g}}$$

and $l = \left(\frac{T}{2\pi}\right)^2 g$ we may write

$$\left(\frac{T}{2\pi}\right)^2 g = \frac{MR^2 + Mr^2 + m(R_1 - r)^2}{mR_1}$$

and find

$$MR^2 = \Theta = wR_1 \left(\frac{T}{2\pi}\right)^2 - \frac{W}{g} r^2 - \frac{w}{g} (R_1 - r)^2$$

These purely mechanical methods for finding the moment of inertia are hardly practicable with machines of large size. For large machines a purely electrical method may be used*. It has the advantage that no special instruments are needed and that it may be applied to the machine *in situ* without any alteration in its electrical or mechanical connections, so that the result applies to the machine in working condition. This is of special importance for motors used for driving rolling mills or other appliances having a widely variable load and where a flywheel is fitted to increase the mechanical output at peak load. Machines of this type are specified to give a certain flywheel effect and the test here described has been devised

* Kapp: "Experimental determination of the moment of inertia of a continuous current armature," *Journal Inst. E.E.*, 1909, p. 248.

for the purpose of finding the flywheel effect of the motor in its working condition.

Electrical Method for Determining the Moment of Inertia. Let the motor be running idle under a constant impressed voltage and let its excitation be also kept constant. There will be dynamic equilibrium between the counter e.m.f. and the impressed e.m.f. Let the latter be measured by pilot brushes (two of the main brushes insulated by paper will serve) so as to exclude brush drop and let the observed value at speed u be E . If at the same time we observe the current I in the armature the back e.m.f. is

$$e = E - \rho I$$

ρ being the armature resistance. Using the previous notation we have also

$$e = \frac{p}{a} \frac{z}{100} \Phi u \quad \text{or} \quad e = \epsilon \Phi u$$

and since the exciting current is kept constant there must be strict proportionality between back e.m.f. and speed. We may therefore dispense with a speed counter or speed indicator and take only voltmeter readings E , but correct them to e for resistance drop. If the impressed voltage be increased the current will increase and the motor will speed up, finally settling down to a new condition of dynamic equilibrium when I and e have assumed larger values. In the same way, if the motor is running at full speed and we suddenly reduce the impressed voltage, the motor will slow down and finally come to a steady condition at a lower speed. In the transition from slow to fast and back again from fast to slow, there is one particular speed for which the back e.m.f. is the same both on the ascent and the descent, and since the losses with constant excitation must be the same in both cases we have the two equations:

Ascending: watts input = watts lost + watts spent in acceleration.

Descending: watts input = watts lost - watts given out in deceleration.

Let the indices 1 and 2 stand for ascending and descending run respectively and L represent the loss. The above equations may now be written

$$P_1 = L + \text{watts spent in acceleration.}$$

$$P_2 = L - \text{watts recovered in deceleration.}$$

Putting in (50) $u = \frac{e}{\Phi\epsilon}$ and $P_1 = e_1 I_1$ and $P_2 = e_2 I_2$ we have

$$e_1 I_1 = L + \frac{387\Theta e_1}{(\Phi\epsilon)^2} \frac{de_1}{dt}$$

$$e_2 I_2 = L - \frac{387\Theta e_2}{(\Phi\epsilon)^2} \frac{de_2}{dt}$$

Since these equations refer to the same speed, $e_1 = e_2$ and we find by subtraction

$$I_1 - I_2 = 387 \frac{\Theta}{(\Phi\epsilon)^2} (\tan \alpha + \tan \delta)$$

$\tan \alpha$ and $\tan \delta$ being irrespective of sign the differential quotients de/dt for the accelerating and decelerating conditions. The value of $\Phi\epsilon = \frac{e}{u}$ may be found from a steady run at the excitation chosen.

Introducing this we get

$$\Theta = \frac{e^2}{u^2} \frac{1}{387} \frac{I_1 - I_2}{\tan \alpha + \tan \delta} \quad \dots \quad (52)$$

in $\frac{kg}{9.81} \times \text{metre}^2$.

The sudden change in impressed voltage may be produced in different ways. Fig. 76 shows a potential slide used for this purpose. A, B are the supply terminals and S is the slider. By shifting it to the left the impressed voltage is reduced; by shifting it to the right it is increased. A fixed resistance R , which might be the starter of the motor, should be kept permanently in circuit to protect the armature from an excessive current if the slider should inadvertently be pushed over right to B when the motor is running very slow. The exciting current may be regulated and adjusted for each test to a definite value by the ordinary shunt rheostat s .

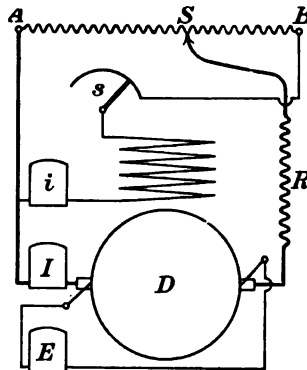


FIG. 76. Electrical test for finding moment of inertia.

When the test has to be applied to a very large motor the potential slide becomes a rather formidable and expensive apparatus. In this case it is advisable to replace it by an auxiliary generator kept running by some prime mover at a constant speed and to

regulate the voltage impressed on the large motor by a field rheostat in the exciting circuit of the auxiliary generator. The output of the latter need not exceed 10 per cent. of the power of the main motor.

In making a test we take time-e.m.f. and time-current readings and plot them as shown in Fig. 77. The e.m.f. to be used in (52)

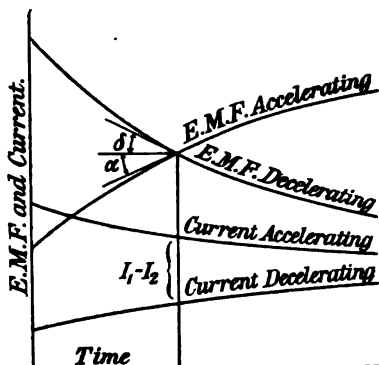


FIG. 77.

is given by the crossing point of the two curves. The difference in the two currents is taken on the same ordinate. The angles α and δ are scaled off the curves and if greater accuracy is required this may be obtained by the galvanometer-condenser method explained on p. 151, but this refinement is hardly necessary since the test is primarily intended for large motors having a considerable flywheel effect and

therefore accelerating and decelerating so slowly that a sufficient number of observations may be obtained in the critical region where the e.m.f. curves cross to make the drawing of the tangents fairly accurate.

Efficiency Tests. The efficiency is sometimes computed by adding to the measured loss when running light the ohmic loss when running loaded and deducting these two quantities from the input when working as motor and adding them to the output when working as generator. If P_1 represents input, P_2 output and P_0 loss when running light, and ρ the resistance, it is supposed that for

$$\text{a generator } P_1 = P_2 + P_0 + \rho I^2$$

$$\text{a motor } P_2 = P_1 - P_0 - \rho I^2$$

In a generator P_2 and in a motor P_1 can be easily measured electrically and on the supposition that the above formulae are correct, we get for the

$$\text{efficiency of a generator } \eta = \frac{P_2}{P_2 + P_0 + \rho I^2}$$

$$\text{efficiency of a motor } \eta = \frac{P_1 - P_0 - \rho I^2}{P_1}$$

But the supposition on which these simple formulae are based is not correct. When the machine is loaded both the hysteric and the eddy current losses increase on account of the distortion of the flux and the result is that the computed efficiency is always higher and sometimes considerably higher than the true efficiency*.

The efficiency of a generator driven by a reciprocating steam engine may be found approximately by indicating the engine when the dynamo is not excited and its brushes are lifted and again when it is working at load. The difference in the indicated power is approximately the power given to the dynamo and since its output can be electrically measured the efficiency may thus be computed. If the engine drives its own airpump it is, of course, necessary to indicate this also.

Small motors may be tested by using some kind of mechanical brake, but as electrical methods of measuring power are far more accurate and more easily applied than mechanical methods, it is generally safer to rely on efficiency measurements carried out electrically. The first suggestion for such measurements was made by the late Major Cardew, and used in testing machines supplied to the Admiralty. In this test three machines were used. The first is a generator driven by any kind of prime mover. The output of this was measured and supplied to a motor to which was mechanically coupled the third machine working as a generator. The output from this was measured. The two measurements, viz., input to motor and output from generator, gave the combined efficiency of the two machines and as these were approximately of the same size and type the efficiency of each may be taken as the square root of the combined efficiency. The drawback to this method is that an error in any power measurement affects the result to an equal extent and even more if these errors are of opposite sign. This difficulty is overcome by Hopkinson's "lost power method" of testing efficiency devised in 1886†. Two equal machines are mechanically and electrically coupled. Their excitation is so adjusted that when the set is driven at normal speed by mechanical power applied from outside one acts as motor and the other as generator. The input to the motor and the output from the generator can be both electrically and therefore very accurately measured. The mechanical

* Epstein, "Testing of electric machinery, etc.," *Journ. Inst. E.E.*, Vol. xxxviii, p. 28.

† Dr John Hopkinson, "Dynamo Electric Machinery," *Phil. Transactions*, May, 1886. Also *Engineering*, 1886, p. 446.

input to the combination is measured by a transmission dynamometer. This represents the power lost in the double conversion. An error in the mechanical power measurement does affect the final result, but to a very small extent as will be easily seen by considering a concrete case. Say that the electrical output of the generator is 100 and that 10 is lost in it. The shaft will therefore have to give the generator a mechanical power of 110. The motor receives 100 electrically and loses 10; it therefore gives to the shaft 90. The difference between 110 taken by the generator and 90 given by the motor must be supplied through the transmission dynamometer. The power to be measured by it is therefore 20. Now suppose that this measurement were wrong by as much as

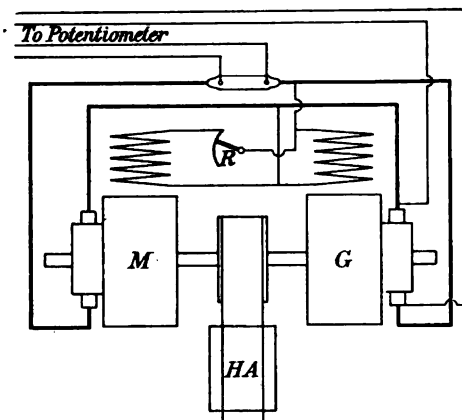


FIG. 78. Hopkinson test for efficiency.

5 per cent. in excess of the true power. We should then debit the combination with 21 instead of with 20. In the example chosen the true efficiency of the generator is $100/110$ and that of the motor $90/100$. On account of the 5 per cent. error in measuring the power passing through the dynamometer the efficiencies come out at $100/110.5$ and $90/100.5$. The error in the final result is only $1/10$ th of the error in the mechanical measurement. With the direct method it might have been of the same order of magnitude. The arrangement of apparatus is shown in Fig. 78. *M* is the machine whose field is weakened so that it may work as a motor, *G* has its normal field and acts as a generator. *HA* is a Hefner-Alteneck transmission dynamometer. The current supplied by *G* to *M* and the potential difference at the brushes of *G* are measured potentiometrically. It is assumed that all resistances, including the leads between the machines and that of the rheostat *R* in the working position, are accurately known. The purely ohmic losses in the whole combination can therefore be calculated. These losses have to be supplied from outside and are part of the power measured in *HA*. Of the remainder it is assumed that one-half is

required by M and the other half by G . Since all ohmic losses are known and the power supplied to G to overcome all other losses is also known, we have all the data required to determine the efficiency of G with great accuracy.

In order to avoid the use of a mechanical dynamometer the author has modified the Hopkinson test by supplying the lost power electrically*. The arrangement with shunt machines is shown in Fig. 79.

The two machines† are coupled mechanically and receive a supply of power from some outside source at their normal voltage. The rheostat R serves merely for starting both machines as motors running light; it remains short circuited during the test. R_1 and R_2 are the usual shunt regulating switches and A is an amperemeter which, with the connections arranged as shown, serves to measure both the current given by the machine acting as generator and the current taken by the machine acting as motor. The voltmeter V is not essential, it is merely used to make sure that the source supplies current at the correct e.m.f., but its reading does not enter into the determination of the efficiency. The conductors joining the brushes of the two machines must be of sufficient sectional area to make the drop negligibly small. Then the brush voltage of both machines will be the same. In the position of the field rheostats shown M acts as motor and G as generator. The current given by the generator G can be read on the amperemeter by opening switch S_1 , that taken by the motor by closing S_1 and opening S_2 . As a check another amperemeter may be put into the supply circuit and this will read $I = I_2 - I_1$. The advantage of using the same instrument for measuring both I_1 and I_2 is that a possible instrumental inaccuracy has less effect since it enters into both readings with the same

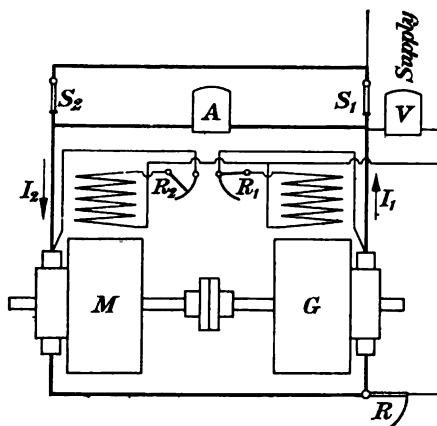


FIG. 79. Kapp test for efficiency.

measures both the current given by the machine acting as generator and the current taken by the machine acting as motor. The voltmeter V is not essential, it is merely used to make sure that the source supplies current at the correct e.m.f., but its reading does not enter into the determination of the efficiency. The conductors joining the brushes of the two machines must be of sufficient sectional area to make the drop negligibly small. Then the brush voltage of both machines will be the same. In the position of the field rheostats shown M acts as motor and G as generator. The current given by the generator G can be read on the amperemeter by opening switch S_1 , that taken by the motor by closing S_1 and opening S_2 . As a check another amperemeter may be put into the supply circuit and this will read $I = I_2 - I_1$. The advantage of using the same instrument for measuring both I_1 and I_2 is that a possible instrumental inaccuracy has less effect since it enters into both readings with the same

* Kapp, "The Determination of the Efficiency of Dynamos," *The Electrical Engineer*, 1892, Jan. 22 and 29.

† For a modification by which this test can be applied to one machine only see W. Lulofs, *Journal Inst. El. Eng.*, No. 196, p. 150.

sign. Let E be the supply voltage and η_2 the efficiency of the motor then the power given mechanically to G is $\eta_2 I_2 E$; and if η_1 is the efficiency of the generator its output will be $\eta_1 \eta_2 I_2 E = I_1 E$. We thus find the combined efficiency

$$\eta_1 \eta_2 = \frac{I_1}{I_2}$$

The machines are not working exactly under the same conditions; M has a weaker field than G and carries a larger current. The weaker field should result in a somewhat smaller hysteric loss, but as the current is larger the distortion must be greater than in G and therefore the difference in hysteric loss between the two machines cannot be very great. On the other hand there is a larger ohmic loss in M , so that on the whole the sum of all the losses in the two machines cannot be greatly different. On this account no very serious error is introduced by the assumption that the two efficiencies are equal and then we have for the efficiency of either machine

$$\eta = \sqrt{\frac{I_1}{I_2}} \quad \dots \dots \dots (53)$$

It is, however, possible to eliminate any source of error which may be due to unequal electric and magnetic loading of the two machines

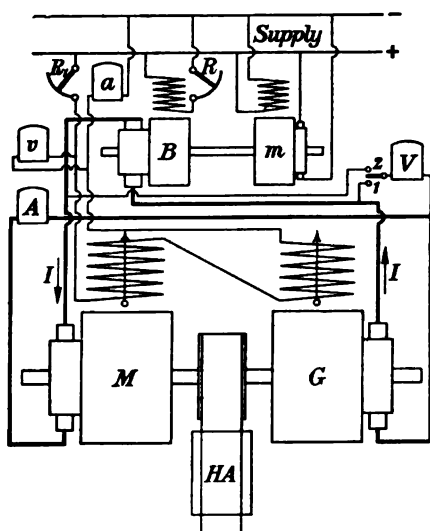


FIG. 80. Blondel test for efficiency.

by so altering the Hopkinson test that mechanical losses are measured by the transmission dynamometer and electrical losses by volt and ampere measurements. This test, devised by Professor Blondel, requires rather more apparatus than either the original Hopkinson test or the author's modification of it. The principle of Blondel's test is shown in Fig. 80. In addition to the transmission dynamometer HA , which measures the purely mechanical losses in the two dynamos, a booster B is required to supply the power necessary to cover the electrical losses in

the two armatures. Since the machines are equal in all respects, the armatures carry the same current and the fields receive the same excitation, the magnetic and electric loading is precisely the same and the efficiencies must also be equal. The booster B is driven by a small motor at constant speed and if desired the same motor can drive the belt of the transmission dynamometer. The circulating current is indicated on A and the exciting current on a . The same voltmeter V is used to indicate the terminal voltage of the machines. If the voltmeter switch is placed on stud 1 the instrument reads the voltage of G , if on stud 2 the voltage of M , the difference being the voltage supplied by the booster. The e.m.f. required for excitation is indicated on v , so that input to M and output of G can be measured. If the mechanical power passing through HA is added to the input we have all the data required for determining the efficiency of the set. Let E_1 be the terminal e.m.f. of the generator, E_2 that of the motor, I the circulating current, i the exciting current, e the exciting voltage measured on v and P_m the power in watts measured on the dynamometer, then the total input the system receives is

$$P_m + ei + E_2 I$$

and the output is $E_1 I$, giving the combined efficiency

$$\eta_1 \eta_2 = \frac{E_1 I}{P_m + ei + E_2 I}$$

Since both machines are electrically and magnetically in the same condition $\eta_1 = \eta_2$ and we have for each

$$\eta = \sqrt{\frac{E_1 I}{P_m + ei + E_2 I}}$$

The excitation is regulated by the rheostat R_1 and the strength of the circulating current by the rheostat R . The necessity for special apparatus and the great care required to make the different adjustments are drawbacks which have prevented this test from coming into general use; for practical work the all electric test shown in Fig. 79 is sufficiently accurate with modern machines having an efficiency not under 85 per cent. and is in general use.

The purely electrical method for testing efficiency can also be used with series machines and is especially convenient in testing railway motors. Into the circuit connecting the two machines in series a booster is inserted and the field of one machine is weakened by shunting part of the exciting current through an adjustable

resistance. This machine acts as a generator, that with the strong field as a motor. The same current circulates through both armatures. The arrangement is shown in Fig. 81. Two machines, M and G , are mechanically coupled and the field of G is shunted by a fixed and a variable resistance R_1 . The terminal voltage of each machine can be read on the voltmeter V , which is provided with a change-over switch. Using the same instrument for both readings has the advantage of minimising calibration errors. B is a booster, the output of which having merely to cover losses need only be a fraction of the output of the machines under test, but it is convenient to use for this also a normal railway motor driven by another M_1 , so that the test requires two pairs of machines, or four machines in all. Since railway motors are always made in quantities this condition presents no difficulty. On the other hand the use of precisely

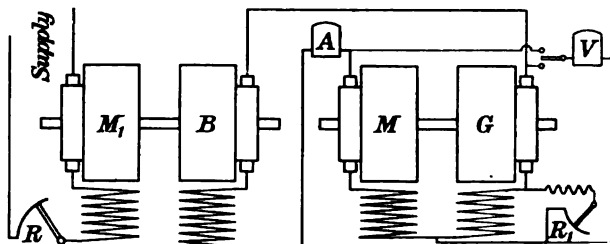


FIG. 81. Efficiency tests for railway motors.

similar machines for the boosting set has the great advantage of ensuring stability. If the boosting set consists of shunt machines the test requires great care and patience in making adjustments so as to get stable running conditions and reliable readings, but if series machines are used the system has inherent stability and the test can be made very quickly. Moreover the constant voltage of the supply need not be the normal voltage of the machines under test, but may be considerably lower. The boosting set is regulated by the rheostat R and the set under test by the shunt resistance and rheostat R_1 . The reading of the amperemeter A does not enter into the formula for efficiency, but this instrument is necessary to determine the load at which the machines work. If E_1 is the voltage of the generator and E_2 that of the motor the efficiency is

$$\eta = \sqrt{\frac{E_1}{E_2}} \dots \dots \dots (54)$$

CHAPTER VII

ALTERNATORS

Principles of construction—Wave form—To predetermine the effective e.m.f. of a three phase alternator—Approximate method for determining k —Harmonic analysis—Apparatus for recording wave form—Influence of star and mesh coupling on higher harmonics—Ripples in the e.m.f. wave due to teeth—Disturbance of telephone circuits by higher harmonics—Error due to higher harmonics in testing for inductance and capacity.

Principles of Construction. Since alternators are essentially machines for high voltage it is expedient to avoid centrifugal forces in the armature winding and for this reason the armature is made the fixed part or “stator” whilst the field is the rotating part or “rotor.” With the usual frequency of 50 cycles per second and the speed of rotation possible with a reciprocating engine or

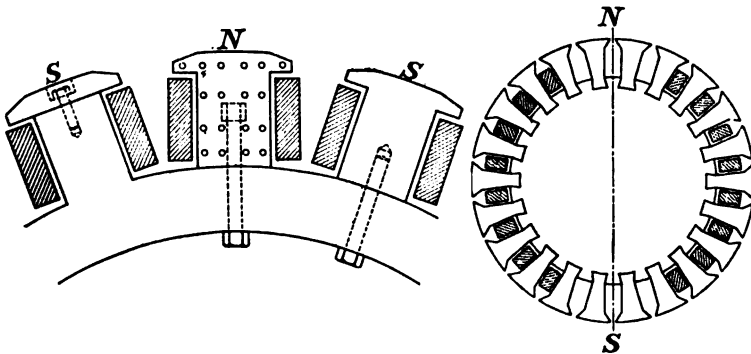


FIG. 82. Field magnet with salient poles and cylindrical field magnet.

a water turbine working under a low or moderate head the number of poles required is large and this leads to a construction with “salient poles” as shown on the left of Fig. 82. The magnet cores are fitted radially to a yoke ring and each core is provided with an exciting coil which is held in place by the pole shoe. Three different types of construction are shown. If yoke ring and magnet cores are cast in one piece the pole shoes must be separate so as to allow of the exciting coils being inserted. The pole shoe is fastened by screws with countersunk heads.

A more usual construction is to make the magnet cores separate from the yoke ring and in this case the pole shoe may be in one

piece with the core, the latter being attached to the yoke ring by bolts as shown. Sometimes the magnets are made of sheet iron as shown for a north pole in the figure. The iron stampings are riveted up to form a solid body and the holding on bolts may be tapped into this as if it were solid metal; a better practice is, however, to provide the punchings with a rectangular window into which a steel key is inserted into which the bolt is tapped.

For turbo alternators the number of field poles that may be used is restricted by the very high speed necessary for steam turbines. Thus at 3000 revs. p.m. and 50 frequency we must use a two pole rotor and as perfect mechanical balance is essential at high speeds it is expedient to avoid salient poles. We must use a cylindrical magnet with the exciting coil subdivided into a number of parts which are housed in slots as shown on the right of Fig. 82. The teeth between the slots are dovetailed into the solid centre part.

Another method of construction is to mill parallel slots out of the solid body and secure the coils by metal wedges dovetailed at the top of the slot. The latter method is generally adopted for four pole rotors of the cylindrical type.

The stator winding is also housed in slots. Fig. 83 shows the arrangement for a three phase slow speed alternator in which the

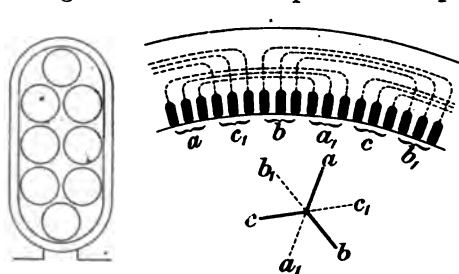


FIG. 83. Three phase armature winding. Three slots per phase per pole.

distance between slots is $1/9$ th of the polar pitch. The end connections are indicated by dotted lines and the lettering a, c_1, b , etc., corresponds with the vector diagram placed beneath the figure. The winding is insulated by a micanite tube as shown to a larger scale on the left. The thickness of this tube has to be chosen not only with reference to its electric but also to its mechanical strength. Hence in machines for moderate voltage the thickness is rather greater than would be required for electric strength alone. The strength of micanite at 50 frequency varies from 15,000 to 40,000 volts per mm.* According to Zipp† the strength of micanite 1 mm. thick is

* Miles Walker, "Specification and Design," p. 176.

† Hermann Zipp, "Hochspannungs technik," page 18.

35,000 and 4 mm. thick 75,000 volts. If the star centre of the machine were always at earth potential corresponding to equality of insulation resistance in all three phases, then the greatest potential difference between any coil and the iron of the armature would be: line voltage divided by $\sqrt{3}$, but a displacement of star potential may occur through a variety of causes, so that the full line voltage may exist between a particular wire of the winding and the frame. The micanite tube should therefore be made thick enough to withstand the extra stress due to an abnormal condition. There must also be an ample margin of safety, especially as the quality of mica may vary, and some allowance is necessary for possible mechanical injury. The following table gives the thickness of micanite tubes according to good modern practice:

Up to 3,000 volts	...	1.5 to 2 mm.
Up to 6,600 „	...	2.5 to 3 „
Up to 8,000 „	...	3.5 to 4 „
Up to 11,000 „	...	4.5 mm.

The lower figures refer to tubes made up beforehand and inserted into the slots before the conductors are drawn in. The upper figures refer to tubes moulded on to the bundle of wires and inserted together with the coils. In this case the tube is not as perfect because it has to be clamped in a vice when hot to compress it into the final shape. During this process creases may be formed in the mica which represent weak spots; hence an extra thickness is advisable.

The combination: coil-insulation-stator iron forms a condenser with a dielectric consisting partly of micanite, partly of the covering of the wire and partly of air. We have here the case of a compound dielectric and since the dielectric constant of micanite is at least six times that of air, the stress on whatever air film separates the conductor from the micanite must be very great; with high pressure machines so great that the air breaks down (see Vol. I, p. 131) and becomes ozonised by a so-called silent discharge. This does not mean a break down of the insulation generally, for the micanite tube is quite equal to withstanding the whole of the stress without assistance from any air space, but it means that some chemical action takes place resulting in the formation of nitric oxide and, if moisture is present, nitric acid which attacks metal and insulation and finally destroys it. Hence the necessity of impregnating the

coil and filling up all internal spaces with insulating compound, or sealing up the ends of the tube to exclude air entering from outside.

Wave Form. The form of the e.m.f. wave of an ideal alternator is a sine curve (Vol. I, p. 298), but in an alternator, such as can be practically made, the wave form may depart more or less from the true sine shape. The wave form must obviously be influenced by the character of the load, but as this is an unknown and variable element it is customary to define wave form as the shape of the e.m.f. curve on open circuit. The wave form depends on various details of design and amongst these the most important are width and shape of polar surface, and width of a coil side, both referred to the pole pitch. Fig. 84 shows the influence of the width or

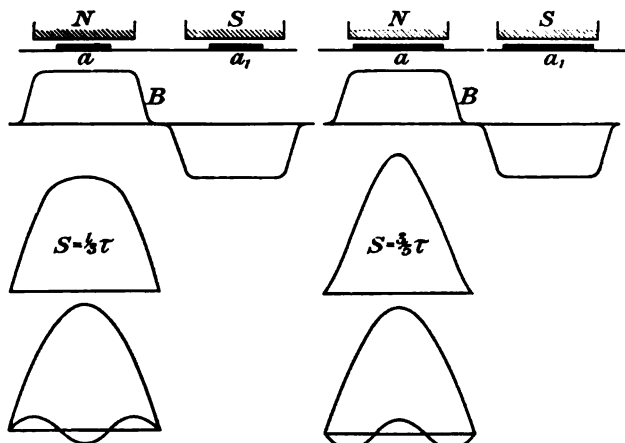


FIG. 84. Showing influence of width of side of coil on wave form.

spread S of the coil side where the width of the pole shoe is $\frac{2}{3}$ of the pole pitch. The left of this diagram refers to a coil the side of which is $\frac{1}{3}$ of the pole pitch and therefore half the width of the pole shoe, whilst the right side refers to a wider coil where $S = \frac{3}{5}\tau$ and therefore only little less than the width of the pole shoe. The thick lines marked a and a_1 represent the band of conductors forming a coil side. The ordinates of the curves marked B represent induction in the air gap and the curves below show the wave form in the two cases. It will be seen that with narrow coils crest value of e.m.f. is kept up for an appreciable time making the wave rather flat at its highest part. With wide coils a slight departure from the

centre position brings some of the conductors out of the influence of the field and therefore lowers the e.m.f. Hence crest value is kept up for a much shorter time and the curve becomes rather peaky. Below these curves are shown two sine curves of the same periodicity and two sine curves of much smaller amplitude but of three times the periodicity. The latter are called "upper harmonics" and the resultant of the "fundamental" and third upper harmonic gives a shape of the same character as the actual wave form. There is however a difference between the two cases of narrow and wide coil sides. Let E_1 be the crest value of the fundamental and E_3 that of the third harmonic, then we have for the instantaneous value of the wave

$$\text{With narrow coils } e = E_1 \sin \omega t + E_3 \sin 3\omega t,$$

$$\text{With wide coils } e = E_1 \sin \omega t - E_3 \sin 3\omega t.$$

There may be and generally are other harmonics of a higher order influencing the shape of the wave, but a study of the third harmonic is important as showing in what direction the design should be altered to approach more nearly to a sine wave. In a three phase machine there is not much choice in the width of the coil side. We must either make it $1/3\tau$ or $2/3\tau$ in order to get a symmetrical winding and utilise the whole circumference. The usual arrangement is to make the width of a coil side one-third of the pole pitch and in this case we shall approach a sine curve by depressing the shoulders of the wave shown. This may be done by making the air gap larger at each polar edge, that is by chamfering the pole faces. The usual practice is to make the middle third with constant and minimum air gap and to widen the air gap on each side by bevelling the face.

If a D.C. armature winding is tapped at points 120 electrical degrees apart and three phase current introduced we get bands of conductors each 120 degrees wide. Conversely if an ordinary D.C. machine is provided with three slip rings and these are connected to points on the winding 120 degrees apart we have a three phase alternator with coil sides of a width equal to $\frac{1}{3}$ of the pole pitch. Fig. 85 shows this diagrammatically for a two pole machine. A, B, C are

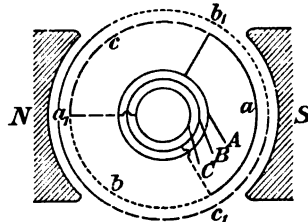


FIG. 85.

the brushes bearing on the slip rings. The conductors being arranged in two concentric circles, namely half of them at the bottom and half at the top of the slots, there is some overlapping of the bands of conductors, but each is 120 degrees wide and we have therefore approximately the case shown on the right of Fig. 84. To get approach to a sine wave the width of the poles should be less than that of the coil side. To make the poles narrow would, however, reduce the total flux and therefore the output of the machine, so that a winding with wide coil sides is not generally used if the machine is intended to work merely as a three phase generator pure and simple; such coils are however used in a special type of machines termed "converters," where the armature (which in this case is the rotor) is traversed simultaneously by a D.C. and an A.C. and serves for the conversion of one kind of current to the other. Machines of this type are treated in a separate chapter; for the present we confine our study to three phase generators with narrow coils. The three windings may be coupled in mesh or star. In the first case the line voltage is the same as the phase voltage, in the latter case it is $\sqrt{3}$ times the phase voltage.

To Predetermine the Effective e.m.f. of a Three Phase Alternator. For the ideal machine discussed in Vol. I, p. 297, we found that the instantaneous value of the e.m.f. is given by the equation

$$e = \omega \Phi \frac{z}{2} \sin \omega t$$

Its crest value corresponding to $\omega t = \frac{\pi}{2}$ is

$$E = \omega \Phi \frac{z}{2} = 2\pi f \Phi \frac{z}{2}$$

Since the wave is a sine curve the effective value is $e = E/\sqrt{2}$ and we have

$$e = \frac{\pi}{\sqrt{2}} f \Phi z 10^{-8} \text{ volts}$$

The coefficient $\frac{\pi}{\sqrt{2}} = 2.22$ is peculiar to a machine giving a true sine wave; if the wave has a different form, or if the winding cannot be considered as concentrated in one plane as in Fig. 148 of Vol. I, then the coefficient will be different, so that we may write generally for a machine of any wave form

$$e = k \frac{f}{100} \Phi z \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad (55)$$

where Φ is the flux from one pole in megalines,

f is the frequency in cycles per second, and

z is the number of active conductors belonging to one phase and coupled in series.

It will be noticed that neither speed (u) nor number of poles ($2p$) enter explicitly into this formula; nevertheless they are both implicitly contained. Since $f = pu$ we may write

$$e = k p \Phi \frac{z}{100} u$$

which is the form for the e.m.f. of a D.C. machine of $2p$ poles with wave winding, but with the difference that the factor k now enters. In an alternator all the z conductors are in series, in the D.C. machine only $z/2$ conductors are in series.

The correct way of predetermining the e.m.f. of an alternator is to map out the curve of induction in the air gap over a pole pitch taking account of the influence of the teeth and their saturation and noting the e.m.f. of each conductor or group of conductors in the slots according to their position in the curve of induction. The e.m.f. (being proportional to the speed) is found from the fundamental equation $e = BvL$, where L is the length of the armature, v the speed and B the gap induction. This gives the instantaneous e.m.f. for the position chosen. Repeat for other positions and take the R.M.S. value of all these e.m.f.'s to get the effective e.m.f. Φ is found by planimentering the B curve so that the value of k in (55) may be determined. The process is somewhat laborious, but having been performed for one particular machine the value of k may be used for any other machine of a similar type even if of different size.

Approximate Method for Determining k . We have seen that by suitably shaping the pole shoes it is possible to obtain an e.m.f. wave which does not materially differ in its general shape from a sine curve. There may be ripples in it due to the effect of open slots, but these cannot materially alter the area, that is, the total flux, nor the general outline which is that of the fundamental curve. In any case crest value of e.m.f. occurs when the coil side is under the centre of the pole and as the general shape of the wave is of a sine character we obtain the effective value by dividing the crest value by $\sqrt{2}$. The problem of the approximate determination

of the e.m.f. is thus simplified to the determination of the crest value for the central position of the coil side. Having mapped out

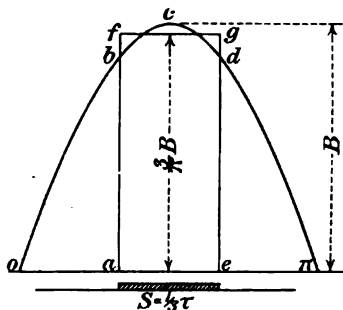


FIG. 86.

the true curve of induction as above described we draw an equivalent sine curve of equal area, Fig. 86. Its ordinates represent the e.m.f. generated in a single turn spanning a pole pitch. If all the conductors were massed in the centre of the coil side the curve would to another scale also represent the e.m.f. of the whole phase, but as the conductors are distributed over the

width $S = \frac{\tau}{3}$ the crest value of the e.m.f. curve must be a little smaller than the value corresponding to a concentrated winding. Its true value is obviously the average taken over the width of the coil side. The average induction B_1 is found by equating area $abcde$ to area $afge$

$$\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} B \sin \alpha d\alpha = \frac{\pi}{3} B_1 \quad \text{or} \quad B_1 = \frac{3}{\pi} B$$

giving for all the z conductors of the phase the e.m.f.

$$\frac{3}{\pi} BvLz$$

Since $v = 2\pi f$ and $\Phi = \frac{2}{\pi} B\tau L$ we get for the effective e.m.f.

$$e = \frac{3}{\sqrt{2}} f \Phi z$$

in c.g.s. units, or

$$e = 2.12 \frac{f}{100} \Phi z \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad (56)$$

if the flux Φ is given in megalines. In this case $k = 2.12$. If the coil side is $\frac{2}{3}$ of the pitch we find in the same way $k = 1.84$; and if the coil side is $\frac{1}{2}$ of the pitch $k = 2$.

Harmonic Analysis. In the section on wave form it has been shown how the general character of a particular wave may be obtained by the superposition of a third harmonic on the fundamental. This is but a special case of a general law formulated by

Fourier to the effect that a wave of any shape may be accurately represented by the addition of sine and cosine terms representing waves the frequency of which are whole number multiples of the frequency of the fundamental. Mathematically stated the ordinate of the wave is therefore the sum of the two series

$$\text{Sine Terms} \quad A_1 \sin \alpha + A_2 \sin 2\alpha + A_3 \sin 3\alpha + \dots$$

$$\text{Cosine Terms} \quad B_1 \cos \alpha + B_2 \cos 2\alpha + B_3 \cos 3\alpha + \dots$$

Since in a machine all the flux coming out of one pole must enter the other and all poles are of the same shape, it follows that except for the sign, ordinates π radians apart must be equal, that is to say, the algebraic sum of any two components π radians apart must be zero. This is obviously the case for the fundamental and any odd harmonic, but it is not the case for any even harmonic unless its amplitude is zero. Thus for the second harmonic we have

$$A_2 \sin 2\alpha + A_2 \sin (2\alpha + 2\pi) = 2A_2 \sin 2\alpha$$

and this can only be zero if A_2 is zero. The same with the cosine terms and all higher even harmonics. We therefore find that the Fourier series expressing the wave form of an alternator contains no even harmonics. Its equation is

$$y = A_1 \sin \alpha + A_3 \sin 3\alpha + \dots + B_1 \cos \alpha + B_3 \cos 3\alpha + \dots$$

The amplitudes A_1, A_3, A_5 , etc., and B_1, B_3, B_5 , etc., may be positive or negative. Since these amplitudes are constants and therefore not influenced by the selection of the starting point from which we count the angle α , we may take for this the point at which y is zero. This is somewhere between (though not necessarily midway between) two polar axes. Counting then the angle from this zero point, two important results immediately follow. Since for $\alpha = 0$ all the sine terms vanish and all the cosines are unity we obtain

$$B_1 + B_3 + B_5 + \dots = 0$$

from which it is obvious that if cosine terms are present at all they must be partly positive and partly negative, but cannot all be of the same sign. For $\alpha = \frac{\pi}{2}$ all the cosine terms vanish and we are left with

$$y_0 = A_1 + A_3 + A_5 \dots$$

where y_0 is the ordinate somewhere near to, though not necessarily coincident with, the polar axis. The A amplitudes may be of the

same sign, though not necessarily so. In certain cases, especially in connection with the problem whether resonance may produce

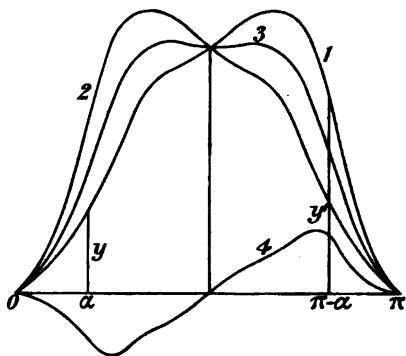


FIG. 87. Harmonic analysis.

a dangerous condition (Vol. I, p. 317), it is important to know the amplitude of any harmonic in the e.m.f. wave and although the wave itself may be easily obtained by oscillograph, Joubert disc or ondograph, these instruments do not indicate any particular harmonic separately and to find the amplitude of any particular harmonic from the curve obtained experimentally it is necessary to employ some method of harmonic analysis. A number of such methods have been devised by mathematicians and are found in mathematical text-books. As an example one of these methods frequently used by practical engineers may here be briefly described. It has the merit of being fairly simple and not too laborious. As a first step it is necessary to separate the sine and cosine terms. Let in Fig. 87 the curve marked 1 be the original wave form as taken by any of the appliances above mentioned. Take two ordinates such as y and y' equidistant from O and π respectively. For these we have

$$y = A_1 \sin \alpha + A_3 \sin 3\alpha + \dots + B_1 \cos \alpha + B_3 \cos 3\alpha + \dots$$

$$y' = A_1 \sin \alpha + A_3 \sin 3\alpha + \dots - B_1 \cos \alpha - B_3 \cos 3\alpha - \dots$$

By addition and subtraction we get two new curves marked 3 and 4 in the figure. These are represented by the equations

$$\frac{y + y'}{2} = A_1 \sin \alpha + A_3 \sin 3\alpha + A_5 \sin 5\alpha + \dots$$

$$\frac{y - y'}{2} = B_1 \cos \alpha + B_3 \cos 3\alpha + B_5 \cos 5\alpha + \dots$$

the first containing only sine and the second only cosine terms. The easiest way to draw these curves is to reverse 1 in respect of the ordinate passing through the point $\alpha = \frac{\pi}{2}$ and thus obtaining its image 2 and taking half the sum and half the difference of the ordinates of 1 and 2. The curve 3 may now be used to find all the

sine terms and the curve 4 to find all the cosine terms. From these may be found the amplitude of any particular harmonic. Thus for that of the 5th harmonic we have

$$\text{Amplitude} = \sqrt{A_5^2 + B_5^2}$$

The method is based on the following theorem of the integral calculus.

$$\int_0^\pi \sin(m\alpha) \sin(n\alpha) d\alpha = \begin{cases} 0 & \text{if } m = n \pm \text{a whole number} \\ \frac{\pi}{2} & \text{if } m = n \end{cases}$$

$$\int_0^\pi \cos(m\alpha) \cos(n\alpha) d\alpha = \begin{cases} 0 & \text{if } m = n \pm \text{a whole number} \\ \frac{\pi}{2} & \text{if } m = n \end{cases}$$

To find the A term of the n th harmonic we multiply the ordinates of the curve 3 with sine $(n\alpha)$ and draw a new curve on the same base. The area of this curve is

$$\int_0^\pi y \sin(n\alpha) d\alpha$$

and can be found by planimeter. Its mean ordinate is

$$a_n = \frac{1}{\pi} \int_0^\pi y \sin(n\alpha) d\alpha$$

Inserting for y the value of the Fourier series we get terms containing the product of two sines of different angles and only one in which the angles are the same and of which the integral is therefore $\frac{\pi}{2}$.

We thus obtain

$$\int_0^\pi y \sin(n\alpha) d\alpha = \frac{A_n}{n} \int_0^{n\pi} \sin^2(n\alpha) d(n\alpha) = \frac{\pi}{2} A_n$$

$$a_n = \frac{1}{\pi} \frac{\pi}{2} A_n \text{ and } A_n = 2a_n \quad . \quad . \quad . \quad (57)$$

The amplitude of the n th harmonic is therefore twice the value of the mean ordinate as found by planimeter. A similar treatment gives the cosine terms. The advantage of this method is that any particular harmonic may be found quite independently of all the others so that the labour is restricted to those harmonics which might become dangerous.

Apparatus for Recording Wave Form. Various instruments are available for this purpose; of these the Joubert disc, the oscillograph and the ondograph are those most used. Any of these

may be used to obtain a graphic record not only of the e.m.f. wave, but also of the current wave. As such investigations are generally necessary on high voltage machines it is advisable to interpose transformers so as to protect observer and instrument from the danger of high voltage. The general principle is illustrated in Fig. 88. T_c is a so-called "current transformer," the primary of which is connected in series with the line. Its secondary carries a current proportional to the line current and of identical wave form. If both windings have the same number of turns, both currents are of equal value; if the secondary winding has more turns, the current through it is reduced in magnitude. Such transformers are also generally used in connection with amperemeters in order to avoid stress on their insulation and bring the current to be

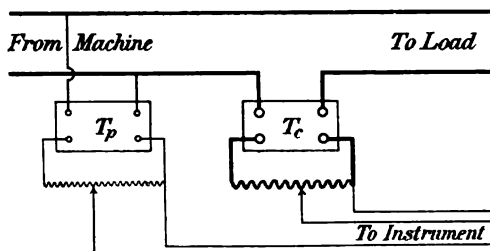


FIG. 88.

measured within the range of the instrument. The current from the secondary is passed through an inductionless potential slide. The instantaneous value of the e.m.f. between any two points on this slide is therefore proportional to the instantaneous value of the current and by obtaining a graphic record of the former we also obtain with a suitable change of scale a graphic record of the latter. To record the e.m.f. wave we use a so-called "pressure transformer" marked T_p in the figure and a similar arrangement of potential slide. The oscillograph* consists of a strong electro-magnet in the air gap on which is placed a loop through which passes the current from the circuit to be measured. Generally there are two such loops, one connected to T_p and the other to T_c so that e.m.f. and current waves may be recorded simultaneously, together with their phase displacement. The plane of the wire loops is parallel to the lines of force in the air gap and each loop

* For a detailed account of the type of oscillograph designed by Mr Duddell and made by the Cambridge Scientific Instrument Co., see a paper by Duddell and Marchant, *Journ. Int. E. E.*, No. 138, p. 1.

carries a light mirror. If a current passes, the ascending wire of the loop is pushed by the field in one direction and the descending wire in the opposite direction producing an angular movement in the mirror which is made visible by the angular deflection of a beam of light from an arc lamp. The moving system is exceedingly light and the controlling force due to the tension in the loop very large, so that the natural period of vibration is exceedingly short. The natural frequency is of the order of 3000 cycles per second, but in the most sensitive type it reaches 10,000. This is so much higher than the frequency of any harmonic which might occur in the wave of an alternator that the deflection of the beam of light is strictly proportional to the e.m.f. on the oscillograph terminals. The horizontal movement of the spot of light is therefore proportional to the ordinates of the wave. If projected on photographic paper moving vertically with constant speed a record of the true wave form may be obtained. Various methods are used to obtain a standing image of the wave. One of these is to employ as a screen on which the light spot falls a cylinder with a base of the shape of an Archimedean spiral. The axis of this cylinder is horizontal and at right angles to the direction of the beam. Viewed from above the distance of the spot of light from the axis will vary according to the position of the spiral. The cylinder is rotated by a little synchronous motor driven by current from the machine to be tested. If the oscillograph is not in circuit the observer sees simply a straight line traced by the spot of light on the revolving cylinder at right angles to its axis; if now the oscillograph is switched on the observer sees the complete wave. There are other methods employing revolving or oscillating mirrors to obtain a standing wave which may be traced or photographed, but in all the principle is maintained that the ordinates are given by the movement of the mirror of the oscillograph and the abscissae by a deflection of the ray of light at right angles and proportional to time.

The Joubert disc, Fig. 89, is used to obtain a point to point record of the instantaneous e.m.f. in the circuit *AB* coming from the transformer. The disc may be driven mechanically from the shaft of the alternator, or by a synchronous motor fed from the alternator. The latter method is more convenient as the instrument need not be set up close to the alternator. The disc is of insulating material, but has a short metal segment let into its rim for the purpose of making momentary contact with two small brushes, one

connected to *B* direct and the other through a potentiometer to *A*. The brushes are fixed to a segment which can be set in different angular positions that are indicated on a graduated circular scale. *G* is a galvanometer with universal shunt and *V* is a voltmeter which indicates the instantaneous value of the e.m.f. corresponding to the angular setting of the brush holder. The potential slide is connected to any convenient source of constant e.m.f. and any

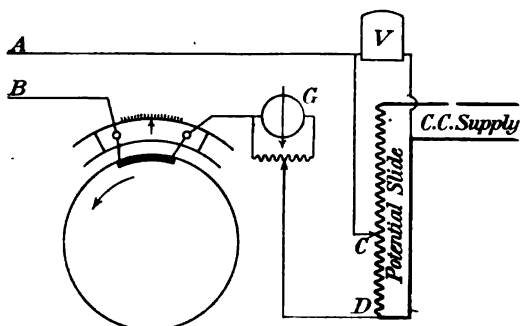


FIG. 89. Joubert disc.

smaller voltage may be taken off by the slide *C* and read on the voltmeter. If the P.D. between *C* and *D* is the same as the instantaneous value of the P.D. between *A* and *B*, the galvanometer will show no deflection. In making a test the brush holder

is set to a certain angular position and *C* is adjusted until the galvanometer shows no deflection. *V* is observed and noted against the position of the brush holder. The latter gives the abscissae and the voltmeter reading the ordinates of the wave.

The Hospitalier Ondograph is also a point to point instrument, but with automatic shifting of the phase and with so many points during a period that the result is a continuous record traced by the pointer of a D.C. voltmeter on a drum revolving once for every 1000 completed cycles. The principle of the instrument is shown in Fig. 90. *SM* is a synchronous motor which is started by hand and kept going by the A.C. when the switch *S* is closed. This motor is geared to the paper drum *D* and to a commutating switch *CS*. This consists of a brass cylinder with insulation inlaid where the shaded parts are. Three small copper brushes are applied. That marked *b* touches the unbroken metallic surface of the cylinder; that marked *a* also touches the metallic surface save for a short interval when the part inlaid with insulation passes under it; that marked *c* touches metal only for a very short time when the narrow metallic strip passes, the rest of the time it touches the insulation. When *c* makes contact a circuit is established through the condenser *C* and this gets a charge corresponding to the value of the e.m.f. at

that instant. The voltmeter V is connected to the condenser and brush a . At the moment when the condenser receives a charge a is insulated, but an instant later a touches metal and is therefore connected to b and the voltmeter receives current by the discharge of the condenser. The capacity is very large so

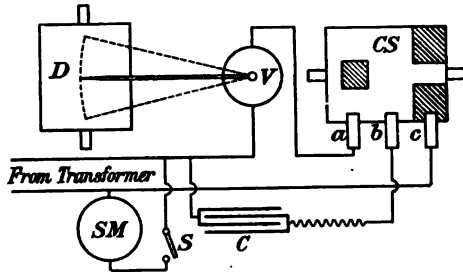


FIG. 90. Connections of ondograph.

that the charge in the condenser is not appreciably reduced in the short time of one revolution of CS when the charge of the condenser is again replenished. If CS were to revolve absolutely synchronously the charge would be replenished always at the same part of the wave and the e.m.f. thrown on V would be constant. The gear between SM and CS is, however, so arranged that CS does not rotate exactly with frequency speed, but with a speed $1/1000$ th short of frequency so that the successive charges given to C and consequently the successive e.m.f.'s impressed on the voltmeter correspond to ordinates of the wave $1/1000$ th of 2π or 0.36 time degrees apart. With 50 frequency CS does not rotate at a speed of 50 r.p.s. but at $0.999 \times 50 = 49.95$ r.p.s. with the result that the pointer of the voltmeter traces a complete cycle in 20 seconds. Its end carries a pen which records the wave form on the paper drum D . This is driven by the synchronous motor at a speed of one revolution in 20 seconds. A clutch is provided for detaching the drum from the motor so that a new paper may be applied without having to stop the motor. The ondograph is essentially a shop instrument; no special skill is required to operate it and the record is given in a thin sharp line.

Influence of Star and Mesh Coupling on Higher Harmonics. It has been shown (Vol. I, p. 332) that with sinusoidal currents the sum of the three e.m.f.'s is zero at all times as also is the sum of the three currents. The question now arises whether this is also true for the higher harmonics. If not, what is the effect on the machine if mesh coupled and on the line if the machine is star coupled? With mesh coupling the line must obviously have the same voltage as the machine terminals and, as all three phases

are equally affected by upper harmonics, the P.D. between any two line wires must be the phase e.m.f. whether upper harmonics are present or not. With mesh coupling we are only concerned with the question whether upper harmonics will produce circulating currents in the machine itself. With star coupling there can obviously be no internal circulating currents, but there may be caused a dissymmetry of the P.D. between any two line wires. In studying this problem we need not take into consideration the fundamental since this balances. It is only necessary to determine the relative position of the vectors of the upper harmonics and to see whether they balance.

Mesh Coupling. Select in all cases the moment when the *A* vector is directed vertically upwards and draw a diagram showing the position of the *B* and *C* vectors at the same instant. From such a diagram it can be seen at a glance whether the vectorial sum of the three vectors of the same order is zero or has a definite value. Fig. 91 is such a diagram. To show how it is obtained take by way of example the 5th harmonic. With the sequence *ABC* we obtain the position of the *B* vector by rotating the *A* vector counter-clockwise through an angle of

$$5 \times \frac{2}{3}\pi = 3\pi + \frac{\pi}{3}$$

3π brings the vector pointing vertically downwards and adding $\frac{\pi}{3}$

brings it into the position shown in the diagram, that is pointing downwards and to the right; its angle to the vertical being 60° . For the *C* phase we have $5 \times \frac{4}{3}\pi = 6\pi + \frac{2}{3}\pi$. This brings the vector into the position where it points downwards and to the left, the angle with the vertical being also 60° . The sum of the three vectors is zero and we conclude that a 5th harmonic does not produce a circulating current. Reasoning in the same way for the 3rd or the 9th harmonic we find that all the three vectors point upwards and their sum is three times the amplitude of one, so that these harmonics do produce circulating currents.

Order of Harmonic	3	5	7	9	11	13	15	17
Phase A.	↑	↑	↑	↑	↑	↑	↑	↑
Phase B.	↑	↘	↙	↑	↘	↙	↑	↘
Phase C.	↑	↙	↘	↑	↙	↘	↑	↙

FIG. 91. Analysis of upper harmonics.

Star Coupling. Where the three vectors are in the same direction they raise or lower the potential of all three lines simultaneously and can therefore not alter the P.D. between lines. We may therefore conclude that the third, ninth, fifteenth and any harmonic whose order is an odd multiple of three cannot get into the line. On the other hand the vectors of all other harmonics not being unidirected shift the potential of the lines in different directions and thus must alter the P.D. between any two lines and moreover alter it differently to that of any other two lines. This means that the fifth, seventh and any other harmonic, not being of the order of an odd multiple of three does get into the line and causes a pulsation in the shape of the vector triangle. These harmonics although quite harmless as far as the machine itself is concerned may nevertheless become objectionable through disturbing telephonic speech. With mesh coupling the third harmonic is, however, distinctly harmful to the machine itself because it gives rise to circulating currents and thus wastes power. For this reason alternators are generally designed for star coupling, but where for some reason mesh coupling must be used, care should be taken in the design of the pole shoes and the arrangement of the winding to avoid the generation of a third harmonic altogether, or at least minimise its amplitude. One obvious way applicable to a drum-wound armature is to make the span of each armature coil not π but only $\frac{2}{3}\pi$, that is adopt chord winding. Each coil side taken by itself has still its third harmonic, but since the time angle of the third harmonic is $\frac{2}{3}\pi$ and the space angle between the two coil sides in series is also $\frac{2}{3}\pi$ the two cancel, so that the coil as a whole does not produce a third harmonic. By using such short pitch coils the e.m.f. obtainable from the machine with a given amount of material is slightly lowered.

Although in a star coupled machine the third harmonic does not get into the line its presence in the phases may become a source of danger under certain conditions. It is common practice to test each machine for short circuit current under fairly large excitation. The terminals are short-circuited and the phase current is measured and plotted as a function of exciting current. Apparently a short-circuited machine should be harmless, but in reality this is not so. If the starpoint is earthed and the three terminals are simply short-circuited amongst themselves, but not earthed, they will, as is clear from Fig. 91, have a P.D. to earth equal to the amplitude of the third harmonic and it may be dangerous to touch them or

handle any measuring instrument in this short circuit. To make the test safe the conductor used to short circuit the terminals should itself be well earthed.

The third harmonic may also become troublesome when alternators are worked in parallel with their star points earthed. In this case any difference in phase or magnitude of the third harmonic must produce circulating currents which heat the armature windings and waste power. The remedy is to connect only one star point to earth through a resistance leaving all other star points free.

Ripples in the e.m.f. Wave due to Teeth. In a slotted armature the reluctance of the air gap cannot be absolutely constant. As teeth enter on one side and issue on the other from the pole face there must be slight and rapid variations of the air gap area and consequently a tendency to a very slight and very rapid pulsation in the strength of the field. This tendency cannot develop, because of the very large inductance of the exciting coils. The flux taken as a whole is practically of constant strength, but its configuration is subject to rapid changes. As teeth enter and issue the flux swings forward and backward with a frequency $2s$ times that of the fundamental, if by s we denote the number of teeth in a pole pitch. This flux swinging superimposes higher harmonics on the wave quite apart from the third, fifth, etc., which are due to the general configuration of pole and coil. The problem of these harmonics is of practical importance because they are of telephonic frequency and might under certain conditions cause trouble on that account. To study this matter we must distinguish between the geometric and magnetic axis of the flux. The former goes through the centre of the pole, the latter passes alternately to the right and the left and very close to it (roughly speaking the distance varies from zero to \pm half a tooth pitch). The effect of this flux swing must obviously be most pronounced in the region where the induction is strongest, that is in the middle of the poles and this deduction is borne out by oscillograms which show the strongest ripples in the middle of the wave whilst they are very small or absent at the points where the wave goes through the axis. But this point is not fixed; it oscillates in the same way as the flux axis to right and left of the true neutral point which is midway between pole centres. In deducing the formula for the e.m.f. we have counted the angle from the neutral point. Now we must distinguish between the geometric and magnetic neutral point. Let α_1 be the angle if referred

to the former and α the angle if referred to the latter, and let δ be the maximum angular deviation between the two axes, then on the assumption that the ripples also follow a sine law we have*

$$\alpha = \alpha_1 - \delta \sin 2s\alpha_1$$

The flux passing through the coil is proportional to $\Phi \cos \alpha$ and the instantaneous e.m.f. is proportional to $-\frac{d}{dt} \Phi \cos \alpha$. Since Φ is constant we have

$$e \equiv -\frac{d}{dt} \cos (\alpha_1 - \delta \sin 2s\alpha_1)$$

$$e \equiv -\frac{d}{dt} [\cos \alpha_1 \cos (\delta \sin 2s\alpha_1) + \sin \alpha_1 \sin (\delta \sin 2s\alpha_1)]$$

Since δ is an exceedingly small angle and the numeric $\sin 2s\alpha_1$ cannot be larger than 1, we may put $\cos (\delta \sin 2s\alpha_1) = 1$, and we may also write δ for $\sin \delta$ and thus obtain

$$e \equiv -\frac{d}{dt} [\cos \alpha_1 + \delta \sin \alpha_1 \sin (2s\alpha_1)]$$

and since $\alpha_1 = \omega t$ we find

$$e \equiv \omega [\sin \alpha_1 - \delta \cos \alpha_1 \sin (2s\alpha_1) - \delta 2s \sin \alpha_1 \cos (2s\alpha_1)]$$

within the important region $\sin \alpha_1 > \cos \alpha_1$ and moreover $\delta 2s$ is so much larger than δ that we may neglect the second term and write

$$e \equiv \omega [\sin \alpha_1 - \delta 2s \sin \alpha_1 \cos (2s\alpha_1)]$$

The first term refers to the fundamental, the second to the superimposed ripple. The instantaneous value of the ripple e.m.f. is therefore proportional to

$$-2 \sin \alpha_1 \cos (2s\alpha_1)$$

From $\sin (2s\alpha_1 - \alpha_1) = \sin (2s\alpha_1) \cos \alpha_1 - \cos (2s\alpha_1) \sin \alpha_1$

and $\sin (2s\alpha_1 + \alpha_1) = \sin (2s\alpha_1) \cos \alpha_1 + \cos (2s\alpha_1) \sin \alpha_1$

By subtraction

$$\sin (2s-1)\alpha_1 - \sin (2s+1)\alpha_1 = -2 \sin \alpha_1 \cos (2s\alpha_1)$$

The ripple e.m.f. contains two harmonics, one of the order $2s-1$ and the other of the order $2s+1$. Thus in a three phase alternator with three slots per pole per phase s is 9 and the ripples represent a 17th and 19th harmonic. With a fundamental frequency of 50

* The theory of ripples here developed is based on that published by Messrs Smith and Boulding; see *Journ. I. E. E.*, No. 241, p. 224.

this means harmonics of 850 and 950 frequency, which is well within telephonic range.

Disturbance of Telephone Circuits by Higher Harmonics. The disturbance of a telephone circuit with earth return is easily explained by electromagnetic induction if the power line is carried overhead and parallel to the telephone line, but disturbance may also occur where the power line is a three phase cable laid underground. In this case there can be no electromagnetic induction; yet there may be earth currents of high frequency. To explain their occurrence we may take a star coupled three phase alternator with the star centre earthed. This is generally done with a view to facilitate the prompt action of safety devices. At some distance from the generator there is a transformer, the case of which is also earthed. This is necessary to ensure personal safety. Each winding has a capacity to case and unless the three capacities have absolutely the same value the capacity currents which must return to the star centre through earth do not cancel. Although in the system taken as a whole (that is, cable and earth) the sum of the currents is zero at any time, it is not zero for the cable or for earth taken separately. The result is that earth currents flow which take their way partly through the earth return of the telephone system.

This phenomenon is not confined to A.C. plant, but may also be observed in D.C. systems, since it is due to the action of the armature teeth and must therefore occur with any toothed armature. Various remedies may be employed. The slots may be closed, the polar edges may be set in a slightly oblique way, that is not quite parallel to the shaft, or some dissymmetry may be introduced into the field system by making the neutral space alternately larger and smaller so that the e.m.f. due to flux swinging is positive in one coil side and negative in the other. Since both coil sides are in series the two ripple e.m.f.'s balance and no outside effect is produced.

Error due to Higher Harmonics in Testing for Inductance and Capacity. Capacity and inductance are important elements of a transmission line and a knowledge of their exact values may be necessary. The tests for capacity given in Vol. I, pp. 109 and 111, cannot be applied, because they are only valid in cases where the circuit tested has negligible inductance, which is certainly not the case in a power line. We must therefore make use of some indirect

method of measuring capacity. This is based on the law that in a transmission line of any configuration the product of capacity in m.f. per km. and inductance in henrys per km. is a constant, namely

$$\epsilon \times 11.1 \times 10^{-6} = \frac{\text{m.f.}}{\text{km.}} \times \frac{\text{henrys}}{\text{km.}}$$

where ϵ is the dielectric constant. Whereas in an overhead line the insulation is air, this constant is unity and we have the law that for any overhead line irrespective of the relative position of the conductors the product of capacity and inductance per km. is 11.1×10^{-6} . If, then, we can find a simple way of measuring the inductance we obtain at the same time a knowledge of the capacity of the line. We know that $e = \omega Li$ and if e were a simple harmonic function we could easily find L by current and e.m.f. readings. It might be thought that using the relation: charging current $= \omega Ce$, we could in the same way find C by readings of a voltmeter and amperemeter, but such a test would be inaccurate. The capacity of an overhead line is very small; hence to get a reliable reading on the amperemeter which measures the charging current a high voltage must be employed. With a high voltage the leakage over the many insulators becomes a disturbing element and although, if the insulation resistance is known, allowance can be made for this, a test where this disturbing element is reduced in magnitude is preferable. If we short circuit the line at the far end a comparatively small e.m.f. suffices to send the full working current through the line and under this small e.m.f., which tapers down to nothing at the far end, the leakage current is very small and may be neglected. The measured e.m.f. is then the resultant of the e.m.f. required to overcome line resistance and that required to balance reactance. Since the former is known and both are in quadrature it is easy to determine by a short circuit test the magnitude of the reactance e.m.f. Let its value be e and that of the current i , then the reactance is $\omega L_1 = \frac{e}{i}$. If the e.m.f. were a simple sine wave then L_1 would

be the inductance of the line. The question now arises whether with some more complicated wave form the measured value L_1 is the true inductance, or merely an apparent value different from the true inductance. To investigate this point we take the simple case that beside the fundamental there is only one higher harmonic (say the n th) present in the e.m.f. wave. The extension to any

number of harmonics will be obvious when we have arrived at a solution of this simpler problem. Let E and E_n be the crest values of the two harmonics, then the voltmeter indicates the R.M.S. of

$$E \sin \alpha + E_n \sin n\alpha$$

or in symbols

$$e_1 = \sqrt{\frac{1}{\pi} \int_0^\pi (E \sin \alpha + E_n \sin n\alpha)^2 d\alpha}$$

The bracket contains the product of the sines of two different angles, one an odd multiple of the other. By a well-known theorem of the calculus the integral between the limits 0 and π is zero, so that we have the simpler equation

$$e_1 = \sqrt{\frac{1}{\pi} \int_0^\pi E^2 \sin^2 \alpha d\alpha + \frac{1}{\pi} \int_0^{n\pi} E_n^2 \sin^2 n\alpha \frac{d(n\alpha)}{n}}$$

$$e_1 = \sqrt{\frac{1}{\pi} E^2 \frac{\pi}{2} + \frac{1}{n\pi} E_n^2 \frac{n\pi}{2}}$$

$$e_1 = \sqrt{\frac{E^2}{2} + \frac{E_n^2}{2}} = \sqrt{e^2 + e_n^2}$$

if we denote the effective value of each wave by a small letter. It is convenient to express the magnitude of the higher harmonics by their relation to that of the fundamental. We may write $e_3 = p_3 e$, $e_5 = p_5 e$, etc., and

$$e_1 = e \sqrt{1 + p_3^2 + p_5^2 + \dots} \quad (58)$$

Since the higher harmonics are comparatively of small magnitude we may develop the square root by the binomial theorem, and neglecting higher terms we get

$$e_1 = e \left(1 + \frac{p_3^2}{2} + \frac{p_5^2}{2} + \dots \right) \quad (59)$$

The effective value indicated by the voltmeter is larger than the effective value of the fundamental.

In the same way we have for a current wave containing the upper harmonics, $i_3 = p_3 i$, $i_5 = p_5 i$, etc.

$$i_1 = i \left(1 + \frac{p_3^2}{2} + \frac{p_5^2}{2} + \dots \right) \quad (60)$$

Assuming then that a current wave containing only one upper harmonic is given and the current is sent through a resistanceless

inductance L , what effective e.m.f. will be indicated by a voltmeter connected to the terminals of L ? We have generally

$$e_1 = L \frac{di_1}{dt}, \text{ or } e_1 = L\omega \frac{di_1}{d\alpha}$$

Inserting the value of i_1

$$e_1 = L\omega \frac{d}{d\alpha} (I \sin \alpha + I_n \sin n\alpha)$$

$$e_1 = L\omega [I \cos \alpha + (nI_n) \cos n\alpha]$$

For the sake of simplicity we have here only dealt with sine terms, but obviously the same reasoning applies to cosine terms. The expression in the bracket is of the same form as that given at the top of p. 186, except that we have now cosine terms instead of sine terms (which does not affect the integration) and that the upper harmonic enters with n times its value. Applying the same deductions we find

$$e_1 = L\omega \sqrt{i^2 + (ni_n)^2}$$

or generally for the e.m.f. indicated on the voltmeter,

$$e_1 = L\omega i \sqrt{1 + 9p_3^2 + 25p_5^2 + \dots}$$

whilst the amperemeter indicates

$$i_1 = i \sqrt{1 + p_3^2 + p_5^2 + \dots}$$

The apparent reactance is

$$\omega L_1 = \frac{e_1}{i_1} = \omega L \sqrt{\frac{1 + 9p_3^2 + 25p_5^2 + \dots}{1 + p_3^2 + p_5^2 + \dots}}$$

The true inductance is therefore

$$L = L_1 \sqrt{\frac{1 + p_3^2 + p_5^2 + \dots}{1 + 9p_3^2 + 25p_5^2 + \dots}} \quad (61)$$

or with sufficient approximation for practical work

$$L = L_1 \left(\frac{1 + 0.5(p_3^2 + p_5^2 + \dots)}{1 + 0.5(9p_3^2 + 25p_5^2 + \dots)} \right) \quad (62)$$

The time inductance is smaller than the apparent inductance. The inverse relation holds good for true and apparent capacity.

The discrepancy between the true and apparent values may be sensible. Thus in a wave having a third harmonic 15 per cent., a fifth 10 per cent., and a seventh 6 per cent. of the fundamental, the true value of inductance would be only 81 per cent. of the respective values calculated from ampere and voltmeter readings.

CHAPTER VIII

ALTERNATORS ON LOAD

Effect of armature current on the field—Armature reaction in a three-phase alternator—Short circuit current—Terminal characteristic of alternator on load—Working condition at constant excitation—Synchronous impedance—Graphic representation of power—Bearing currents—Experimental determination of armature ampere turns and armature reactance.

Effect of Armature Current on the Field. The current flowing in the bands of conductors of the coils grouped round the

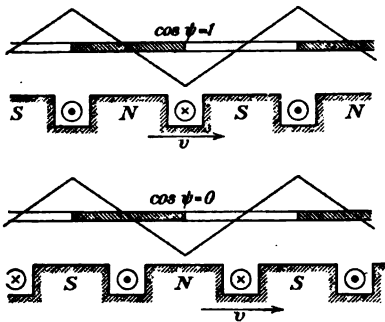


FIG. 92.

circumference of the armature produces a flux which is superimposed on the flux produced by the exciting winding. The resultant flux, *i.e.* that which actually exists and induces the e.m.f., is therefore due to the vectorial sum of the impressed excitation and that represented by armature ampere turns. The actual induction at any point in the air gap is due to the

resultant ampere turns. A graph showing resultant ampere turns in combination with the O.C. characteristic makes it possible to map out the field actually existing. This is different from the field on open circuit in two ways, namely by distortion and by alteration in magnitude. In order to obtain a general conception of these effects we take the case of a single phase alternator with salient poles two-thirds of a pole pitch, and the whole circumference of the armature covered by winding, so that the band of conductors forming a coil side has a width equal to the pole pitch. The upper part of Fig. 92 represents the case of a machine working with an internal lag of 0. Maximum current will occur when the poles are opposite to the centres of the coil sides. The band of conductors carrying at that moment maximum current downwards is shown by hatching, the direction of motion of the poles by the arrow v . The excitation of the poles is shown by the circles with dots and crosses. The armature ampere turns are represented by the zig-zag line. Positive ordinates represent ampere turns acting vertically upwards and therefore pro-

ducing an upward flux that is in the same direction as would be impressed by a north pole. In the position shown the centre of the physical north pole of the field magnet coincides with the point at which the excitation produced by the armature ampere turns is zero. In that point the impressed flux is on the whole neither strengthened nor weakened. To the left of the geometrical centre line of the pole it is strengthened, to the right weakened, but if we abstract from saturation the two effects cancel, so that the magnitude of the impressed flux is not altered. The centre of magnitude of the resultant flux is, however, shifted slightly to the left of the geometrical centre of the pole and this causes a slight increase of lag. This increase we neglect for the present.

In the lower part of the figure the case for an internal lag of 90 degrees is drawn. Here we see that the armature ampere turns oppose directly the field ampere turns, thus weakening the resultant flux and therefore also the induced e.m.f. The centre of magnitude remains in the geometric axis of the pole and there is no distortion producing dissymmetry of the flux. For internal phase angles between 0 and 90 degrees there will be some weakening of the resultant flux, less for small and more for large phase angles. If the machine acts as a motor the current is reversed and then its lagging component will strengthen the field. The ordinates of the zig-zag line representing armature ampere turns vary with the instantaneous value of the current, but the zero points or nodes of this line remain in the centres of the coil sides, that is to say, they are fixed in space since the armature is the stator. In the case $\cos \psi = 0$ we have the maximum demagnetising action of a generator armature at the moment when the current has crest value. Using the previous notation of q as the number of wires in a coil side, we have $qi \sqrt{2}$ for the apex of the triangle and the average value over the face of the pole is $\frac{2}{3} qi \sqrt{2}$. Assuming the current to vary according to a sine law, then the average over the whole period is $2/\pi$ of this, so that the average of the fluctuating armature ampere turns is $\frac{2}{\pi} \frac{2}{3} qi \sqrt{2}$, or $X_a = 0.6 qi$ demagnetising ampere turns of the armature of a generator if the internal lag is 90° . We are justified in taking the average because of the very large inductance of the exciting coils which prevents any sensible fluctuation in the resultant flux. For any internal phase angle ψ we may assume

$$X_a = 0.6 qi \sin \psi \quad . \quad . \quad . \quad . \quad . \quad (63)$$

The above reasoning may be summarised as follows: The armature current produces both distortion and change of magnitude of the original flux. Distortion is produced by the watt component and change of magnitude by the wattless component of the armature current. In a generator the distortion produces a slight increase, in a motor a slight decrease, in the lag of current. The wattless component if lagging weakens the flux in a generator and strengthens it in a motor. It follows that the more inductive the load on a generator the more must the excitation be increased to keep the terminal e.m.f. constant.

Armature Reaction in a Three-Phase Alternator. A more detailed study of the effects generally described in the last section for a single phase armature is hardly necessary because single

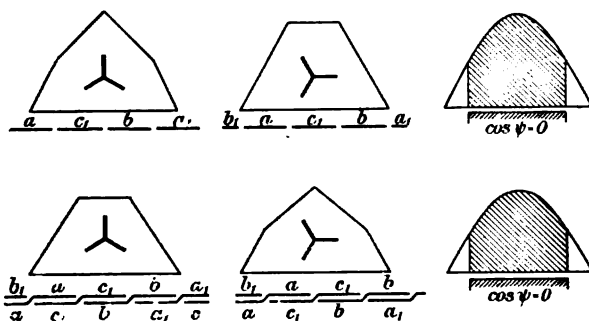


FIG. 93.

phase machines have not much practical importance. Most alternators are built on the three phase principle and for this reason we shall limit the investigation to this type of machine. The armature current reacts in two ways on the terminal voltage; by true impedance and by altering the magnitude of the field. The first effect, which is due to resistance and inductance of the armature taken by itself, is not very important and is generally within the limits of 5 and 10 per cent. of the open circuit voltage. The second effect is more marked and may with a low power factor of the load necessitate a considerable increase of excitation in order to keep the terminal voltage at its normal value. Both effects taken together are comprised under the term "armature reaction" and are felt the more strongly the heavier the load and the lower the power factor.

To study the magnetising effect of the armature current we draw diagrams (Fig. 93) of the exciting force due to the armature current

for two phase positions of the coils. The top line in the figure refers to narrow coil sides ($S = \frac{1}{3}\tau$) and to the two conditions that one phase has crest value and one phase has zero. Any intermediate phase conditions fall between these extremes, but as the area of these two figures is very nearly the same we need not draw diagrams for intermediate phase positions. Let I be the crest value of the current in one wire and q the number of wires in a coil side, then $x = qI$ represents the ampere turns due to the crest value of the current in that coil. At the same time each of the other coils carries $I/2$ and the total ampere turns, that is, the highest point in the figure is $2qI$. At the time that one phase goes through zero each of the other two phases carries $\frac{I\sqrt{3}}{2}$ amperes and the highest part of the corresponding graph is $qI\sqrt{3}$. The areas of the two figures are

$$\frac{7}{6}\tau x \text{ and } \frac{2}{\sqrt{3}}\tau x$$

They are very nearly equal and their average value is $1.16\tau x$. On the right is drawn a sine curve of the same area. Its highest ordinate is

$$X_0 = \frac{\pi}{2} 1.16x$$

Substituting this sine curve for the true and ever changing outline of the exciting force diagram we can determine by integration the average exciting force acting over the pole face and therefore interfering with the excitation applied by the field circuit. The relative position between the sine curve and the pole face remains unaltered; both travel through space with a speed corresponding to the frequency. For an internal phase angle of 90° the armature exciting force is the mean ordinate of the shaded area. With the usual pole face of two-thirds of the pole pitch we must integrate between the limits of 30° and 150° and we thus obtain for the armature exciting force

$$X = X_0 \frac{3\sqrt{3}}{2\pi}$$

and inserting the value of X_0 and instead of the crest value I the effective value i , we obtain for the armature ampere turns

$$X = 2.12qi$$

The coefficient 2.12 is only valid for the special case here treated, namely narrow coils and pole face of two-thirds of pole pitch. If

we make the pole face narrower say one half the pole pitch the mean ordinate of the shaded area becomes a little larger. In this case the shaded area is

$$\frac{1.16}{\sqrt{2}} \tau x$$

and the mean ordinate is $2.32qi$. By narrowing the pole face we have slightly increased the coefficient. If we were to reduce the polar face to one-third of the pole pitch the coefficient would be 2.45. To cover all cases we may write generally

$$X = aqi \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

The bevelling of the polar edges has the effect of concentrating more of the flux in the middle and this has the same result as narrowing the poles. Thus with bevelled edges and a polar angle of 120° the coefficient will be a little greater than 2.12. The value assumed in practice is 2.2, so that the formula for armature ampereturns with coils sides of one-third and pole faces of two-thirds of the pole pitch is

$$X = 2.2qi \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Similar calculations may be made for the case of wide coil sides. The graph of exciting force is shown in the lower part* of Fig. 93. The outlines have the same shape, but the maxima are in both figures reduced in the ratio of 2 to $\sqrt{3}$. It is therefore not necessary to go through the calculation in detail; all we need do is to reduce the value of a in this ratio. This gives $a = 1.9$.

With a lag of 90° X is a back excitation reducing the flux and therefore lowering the voltage of the generator. The back excitation is proportional to the slice cut out from the equivalent sine curve of the graph of total armature ampere turns. By a well-known theorem of trigonometry the area of such a slice is itself a sine function of the position where it is taken. In the case illustrated in Fig. 93 the internal phase angle ψ is 90° and the area of the slice is a maximum; if the internal phase angle were zero the area (being one half positive and the other half negative) would be zero and for any intermediate phase angle we have for the armature ampere turns demagnetising the field

$$X = aqi \sin \psi \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

* Such a winding may be obtained by tapping an ordinary D.C. drum armature at points 120° electrical degrees apart. The three circuits are then mesh connected, but star connection is also possible by cutting the winding. Since the winding is in two layers there is overlapping and this is shown in the figure. The sequence of the coils is $a a_1 : b b_1 : c c_1$ and that of the coil sides is $a : c_1 : b : a_1 : c : b_1$ and so on.

In a two phase machine the width of a coil side is half the pole pitch and a similar investigation as above shows that in this case $a = 1.37$. It is convenient to express armature ampere turns in relation to total number of active wires Z and number of poles. Let as before i be the effective current in one wire and $2p$ represent the number of poles, then for a pole face of two-thirds pole pitch and chamfered edges we have in round figures the armature ampere turns for a lag of 90° in a

Three phase machine with narrow coil sides	$0.36 Zi/p$
Three phase machine with wide coil sides	$0.31 Zi/p$
Two phase machine	$0.34 Zi/p$

The demagnetising effect of the armature is not very different in these different types of winding.

The inductance of the armature is not absolutely constant, but varies a little with the position of the coil side relatively to the pole. It is greatest when the centres coincide, because in this position the face of the pole shoe can take some of the lines passing through the head of one tooth to that of the next, but unless the air gap is very small and the tooth pitch rather large the reduction in reluctance is not very sensible and as it only affects one of several paths of the self-induced flux the influence on the total is not very important. The flux along the different paths can be calculated by the well-known rules of the magnetic circuit, account being taken of the saturation of the teeth. As far as paths through air are concerned, full areas must be taken for the flux across the shelf, lip and over the heads of the teeth, but for that passing from side to side of slot only one-third of the full area must be taken, because of the even distribution of conductors in the slot. To calculate the flux surrounding the heads of the winding outside the armature iron by Hobart's rule would give rather too large a figure, because the conductors are not bunched together. For this part of the winding the following empirical formula derived from one given by Dr Kloss* may be used. Let h be the distance over which the head projects beyond the core and τ_1 the span of the coil (in full pitch winding τ_1 is a little greater than τ) and w the number of conductors in a slot, then the leakage lines through the head on one side produced by one ampere (crest value) number

$$\Phi_A = (1.4h + 0.24\tau_1) 1.25w.$$

* *Elektrotechnik und Maschinenbau*, 1910, No. 3.

Denoting the slot leakage also produced by one ampere by Φ_s , then the $\frac{z}{2}$ turns in one phase are interlinked with the flux

$$\Phi_A = 2 (\Phi_A + \Phi_B)$$

The inductance is $\lambda = z(\Phi_a + \Phi_s) 10^{-8}$ henrys; and the e.m.f. $\omega \lambda I$ due to inductance can now be found. In modern machines it is generally from 5 to 10 per cent. of the terminal e.m.f., unless for the protection of the machine against too large a short circuit current special provision is made for increasing the slot leakage by making the slots deeper than necessary and placing the conductors at the bottom of the slot.

The e.m.f. required to overcome armature resistance is calculated according to Ohm's law. It is advisable to add about 50 per cent. to cover the effect of eddy current losses which act in the same way as an increased ohmic resistance.

Short Circuit Current. Specifications generally contain a clause stating what the short circuit current at full excitation shall be

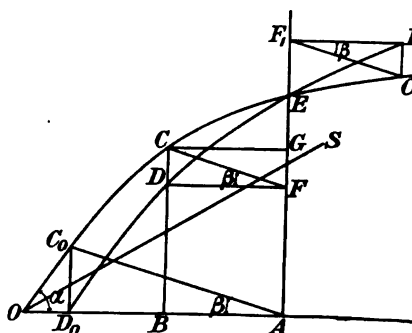


FIG. 94.

relatively to the normal full load current. The customary figure lies between 2 and 3. In this condition the machine is doing no outside work and if for the sake of simplicity we neglect internal losses, we may say that the power required to drive the machine is zero, which means that current and induced voltage must be in quadrature. We

may, as a preliminary, study the working of a machine connected to bus bars which by some source of unlimited current are kept at a constant voltage. The short circuit condition is then merely a special case where this voltage is zero, but in all cases there must be quadrature between induced e.m.f. and current and no power is required to keep the machine running. Since the whole of the current is wattless, $\sin \phi$ in (66) is unity and the armature ampere turns are $X = bI_0$ where I_0 is the effective value of the short circuit current and $b = aq$. I_0 is easily found from the O.C. characteristic. Let in Fig. 94 OE be this curve and OA the excitation, then AE is

the O.C. voltage. Let AB represent the armature back excitation produced by a certain current, then the induced e.m.f. has the value BC . One component of this, namely BD , is required to balance the e.m.f. AF at the bus bars and the other DC is absorbed by $\omega\lambda I_0$, where λ is the inductance in henrys of the phase. We neglect here the resistance of the phase since this is very small in comparison with the reactance and the two e.m.f.'s are in quadrature. Let n be the number of exciting turns per pair of poles and i the exciting current producing the excitation OA , then the resultant excitation is $OB = ni - bI_0$. Since $DC = \omega\lambda I_0$ and $FD = bI_0$ we have

$$\tan \beta = \frac{\omega\lambda}{b}$$

a constant. The slope of the line CF is therefore the same wherever the working point C may lie. Its position depends on the voltage AF of the bus bars and the current corresponding to any bus bar voltage is proportional to DF , where D lies on the line D_0DED_1 . If the bus bars have zero voltage, that is, if the terminals of the machine are short circuited, the working point is C_0 and the current is

$$I_0 = \frac{AD_0}{b}$$

If the bus bars carry the same voltage to which the machine is excited the working point is E and there is no current. If the bus bar voltage is higher, say AF_1 , the working point is C_1 and the current is now of the opposite sign, its magnitude being F_1D_1/b . By the construction shown in Fig. 94 we are thus able to determine the current of an idle running machine if the bus bar voltage is varied. What is usually understood by the term "machine short circuited" is merely a special case when the bus bar voltage is zero.

Since the angle β is independent of excitation the construction may be repeated for any other position of A . Within the limit to which the machine may be excited in practical work the inclined line AC_0 must strike the characteristic somewhere on its straight part and therefore the short circuit characteristic, that is the graph connecting excitation and short circuit current must be a straight line, such as OS drawn to such a scale that the ordinate in A is AD_0/b . Having found the short circuit current I_0 we can determine the reactance from

$$\omega\lambda = \frac{C_0D_0}{I_0}$$

It is here assumed that the armature back turns are known, that is to say that the precise value of b is known. Our assumption of a definite value of b is however only an approximation and it will be seen that a comparatively small error in b makes a large difference in the length D_0C_0 , so that an experimental method of finding b and $\omega\lambda$ is desirable; in other words it is desirable to be able to separate true reactance from the demagnetising force of the armature. Several methods for the experimental determination of b and $\omega\lambda$ are given at the end of this chapter.

Terminal Characteristic of Loaded Alternator. The e.m.f. at the terminals of the machine may be found from the O.C.

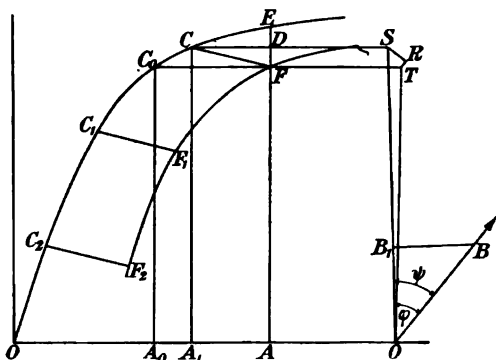


FIG. 95. Terminal characteristic.

characteristic by the method shown in Fig. 95. The abscissae represent excitation required for air gap and armature, therefore exclusive of the excitation required for the field magnets. In the vector diagram on the right OT represents the terminal e.m.f., $TR = \rho I$ the ohmic drop with a

definite current I and SR the inductive drop $\omega\lambda I$. It is customary to take ρ a little larger than the true ohmic resistance of the armature winding when hot in order to allow for any possible eddy current losses in the poles or other parts. The effect of such currents is of the same kind as that of an increased resistance in the A.C. circuit and must make itself felt in the same way, namely by the lowering of the terminal e.m.f. The influence of armature resistance, even if augmented by 25 or 50 per cent. to allow for eddy currents, is, however, very small in comparison with the e.m.f. of inductance and that due to armature back turns, something of the order of one-fifth to one-tenth, so that a slight error in estimating the percentage to be added to the armature resistance on account of eddy currents does not greatly matter. The problem may now be stated as follows: Given the O.C. gap and armature characteristic and the data mentioned, find the exciting current required to produce a definite terminal

voltage at any current and any power factor. Draw in Fig. 95 the current vector under the external phase angle ϕ and measure off the length OB to represent bI to the same scale as the abscissae of the O.C. characteristic represent exciting ampere turns. On open circuit the excitation is OA_0 and the working point is at C_0 . Under load the working point is at C and the effective excitation, namely total impressed ampere turns minus armature ampere turns, is OA_1 . The length of the vector OS and therefore the position of the working point C is found by drawing $TR = \rho I$ parallel and $RS = \omega \lambda I$ at right angles to the current vector. The armature back turns being proportional to the sine of the internal phase angle ψ are found by drawing BB_1 at right angles to OS . If we add $CD = BB_1$ to the effective excitation we find the impressed or field excitation OA . This on open circuit would give the voltage E , but on load gives the terminal voltage F . We have thus found one point of the terminal characteristic. To find other points for the same current and phase angle we may repeat the construction choosing a new value of OT . Since with a definite current and power factor the lines TR and RS shift up and down, but do not alter in inclination or length, the point S moves on a vertical and unless the terminal voltage be chosen very small there is no appreciable change in the very small angle included between OT and OS . Consequently the length BB_1 and therefore also CD remains the same. The difference in height of T and S , that is the length FD also remains the same, so that $\triangle CDF$ retains its size and shape and simply shifts along the O.C. characteristic. By shifting CF parallel to itself, say to C_1F_1 , we find a new point on the terminal characteristic and the whole curve can thus be plotted very quickly. Only for a very low terminal voltage such as F_2 need the construction be repeated in detail because then $\angle TOS$ becomes sensibly larger and BB_1 also a little larger. Thus $F_2C_2 > FC$ and the lower part of the terminal characteristic becomes a little steeper than the O.C. line.

We can draw such curves, each for a particular current and phase angle, and thus produce a chart which will show at a glance the working condition of the machine under any load and power factor. In Fig. 95 the terminal voltage is F and if the load is switched off it rises to E . The ratio of FE to AF is called the "regulation" and is generally specified as a percentage and referred to full load at a definite power factor. The closer the regulation, that is the nearer the two characteristics are, the less adjustment of excitation

is required when the load varies; but on the other hand the greater is the short circuit current and consequently the danger of damage to the machine in the case of an accidental short circuit. It should be noted that in an accidental short circuit the current is much larger than the two or three times normal value generally specified and obtained on the regular short circuit test. In such a test the machine is first short circuited and then excited. If the short circuit is accidental the machine is already excited and since the armature back turns cannot instantly demagnetise the field because of its enormous inductance, the short circuit current during the first few cycles is only limited by the comparatively small impedance of the armature winding. The first rush of current may therefore be not two or three times, but 10 or 15 times the normal value, and damage the machine mechanically. For this reason the modern tendency in large machines is in the direction of bad regulation obtained by increase of true reactance (length of line RS in Fig. 95) because this acts from the very first moment the short circuit occurs. Machines with bad regulation are also cheaper than those with very close regulation. Some device to automatically regulate the excitation for constant terminal voltage may be added in cases where the load is subject to frequent changes.

Working Condition at Constant Excitation. To predetermine what will be the terminal voltage at any current and power

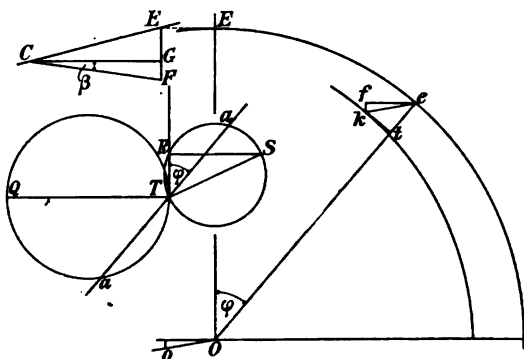


FIG. 96. Terminal e.m.f. at constant excitation.

factor if the excitation is kept constant a series of terminal characteristics as in Fig. 95 may be used, but as the preparation of such a chart is rather a laborious process it is desirable to find an easier

method. This is possible if the working point lies well above the knee of the O.C. curve where the latter has very little curvature. Let in Fig. 96 CE be this upper part of the O.C. curve and E the working point at no load to which the machine is excited. In the vector diagram on the right OE is the open circuit voltage. If there were no armature reaction and no loss, the locus of the volt vector at any load and phase angle would be the circle described from O as centre through E . Thus for the phase angle ϕ the terminal voltage would be Oe , but in reality the terminal voltage is less. To find it we make $CG = bI$ and $GF = \omega NI$, then EF is the voltage drop due to armature reaction for a phase angle of 90° . For a phase angle ϕ the drop is $EF \sin \phi$ and the drop due to armature resistance is $\rho I \cos \phi$. To get the terminal voltage we have to deduct these two components from the voltage Oe . Make $ef = EF$ and $fk = \rho I$, then the projection of ek on the vector Oe gives et as the drop from open circuit to terminal voltage and Ot is the terminal voltage. The point t may be also found with close approximation (at any rate for power factors between the usual limits of 0.8 to 0.5) as the point where the vector Oe cuts a circle drawn with the same radius from o as centre, where Oo is equal and parallel to ek . Since Oo is proportional to the current, a series of t circles may be drawn for different values of the current and the terminal voltage at any power factor within those limits may be read off on the respective circle.

Another method of determining the drop graphically (which is fairly correct also for power factors above 0.8) is shown on the left. Make $TQ = EG$, $TR = fk$ and $RS = GF$ (in the diagram the scale is quintupled for greater accuracy). Draw a line under the phase angle ϕ to the vertical, then the piece aa cut off this line by the two circles is the drop.

We have here assumed that the O.C. curve referring to the air gap and armature (that is exclusive of field magnets) is known. It cannot be directly found by a test as is the case with the characteristic of the whole machine, but as this partial O.C. curve has in any case to be determined by the designer the pre-determination of the drop is quite feasible. In the case that only the complete characteristic is available the same methods can be employed as above explained, but with less accuracy because the leakage flux is not a constant percentage of the useful flux and the excitation for the whole machine increases at a rather faster rate than that portion of it which is required for air gap and armature.

Synchronous Impedance. This name has been given to the ratio between the open circuit voltage and the corresponding current at short circuit if the excitation is kept at that particular value at which the O.C. voltage is measured. Here again we have to distinguish between the two kinds of characteristic values. That referring to air gap and armature alone lies higher than that referring to the whole machine. The synchronous impedance is therefore larger in the former case than in the latter. It should also be noted that the synchronous impedance is not a constant, but owing to the curvature of either characteristic diminishes with increasing excitation. In stating a definite value of synchronous impedance it is therefore necessary to also state to which of the two characteristics and to what open circuit voltage this value refers.

Graphic Representation of Power. The conception of synchronous impedance makes it easy to draw a diagram in which

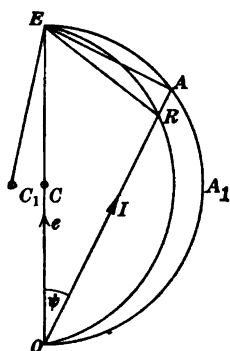


FIG. 97. Cycle diagram of A.C. generator.

the power, both of input and output, is represented by an area. Let in Fig. 97 the induced open circuit voltage be OE and OA its projection on the current vector. The mechanical power taken by the alternator is for each phase $OA \times I$. Let ωL be the total reactance of the circuit, namely that of the load plus the synchronous reactance of the machine, then $EA = \omega LI$ and with a suitable change of scale EA may be considered to represent the current. The power input is therefore twice the area of the triangle $OE A$. To get the output we must deduct from the watt-component OA the ohmic drop in the machine. This is ρI and is represented in the diagram by AR . For a constant total reactance, both EA and AR are proportional to the current, whatever the ohmic resistance of the load may be and the triangle EAR does not alter its shape, though it grows and shrinks with the current. The obtuse angle at R is therefore constant and the point R has its locus on a circle from the centre C_1 which is found by making angle $CEC_1 = \text{angle } AER$. The power output is $OR \times I$ and therefore proportional to the area of the triangle ORE . The area of the two triangles representing input and output is proportional to the product of the vertical OE and the horizontal distance from it of the points A and R respectively.

With total reactance constant there is a limit to the power the machine can take in. It is reached if A is at A_1 when the internal phase angle $\psi = 45^\circ$ and the total reactance is equal to the total resistance. Maximum output is reached at a slightly smaller internal phase angle and the total resistance slightly greater than the total reactance.

Bearing Currents. This name has been given to a phenomenon which sometimes causes trouble in the bearings of alternators. As it is due to the load it may be discussed in this chapter which treats of alternators on load. Its cause is an induced alternating e.m.f. in the shaft of the machine. An alternating current of a frequency equal to that of the generated current flows along the shaft and through the bearings, the return path being through the bedplate. This current passing through the film of oil in the bearing carbonises it and the result is a hot bearing and scoring of the journal if the current density exceeds about 15 amperes per sq. dm. of journal surface. Fig. 98 shows how these bearing currents are generated. The stator is the armature and is generally made in halves

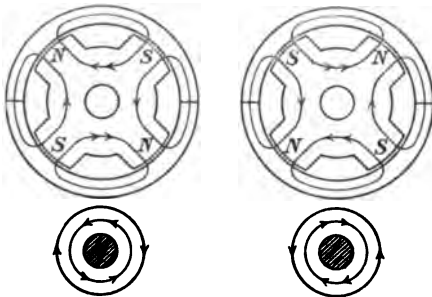


FIG. 98. Illustrating the generator of bearing currents.

with a horizontal joint. The magnetic reluctance of the joint causes a slight dissymmetry in the distribution of the flux. The figure shows the rotor in two positions differing by one half period. Since in the illustration the increased magnetic reluctance is supposed to be due to the horizontal joint in the stator, the flux in a top or bottom quadrant of the stator is slightly larger than that in a left or right hand quadrant. This is indicated by the double arrows. All these fluxes reverse twice in a period and the effect on the shaft as shown below is a resultant alternating flux which reverses twice during the periodic time. The shaft thus becomes the seat of an alternating e.m.f. having the same frequency as the main current. The remedy is either to insulate the bearings, or where that is not feasible, to fit the shaft with slip rings and brushes so as to provide an easy path for the shaft current and thereby reduce the amount which otherwise would pass through the oil film in the bearing.

Experimental Determination of $\omega\lambda$ and b . A very obvious way of finding the reactance of an armature experimentally is to impress a known e.m.f. on it from some outside source and measure frequency and current. The resistance component of the e.m.f. may be taken into account, but it is so small that the error committed on neglecting it is insignificant. If the armature is at rest the inductance of the three phases will be found not to be quite the same. This is due to the position of the coil sides in respect to the polar faces. If the coil sides of one phase happen to be well covered by the poles, their inductance will be a little greater than that of the other phases whose coil sides occupy mid positions between poles. It is therefore advisable when making this test to revolve the rotor slowly and unexcited. We shall then obtain the average value of the inductance. It is also advisable to use a ballast resistance to lessen distortion of wave form by hysteresis and to use a current of sine shape.

When current from an independent source and of suitable wave form is not available it is also possible to let the machine itself provide the testing current and several methods have been devised for this purpose. All have this in common that the machine is run on short circuit and therefore with a current lagging very nearly 90° behind the e.m.f. actually induced. Crest value of current occurs therefore in every coil side when it is midway between poles, that is, in a position where the self-induced flux owing to the increased reluctance in the space outside the heads of teeth is a little smaller than when under the pole. The total change in linkage is therefore slightly less than if a sine current is impressed from the outside whilst the poles are slowly revolving. The error is, however, slight, especially in machines with a fairly large air gap, for in this case the additional magnetic conductivity provided by the polar face can add only little to that of the air gap. Moreover, lip, shelf, flank and head leakage is so much greater than the leakage outside tooth heads that a slight increase or decrease of the latter has only a small effect on the total leakage. We may therefore conclude that the inductance measured by any of the three tests now to be described is very nearly the true inductance of the machine*.

The top of Fig. 99 shows the apparatus required for the so-called "Fischer-Hinnen Test." The alternator represented by the circle is run at normal speed and short-circuited, either directly on itself

* *Journal Inst. E. Eng.*, 1909, Part 195.

if the switch is closed or through a choking coil L if it is open. The current is indicated on an amperemeter and the e.m.f. over the inductance L on a voltmeter.

The excitation is regulated by a rheostat and the exciting current is indicated by an amperemeter in the field circuit. It is so adjusted that whether the switch be open or closed the same short-circuit current I flows through the armature. For simplicity the diagram shows the test on a single phase machine, but it is obvious that by duplicating or tripling the choking coil and its switch the test can also be made on a two or three

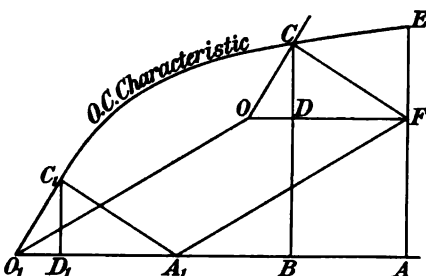
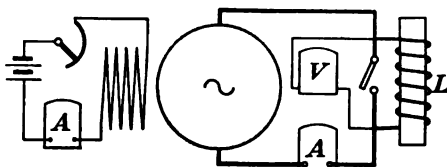


FIG. 99. Fischer-Hinnen's test for armature reaction.

phase machine. The lower part of the Fig. shows the open circuit characteristic. O_1A is the excitation of X ampere turns produced by the current i if the choking coil is in circuit and O_1A_1 the excitation X_1 produced by the current i_1 if the machine is short-circuited on itself. The e.m.f. over the choking coil is AF . If we knew the back turns of the armature we could determine at once the point C on the characteristic giving the e.m.f. actually induced and then we would find $\omega\lambda$ from $DC = \omega\lambda I$. But we do not yet know the back turns, so we must first find the condition governing the position of B . The triangle FDC (compare Fig. 94) retains its shape wherever the working point C may lie. It will therefore also have the same shape if C is at C_1 which corresponds to the exciting current i_1 when the switch is closed. Then $A_1D_1 = FD$ and $O_1A_1 = OF$. Now $O_1A_1 = X_1$, the excitation when the machine is short-circuited on itself. If then we shift the characteristic and with it the figure $O_1C_1A_1$ parallel to itself so that A_1 comes to F we obtain OCF . Having thus found the position of C we have

$$DC = \omega \lambda I \text{ and } FD = bI$$

Blondel's test requires the same apparatus as Fischer-Hinnen's, but the excitation is kept constant. Hence when the switch is closed

the armature current is larger than when it is open. Let I and I_1 be the two values respectively and let e be the e.m.f. over the choking coil indicated on the voltmeter when the switch is open. If we knew reactance and back turns we could find the working point on dead short circuit by drawing in Fig. 100 the line AC under an angle β so that $\tan \beta = \frac{\omega \lambda}{b}$. The inductive drop being proportional to the current we determine a point C_1 so that

$$\frac{AC}{AC_1} = \frac{I}{I_1}$$

This ratio is obtained by the amperemeter indications. The working point with the choking coil in circuit would then be C_0 . But C_0C_1

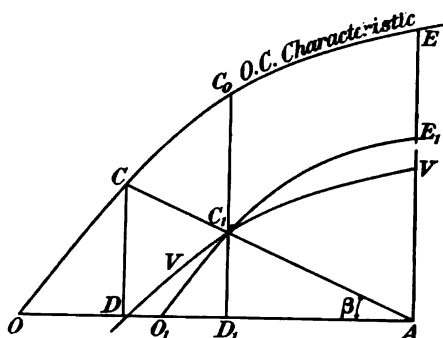


FIG. 100. Blondel's test for armature reaction.

is the inductive drop over the choking coil which is indicated on the voltmeter, so that the point C_1 has to satisfy two conditions. It must lie on a curve which everywhere is lower than the O.C. characteristic by the amount e and it must also lie on a ray such as AC and at such a distance from A as

$$DC = \omega \lambda I \text{ and } AD = bI$$

Both the methods here described require for three phase machines three choking coils and this makes the test if applied to large machines rather expensive. To get reliable figures the choking coils should have about the same inductance as the armature phases and as they have to pass the same current they become rather formidable pieces of apparatus. There is also a possibility of error due to the upper harmonics causing the voltmeter to indicate rather too large an e.m.f. with the consequence that the VV curve is drawn a little too low thus making the reactance appear a little

too small and the back turns a little too large. The last named defect is not a serious matter, but the necessity to provide three large choking coils especially if a test is to be made on site is a drawback. This is overcome in the author's method, which however is only applicable to two or three phase machines*. As three phase star coupled machines are those most commonly used it is convenient to discuss the test in relation to these. The application to two phase machines will then be quite obvious.

The machine is run at normal speed and two short circuit characteristics are taken. One with all three phases short-circuited and the other with only one phase short-circuited. In

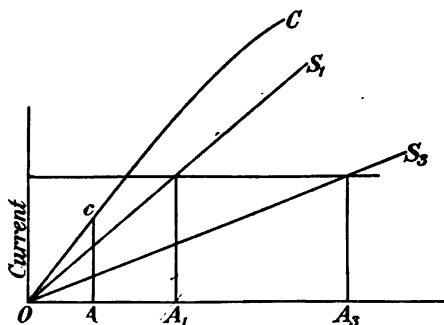


FIG. 101. The author's test for armature reaction.

Fig. 101 the line OC is the lower part of the O.C. characteristic, the straight line OS_1 is the graph connecting excitation and current if only one phase is short-circuited, and the straight line OS_3 is the ordinary short-circuit characteristic with all three

phases short-circuited. Let bI be the back ampere turns in the latter condition, then with only one phase short-circuited the back ampere turns are $bI/3$. The ampere turns required to produce that e.m.f. which represents $\omega\lambda I$ in each phase are not materially affected by the number of phases which are actually short-circuited and may therefore be considered constant if the excitation be so adjusted that the same current flows whether one phase is short-circuited or all three†.

* *Journal Inst. El. Eng.*, 1909, Part 195.

† This statement is strictly true for a two phase machine because the two armature circuits are in quadrature and the self-induced flux of one phase does not thread the coils of the other phase. In a three phase machine there is some interference, but only as regards a very small part of the self-induced flux. If the magnetic interlinking of the phases were so perfect as to make the mutual induction equal to the self-induction then the flux with only one phase short-circuited to star point would be augmented by 50 per cent. if all three phases were so short-circuited, but the interlinking is by no means perfect. Most of the self-induced flux encircles each group of wires closely without threading any other wires, so that the increase of $\omega\lambda$ in one phase due to the other two phases is only of the order of a few per cent. This matter can easily be tested by sending A.C. from some outside source through the armature whilst the field is rotating, but not excited. In one test on a commercial machine I have found that the increase in $\omega\lambda$ is only 6 per cent. The error thus introduced makes b come out a few per cent. too large and $\omega\lambda$ a few per cent. too small.

Let X_3 be the excitation required to produce I on complete short-circuit and X_1 the excitation for the same I on partial short circuit. Let further X_s be the excitation required for the production of $e_s = \omega\lambda I$, then the two conditions may be represented by

$$X_3 = X_s + bI \quad \text{and} \quad X_1 = X_s + \frac{bI}{3}$$

In the figure $X_1 = OA_1$ and $X_3 = OA_3$. From these equations we find

$$bI = \frac{3}{2}(X_3 - X_1) \quad \text{and} \quad X_s = \frac{3X_1 - X_3}{2}$$

In the figure $X_s = OA$ and the corresponding e.m.f. AC can be found from the O.C. curve. We have then

$$\omega\lambda = \frac{AC}{I}$$

For the determination of $\omega\lambda$ a knowledge of the field winding is not required. The abscissae in Fig. 101 may be ampere turns or simply exciting amperes. If the latter, then the formula for X_s is replaced by

$$i_s = OA = \frac{3i_1 - i_3}{2}$$

i_1 and i_3 representing the exciting amperes for partial and complete short-circuit respectively. To find the value of b it is necessary to know the number of turns on the magnet poles. If, however, the object of the test is to obtain the necessary data for finding the terminal voltage under load, then an exact knowledge of b is not required and it is not necessary to know the number of turns on the field. In constructing the diagram as in Figs. 95 and 96 we reduced armature ampere turns to the equivalent exciting current. Thus to an armature current of I_0 amperes corresponds an exciting current of i_0 amperes. The meaning of this is that to neutralise the excitation given by I_0 amperes in the armature an exciting current of i_0 amperes is required in the field winding, or in symbols

$$bI_0 = ni_0 \quad \text{or} \quad i_0 = \frac{b}{n} I_0$$

Since $X_3 = ni_3$ and $X_1 = ni_1$ we have

$$bI = \frac{3}{2}(X_3 - X_1) = n \frac{3}{2}(i_3 - i_1)$$

$$\frac{b}{n} = \frac{3}{2} \left(\frac{i_3 - i_1}{I} \right)$$

The field current corresponding to an armature current I_0 is therefore

$$i_0 = I_0 \left(\frac{3(i_3 - i_1)}{2I} \right)$$

the bracket being the reducing factor which is obtained by the above described test. It will be seen that to obtain this reducing factor a knowledge of the field winding is not required.

Incidentally the test also gives us $\tan \beta$ (compare Figs. 94 and 96). We had $\tan \beta = \omega\lambda/b$. This may be written

$$\frac{\omega\lambda I}{bI} = \frac{AC}{bI} = \frac{X_s \tan \alpha}{\frac{3}{2}(X_3 - X_1)} = \frac{2}{3} \tan \alpha \left(\frac{X_s}{X_3 - X_1} \right)$$

Since $OA \tan \alpha = AC$ and $AC = AA_3 \tan \beta$,

$$\text{we have} \quad \tan \beta = \frac{OA \tan \alpha}{AA_3} = \frac{OA \tan \alpha}{OA_3 - OA}$$

We found on p. 206

$$2(OA) = 3i_1 - i_3.$$

Inserting this and remembering that $OA_3 = i_3$ we obtain

$$\tan \beta = \frac{\tan \alpha}{3} \left(\frac{3i_1 - i_3}{i_3 - i_1} \right)$$

CHAPTER IX

COUPLED ALTERNATORS

Alternators in series and in parallel coupling—Mutual control—Natural period of vibration—Parallel operation—Critical value of WD^2 —Surging power in parallel operation—Parallel operation represented by vector diagram—Theory of dampers—Determination of damping factor b —Sectional area of damping grid—Necessity of dampers in converters—Bus bars not infinitely strong—V curves—Synchronisers.

Alternators in Series and in Parallel Coupling. Fig. 102 shows simplified vector diagrams for these two cases. The simplification consists in neglecting internal losses and assuming that L is constant. Let the machines be series coupled and mechanically connected in exactly the same phase position. They will behave as one machine of double voltage, each engine supplying half the power.

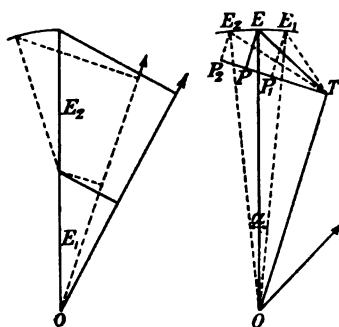


FIG. 102

In the diagram on the left E_1 and E_2 are the O.C. volt vectors of the machines I and II and the power required from each engine is proportional to the projection of the volt vectors on the common current vector. Now loosen the mechanical connection and assume that engine II exerts a slightly greater torque. This will push the e.m.f. vector of machine II into the dotted position and since its projection on the current vector is thereby shortened the resisting torque of machine II will be lessened and that of I increased. But as the driving torque of II is larger this machine will be pushed still further forward so that its condition will be unstable. Machines independently driven cannot work in series.

Now take the case of parallel coupling shown on the right. Assume at first both engines to exert the same torque and the machines to be equally excited. Their O.C. voltage is OE and their terminal voltage OT . The power required by each is proportional to the area of the triangle OTE and since the terminal voltage is constant the power is also proportional to the height of E over

the base line OT . In other words TP is a measure for power and torque. Now let engine II push its machine ahead so that the e.m.f. vector occupies the position OE_2 . The torque is now TP_2 , that is, greater than before. Since the power supplied to the load has not changed, machine I must give less power than before, namely TP_1 instead of TP . The engine pushing ahead is more heavily loaded and the engine deficient in power and therefore lagging is relieved of some of the load.

The condition is therefore perfectly stable and machines can work together if coupled in parallel. For simplicity Fig. 102 has been drawn for machines excited to the same voltage, but as can easily be seen, the same deduction follows if the excitations should be different.

Mutual Control. The additional torque required by the leading machine and the reduction in torque experienced by the lagging machine constitute a mutual control, tending to bring the machines back into the same phase relation, that is to ensure synchronous working. The power corresponding to the additional torque or that corresponding to the deficiency in torque is called the "synchronising power"; and the current which surges between the two machines over and above that which each delivers into the external circuit is called the "synchronising current." Its strength is determined by the resultant e.m.f. of the two vectors (such as the length E_1E_2 in Fig. 102) and the total impedance of the machines including connections to the switch board, also in the case of the machines being in two different generating stations the impedance of the connecting line. If α is the angular deviation of the two e.m.f. vectors in radians, then $e = \alpha E$ is the resultant e.m.f. driving the synchronising current through the two machines. Its strength is

$$i = \frac{e}{2 \sqrt{(\omega L)^2 + R^2}}$$

where ωL is the synchronous reactance of one machine in that working condition which corresponds to load, power factor and excitation and R the ohmic resistance. When the connections have a sensible impedance ωL and R must be suitably increased. In relation to the voltage e which produces it the synchronising current i has a watt- and a wattless component; only the latter interests us at present, because it is nearly in phase with the e.m.f. vector of the machine and the product of the two is the synchronising power of

one phase. The wattless component of i is $i\omega L/\sqrt{(\omega L)^2 + R^2}$ and the corresponding power is $Ei\omega L/\sqrt{(\omega L)^2 + R^2}$. Inserting the value of i and multiplying by 3 we get the synchronising power of each machine for a deviation of $\frac{\alpha}{2}$ radians from the position both would occupy if there were no deviation

$$P_{syn.} = \frac{3}{2} \alpha E^2 \frac{\omega L}{(\omega L)^2 + R^2} \text{ watts}$$

ωL and R being synchronous reactance and resistance of one phase. This expression shows that resistance reduces the synchronising power and that with a given resistance maximum synchronising power is obtained if the reactance is equal to the resistance. A further reduction of reactance would decrease the synchronising power; and in fact, if it were possible to build machines which have no reactance (a physical impossibility) such machines could not work in parallel.

It is convenient to reduce synchronising power to some standard condition. As such we may take an advance or retardation of $\frac{1}{2}^\circ$ electrical, or $\frac{1}{2p}$ mechanical, of the vector from a datum vector indicating the mean position. Inserting the phase e.m.f. actually induced in kv. we get the synchronising power in kw.

$$P_{syn.} = 26 (kv.)^2 \frac{\omega L}{(\omega L)^2 + R^2} \dots \dots (68)$$

The internal resistance of a machine is very small in comparison with the reactance and if the machines are standing side by side in the same station the resistance of the switch board connections is also negligible, so that we have the simpler formula

$$P_{syn.} = 26 \frac{(kv.)^2}{\omega L} \text{ kw. for } \frac{1}{2}^\circ \text{ deviation} \dots \dots (69)$$

This applies to machines running light. When under load the e.m.f. actually induced is a little greater than the terminal e.m.f., so that by inserting for (kv.) the terminal e.m.f. we shall slightly underestimate the synchronising power. The error can be avoided by drawing a vector diagram such as Fig. 95 and inserting for (kv.) the vector OS instead of the vector OT .

Natural Period of Vibration. If the exact coincidence between the e.m.f. vectors of two machines has been disturbed so

that one is a little ahead and the other a little behind the datum vector, and if immediately after the disturbance the driving torque of both machines is restored to the same and constant value, the machines will by mutual control pull each other into step again with a torque which is proportional to the deviation from the datum vector. When arrived at the datum vector they must pass it owing to the inertia of the mass of the rotor and thus a vibration around the datum vector is set up which on account of friction and other damping forces dies down eventually. If on the other hand the disturbing impulse should be repeated with such sequence that there is coincidence with the natural period of vibration, the amplitude of the vibration, instead of diminishing by degrees, will increase until the machines come out of step. Such disturbing impulses are always present if the prime mover is some type of piston engine, but they may also be caused by the governor of any type of prime mover or by some consuming device fed from the circuit. Whatever the cause, it is important to know the sequence of disturbing forces and to be able to calculate beforehand the natural period of vibration of the alternator so as to make sure that the two do not coincide.

Since the controlling force is directly proportional to the angular deviation from the datum vector we may apply a well-known law of mechanics to determine the periodic time. For slightly damped systems this is almost the same as for undamped systems, namely,

$$T = 2\pi \sqrt{\frac{m}{c}}$$

Here m represents the rotating mass reduced to a definite radius and c is the control factor reduced to the same radius. For convenience we may take 1 metre as the radius and m in units of 9.81 kg.; then c is the force corresponding to a linear deviation of 1 metre. Since engine speed is generally given per minute it is convenient for comparison to state not the periodic time in seconds for one cycle, but the number of cycles of to and fro swings performed in one minute. Let S be this number then

$$S = \frac{60}{2\pi} \sqrt{\frac{c}{m}}$$

The problem is now reduced to finding the value of c as determined by the synchronising power given by (68). The mass m , since it refers to one metre radius, is numerically equal to the moment of inertia. Instead of the true moment of inertia the product of weight

multiplied by the square of the diameter is sometimes used as a measure of the flywheel effect, or

$$WD^2 = 9.81 \cdot (2R)^2 m$$

For $R = 1$ metre and W in kg. this gives

$$m = \frac{WD^2}{4g} \text{ or } m = \frac{WD^2}{39.24}$$

The most usual way is, however, to specify the flywheel effect as so many ft.-lbs. or metre kilograms or metre tons of stored energy at the normal speed of U revs. p.m. Let A be the energy in m kg. thus represented, then the moment of inertia and also m at one metre radius is

$$m = \frac{2A}{U^2} = 182.5 \frac{A}{U^2}$$

Since V is the linear speed per second at one metre radius or 2π times revolutions per second, we also have

$$V = 2\pi \frac{f}{p} \text{ and } m = \frac{2A}{(2\pi)^2} \left(\frac{p}{f}\right)^2$$

To find c we establish the relation between synchronising torque (at 1 m. radius numerically equal to tangential force F) and the linear advance corresponding to $\frac{1}{2}^\circ$ or $\frac{\tau}{360}$ where τ is in metres.

Let $P_{\text{syn.}}$ be synchronising power in watts, then

$$FV = \frac{P_{\text{syn.}}}{9.81} \text{ or } F = \frac{P_{\text{syn.}}}{9.81} \frac{p}{2\pi f}$$

Since $F = c \frac{\tau}{360}$ and $\tau = \frac{\pi}{p}$ we also have $F = \frac{c}{360} \frac{\pi}{p}$ and from the two equations for F we find

$$c = \frac{P_{\text{syn.}} \cdot 360 p^2}{9.81 \cdot 2\pi^2 f}$$

or combining with (69) for $P_{\text{syn.}}$ in watts

$$c = 0.048 \left(\frac{E}{\omega L}\right) E \frac{p^2}{f}$$

$$\frac{c}{m} = \frac{360 P_{\text{syn.}} f}{9.81 A} \text{ and } \sqrt{\frac{c}{m}} = 6.058 \sqrt{\frac{P_{\text{syn.}} f}{A}}$$

Combining with S and inserting $P_{\text{syn.}}$ from (68) we find with near approximation for A in metre tons

$$S = 0.3 E \sqrt{\frac{f}{A} \frac{\omega L}{(\omega L)^2 + R^2}} \quad \dots \quad (70)$$

In this formula S is the natural number of complete swings per minute, E is the voltage actually induced per phase and A is the stored energy in metre tons. The frequency f is to be given in cycles per second. When the machines are side by side the resistance of the connections is negligible and that of the winding is very small in comparison with the reactance so that we may write with close approximation

$$S = \frac{0.3E}{\sqrt{\omega L}} \sqrt{\frac{f}{A}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

This formula may be written

$$S = 0.3 \sqrt{\frac{E}{\omega L} E \frac{f}{A}}$$

The first term under the root is an ideal short circuit current in one phase and the second term is an e.m.f. The product represents volt-amperes in one phase. The total voltamperes are three times this amount or $1000 P_m$, if by P_m we denote the kva. of the short circuited machine. Working out we obtain the usual formula for the swings

$$S = 5.4 \sqrt{\frac{f}{A} P_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

where A , the stored energy in the rotor, is to be inserted in metre-tons. If this energy is to be reckoned in foot-tons we have

$$S = 9.76 \sqrt{\frac{f}{A} P_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

This formula is strictly correct, but there is the difficulty of accurately estimating the value of P_m . According to one method, generally called the "short circuit method," P_m is found as follows: Let X be that part of the excitation which is required for the air gap and armature when the machine is working under normal conditions at full load P and X_s the corresponding value when the machine is giving full load current on short circuit, then

$$P_m = \frac{X}{X_s} P$$

This method gives S too small. A nearer approach to the correct value is obtained by what is known as the "distortion method" of Hobart and Punga*. According to this the geometric displacement between datum vector and centre of pole is divided into two parts.

* Everest, *Journal Inst. El. Eng.*, No. 219, p. 520.

The larger part ζ is the distortion due to the cross turns and the smaller part η (which is the difference between internal and external phase angle) is due to self-induction. The distortion angle depends on armature current, field excitation and the ratio of pole arc to pole pitch. It may be calculated from the formula

$$\zeta = \text{distortion angle} = \frac{\text{Armature ampere turns}}{\text{Field ampere turns}} \times K$$

where K is a coefficient for which Everest gives the following table:

Ratio pole arc pole pitch	K in degrees
0.5	10
0.6	13.5
0.7	18
0.8	24
1.0	40

The last line refers to turbo machines where no salient poles are used.

The angle η can be calculated from the ratio of self-induced to effective flux. Everest assumes that only about one-half of the self-induced flux is interlinked with the exciting coils and he adds therefore $\frac{\eta}{2}$ to the distortion angle to get the total angular deviation

between datum vector and polar axis. Let $\alpha = \zeta + \frac{\eta}{2}$, then $P_m \sin \alpha$ is the synchronising power for the total deflection α . This becomes a maximum, namely P_m for 90° , but as we substitute the angle for its sine the maximum corresponds to an angle of 1 radian or 57.3° . If P is the power for which the deflection α has been obtained as above explained we have

$$P_m = P \frac{57.3}{\alpha}$$

The author's method of finding S is based on the following reasoning. The synchronising power being alternately positive and negative does not affect the average power given out by the engine. We must consider the output of the machine to be balanced by the power of the engine, so that one cancels the other and as far as the synchronising current is concerned the machine is in the same condition as an unloaded machine, but only excited to the working point C (compare Fig. 95). The reactance corresponding to that working point is the value of ωL to be inserted in (71) in order to find S . If load and power factor are prescribed the position of the

working point can be found by the construction of Fig. 95, or approximately by that shown in Fig. 96. Let in Fig. 103 OE be the open circuit e.m.f. characteristic of air gap and armature and let for a given load condition C be the working point. Imagine now a very small and wattless demagnetising current flowing through the armature. This will push the working point a little below C . The reduction in the e.m.f. is that due to demagnetisation, and dividing it by the current we obtain the value of a kind of spurious reactance $\omega\lambda_1$, which must be added to the true reactance $\omega\lambda$ to make up the total ωL . But as the construction owing to the small scale would be inexact, we assume an armature current of any convenient magnitude and run the point C down not on the characteristic

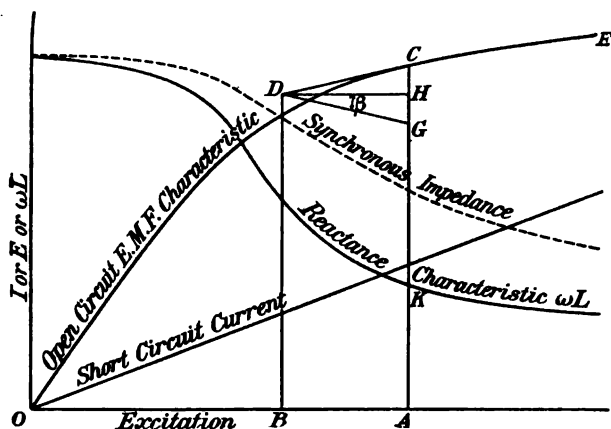


FIG. 103. Construction of reactance characteristic.

itself, but on its tangent. Thus for an armature current I which gives the demagnetising force AB we find the point D . The drop due to demagnetisation is therefore CH and to this must be added the true inductive drop $I\omega\lambda = HG$. The total reactance is therefore

$$\omega L = \frac{CG}{I}$$

which plotted to any convenient scale gives the ordinate AK . The construction repeated for different working points gives the curve marked "reactance characteristic." As it is independent of load, it is in the strict sense of the word a characteristic curve for that particular machine at its normal speed. A study of the open circuit

and reactance characteristics shows certain interesting facts which are quite in accordance with practical observation. The same machine used at different voltages shows a sensible difference in the time of swing. At high voltage it swings more quickly. There is also a slight speeding up of the swing at heavy load and especially if combined with a bad power factor, because in this case the working point lies a little higher. Machines with "weak armatures" and strong fields swing more quickly than machines of the same output, but having weaker fields, and to compensate for this, more conductors on the armature. A slow swing is generally desirable, but there is a disadvantage in a "strong armature" because, as will be seen from (69), the synchronising power is reduced. The dotted curve shows the synchronous impedance. It will be noticed that within the working range this curve lies above the ωL curve and this explains why the swing if calculated by the short circuit method (which is based on synchronous impedance) comes out too slow.

Parallel Operation. When a number of machines deliver current to the same bus bars they are in parallel operation. Irregularities in driving torque tend to disturb the synchronous running and the question arises whether in any particular machine this disturbance may become so great as to throw it out of step. Even short of this a large amount of power surging between the machine and the bus bars would be objectionable on account of causing fluctuations in the voltage over the whole system, so that we have to investigate two things, first what are the conditions making parallel operation altogether impossible and secondly what amount of synchronising power has to be given and taken by the machine to keep it in step. In order to restrict the investigation to one machine we assume that it works on a system of such magnitude that the general voltage is not affected by the action of this one machine. This means in mathematical language that the machine is connected to infinitely strong bus bars which can take any power at any power factor and yet have a datum vector of constant magnitude and revolving at constant speed.

The first question is easily answered. If there is no provision made for reducing the violence of the swing by some damping arrangement, the machine must break step if the disturbing impulses due to the engine synchronise with the natural frequency of oscillation as given by formula (72). In a single cylinder gas engine working

on the Otto cycle we have one impulse at every second revolution, so that if U is number of r.p.m. there are $U/2$ disturbing impulses per minute and the machine cannot keep step if $S = \frac{U}{2}$. In a single crank steam engine there is one impulse per revolution if single acting and two if double acting. In a two crank engine with cranks at 90° there are four impulses per revolution, in a three crank engine there are six and so on, the dangerous conditions being respectively S equal to U , $2U$, $4U$, $6U$. These are strong impulses, but not necessarily all equally strong. Thus in a tandem engine the out stroke may represent more or less energy than the instroke, so that upon two equally strong impulses occurring twice a revolution we may have superimposed one weak impulse occurring once a revolution. The same holds good for a three or four crank engine. The diagrams of all cylinder ends are never quite equal. We may therefore expect that whatever type of piston engine is used, it will have, besides several quick and strong disturbing impulses, one weaker impulse in every revolution and in order to ensure good parallel operation the number of natural swings of the machine should be sensibly lower than the number of r.p.m. In a single cylinder gas engine there is one explosion every two revolutions. We have therefore a disturbing force enormously stronger than the average driving force and to make parallel operation possible the periodic time of natural swing must be longer or shorter than the time occupied in two revolutions. $S = \geq \frac{U}{2}$. But even in a multi-cylinder gas engine with two or more cranks it is probable that one cylinder gives a stronger impulse than the others, so that there will be not a very large, but still a sensible impulse superimposed on the strong impulses of the different cylinders once in every two revolutions, so that also in this case, apart from a possible damping arrangement, S should be smaller or sensibly larger than $U/2$. In addition to disturbances inherent in the engine itself there may be disturbing impulses introduced by the action of the governor. In any modern governor we have a system of rotating masses controlled by springs and this system has itself a periodic time; if this happens to be near the periodic time of the free oscillations of the machine, parallel operation becomes impossible even if the normal turning moment of the prime mover should as in steam or water turbines be free from disturbing impulses.

Critical Value of WD^2 . Let n be the number of disturbing impulses per revolution, then nU is the number of disturbing impulses per minute and in order to avoid resonance S must be larger or smaller than nU , whilst

$$S = nU$$

constitutes the condition of resonance which in the absence of a strong damping force must throw the machine out of step. It should be noted that n may in one and the same prime mover have different values. In a tandem steam engine we have $n = 2$ for the strong impulses and also $n = 1$ for weak impulses. In a two cylinder gas engine the figures are $n = 1$ and $n = \frac{1}{2}$ respectively. With more cylinders there will be a greater variety in the values of n which may be regarded as the order of upper harmonics defining the complicated curve representing the crank pin effort during one cycle of the prime mover. Since the periodicity of these upper harmonics is certainly much greater than that of the natural swing we need only consider the two lowest values of n , namely $\frac{1}{2}$ and 1 for a gas engine and 1 and 2 for a steam engine. The synchronising power for a deviation of α radians (electrical) from the datum vector is $\alpha P_m = FVg$ where F is the tangential force at 1 m. radius and $g = 9.81$.

$$F = \frac{\alpha}{p} c; \quad V = 2\pi \frac{U}{60}; \quad P_m = \frac{2\pi U}{60} \frac{cq}{p} \quad \text{and} \quad c = \frac{P_m 60p}{2\pi Ug}$$

Previously we found $m = \frac{WD^2}{4g}$ and we have therefore

$$\sqrt{\frac{c}{m}} = \sqrt{\frac{P_m 60p 4g}{2\pi Ug WD^2}}$$

Combining with $S = \frac{60}{2\pi} \sqrt{\frac{c}{m}}$ we find for the resonance condition of $S = nU$

$$WD^2 = \frac{P_m p}{2\pi^3 n^2 \left(\frac{U}{60}\right)^3}$$

and since $U = \frac{60f}{p}$ and $2\pi^3 = 61.5$ we also have

$$WD^2 = \frac{P_m p}{n^2 61.5} \left(\frac{p}{f}\right)^3$$

If P_m is in voltamperes WD^2 is in $\text{kg.} \times \text{m.}^2$. If P_m is in kva., WD^2 is in $\text{tons} \times \text{m.}^2$. Adopting the latter notation we find

$$WD^2 = \frac{\text{kva.}}{n^2} \frac{p}{61.5} \left(\frac{p}{f}\right)^3 \quad . \quad . \quad . \quad (74)$$

and with a slight transformation we have also

$$WD^2 = f \frac{\text{kva.}}{n^2} \frac{210,000}{U^4} \quad . \quad . \quad . \quad (75)$$

To avoid resonance the flywheel must be either heavier or lighter than corresponds to these formulae. To make it lighter than the critical value of WD^2 which corresponds to $n = 1$ would be mechanically impracticable because a fairly large flywheel effect is necessary to insure steady running. To make the flywheel heavier than corresponds to $n = \frac{1}{2}$ in a gas engine is not always practicable. If the speed is low, say of the order of 100, and the frequency normal the weight of the flywheel would become prohibitive on account of cost, space and bearing friction. In such cases the WD^2 must be so chosen that it is well over the critical value for $n = 1$ and well under the critical value for $n = \frac{1}{2}$. Since these two values are in the ratio of 1:4 there is ample margin for a suitable choice. With high speed gas engines there is no difficulty in providing a WD^2 well over the critical value for $n = \frac{1}{2}$.

Surging of Power in Parallel Operations or Hunting.

In the foregoing we have only studied the way in which absolute resonance can be avoided, but even if this is done, some power must flow into and out of the machine to keep it in step. This involves a certain amount of irregularity in working which is technically termed "hunting." We shall now proceed to investigate this problem quantitatively, first by neglecting damping and secondly by taking it into account. Let F be the disturbing force which we assume to follow a sine law. It may then be represented by a rotating vector the angular speed of which is

$$\Omega = \frac{2\pi Un}{60}$$

The disturbing force at time t is

$$F \sin \Omega t$$

This force has to accelerate the mass m and overcome the controlling

force which for a linear deviation x is cx . c has the dimensions MT^{-2} . We have therefore the equation of motion

$$F \sin \Omega t - m \frac{d^2 x}{dt^2} - cx = 0 \quad . \quad . \quad . \quad (76)$$

We try whether $x = \frac{F}{c - m\Omega^2} \sin \Omega t$ is a solution, differentiating once, and once more we find

$$\begin{aligned} \frac{dx}{dt} &= \Omega \frac{F}{c - m\Omega^2} \cos \Omega t \\ \frac{d^2 x}{dt^2} &= -\Omega^2 \frac{F}{c - m\Omega^2} \sin \Omega t = -\Omega^2 x \end{aligned}$$

Inserting into (76)

$$F \sin \Omega t + \frac{m\Omega^2 F}{c - m\Omega^2} \sin \Omega t - \frac{cF \sin \Omega t}{c - m\Omega^2} = 0$$

This proves that $x = \frac{F}{c - m\Omega^2} \sin \Omega t$ is a solution of (76). The maximum deviation occurs when $\Omega t = \frac{\pi}{2}$ and its value is

$$x = \frac{F}{c - m\Omega^2} \quad . \quad . \quad . \quad . \quad (77)$$

If the machine were working alone and therefore not under the controlling influence of infinitely strong bus bars c would be zero and the deviation of its vector from a datum vector rotating with constant speed would be $\frac{F}{m\Omega^2}$. The speed of the machine vector is not constant, but varies by $\pm \epsilon \Omega$ and the ratio between this value and the mean speed is a measure of the unsteadiness of the motion. It is called the coefficient of fluctuation of speed or the coefficient of irregularity. If U_1 and U_2 are highest and lowest speed respectively, the coefficient of irregularity of the engine working alone is

$$\epsilon = \frac{U_1 - U_2}{U_1 + U_2}$$

Some engineers take double this value (namely difference of speed divided by mean speed) as the coefficient of irregularity, but for our purpose it is more convenient to take for ϵ the value as defined above. The numerical value of ϵ lies then between 1/200 and 1/500 according to type of engine and weight of flywheel.

Let ϵ_1 be the coefficient of irregularity if the machine is working on bus bars, then we have

$$\epsilon_1 = \epsilon \frac{m\Omega^2}{m\Omega^2 - c}$$

The irregularity is greater than when the machine is working alone.

Equation (77) shows the relation between disturbing force F and deviation x as affected by mass and strength of control. If there is no control as in a machine working alone x has the opposite sign to F . This means that at the moment when the engine exerts the strongest driving torque the machine offers the smallest resisting torque, the difference being used in acceleration. The same applies if the controlling force is weak in comparison to the mass effect, that is if $c < m\Omega^2$. If the control is strong as compared to the mass effect, or $c > m\Omega^2$, then x and F have the same sign and at the moment when the engine exerts greatest tangential force the machine is in a phase position corresponding to greatest mechanical resistance. Thus the relation between c and $m\Omega^2$ determines the character of the vibration. If the two are equal x becomes infinite, that is to say, the machine falls out of step. This is in accordance with the formula for the periodic time given on p. 211.

Power and tangential force being proportional, we may consider the tangential force corresponding to a deviation x as a measure for the power which surges between machine and bus bars over and above the power normally delivered. The crest value of the surging power in watts is

$$P_s = 9.81xcV$$

where V is the speed in metres. The original disturbing power due to the engine alone has the crest value

$$P_d = 9.81FV$$

$\frac{P_s}{P_d} = \frac{x}{F} c$ combining with (77) we have the surging power

$$P_s = P_d \frac{c}{c - m\Omega^2}$$

Let $\frac{c}{m\Omega^2} = q$ then

$$P_s = P_d \frac{q}{q - 1} \quad . \quad . \quad . \quad . \quad . \quad (78)$$

$\frac{q}{q - 1}$ is a numeric showing the relation between the original disturbing power and the surging power it produces. Some writers call it the "wobble factor"; I prefer the term "surge factor" as more directly indicating its nature. For $q = 1$ (mass effect equal to controlling effect) the surge factor becomes infinite and the machine comes out of step. For $q = \frac{1}{2}$ (mass effect twice as great as controlling effect) the surge factor is unity and negative. The physical

meaning of the negative sign is that there is a time lag of $\frac{1}{2}$ period between disturbing and surging power. For $q < \frac{1}{2}$ the surging power is less than the disturbing power. For $q = 2$ (control effect stronger than mass effect) the surge factor is also 2 and there is no time lag between disturbing and surging power. For larger values of q the surge factor becomes less than 2. To make parallel operation possible q must be either smaller or larger than unity and the farther away it is from unity the smaller is the surging power. As a rule engines require for the purely mechanical condition of steady running so heavy a flywheel that in a generating set the

mass effect predominates over the controlling effect, so that q is smaller than unity. A possible exception is a very large and slow speed multicrank gas engine where the lowest value of Ω corresponding to a weak impulse every second revolution would necessitate too heavy a flywheel. In such a case q would be larger than unity for the slow and weak impulse, but smaller than unity for the stronger and quicker impulses.

The use of a heavy flywheel may be considered normal for generators. Fig. 104 shows a chart illustrating the case of a tandem steam engine. The top line represents crank pin effort, the next the equivalent sine curve. The ordinates are either driving torque or force at 1 metre radius or power. The average force is F_0 , which also represents the mean power. Superimposed

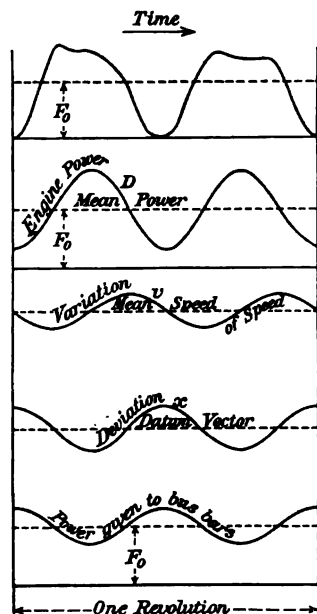


FIG. 104. Heavy flywheel tandem steam engine.

on this is the disturbing power. The third line shows speed variation and the fourth the resulting deviation of the machine vector from the datum vector corresponding to the mean power. The last line shows the surging power for a set in which the mass effect is three times as strong as the controlling effect.

We have hitherto considered the machine as a generator, but since there must be balance between the average power of bus bars and machine we have to deal only with power fluctuation

and therefore our reasoning applies equally to the case where the machine is used as a motor, the only difference being that x and P_s are of opposite sign. The primary source of surges is then not the engine driving the alternator, but the tool which the latter drives. A chart for such a case is shown in Fig. 105. The machine is supposed to be driving a two-crank double-acting pump with an air vessel large enough to eliminate any inertia effect of the water in the delivery main. We have then only to consider the mass effect of the rotor and the variation in the torque due to the constant head and variable positions of the cranks. We assume that the pump has no flywheel or only one of so little mass that $c > m\Omega^2$ which means $q > 1$. It is convenient to start the explanation with the speed curve; that is the top line in the figure. The phase relation between speed and deviation, being purely geometric, is the same as in Fig. 104. The deviation lags 90° behind the speed variation. The disturbing power P_s is known from the work the pump has to do. The surging power being negative for a positive value of x , we have $P_s = -P_a \frac{xc}{F}$

or from (76) $P_s = -P_a \frac{q}{q-1}$.

At the moment the machine vector has the greatest lag behind the bus bar vector the machine receives a maximum of surging power. Part of this is absorbed to produce acceleration and only the rest is given to the pump. This is represented by the last line of the diagram, the ordinates of which are the differences of those of the two curves above. In this case the surging power is greater than the irregularity in the power demand of the pump. The latter is at least 40 per cent. of the power represented in water lifted and the irregularity in the power drawn from the bus bars must be more than 40 per cent. A slight increase of the mass would

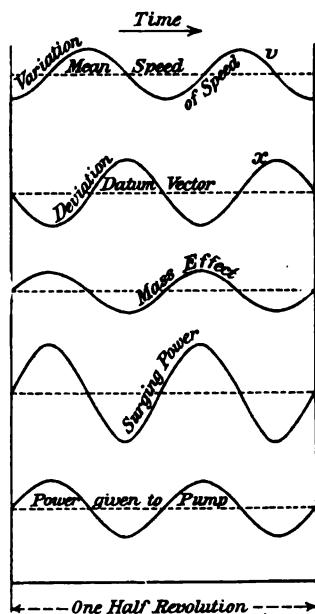


FIG. 105. Small flywheel effect. Two-crank double-acting pump.

make matters worse because increasing the ordinates of the curve marked "mass effect." Since the power fluctuation demanded by the pump is a given amount and equal to the difference between the ordinates of the two curves above, it follows that any increase in the amplitude of the mass effect must be accompanied by an increase in the amplitude of the surging power. Thus a slight increase in the weight of the flywheel would be worse than useless. A very large increase might do good; for instance if we added mass enough to make $q = \frac{1}{2}$ when the surging power would not exceed the disturbing power, but very nearly the same effect might be obtained by using no flywheel at all when q would become large and the surge factor $q/(q-1)$ approach unity. The practical conclusion to be drawn from this study is that the indiscriminate use of a flywheel on a synchronous motor may do harm and that on the whole such a motor (especially if the periodic time of its disturbance is variable) had better be left without a flywheel.

Parallel Operation Represented by Vector Diagram.

By drawing diagrams such as Figs. 104 and 105 the relative position

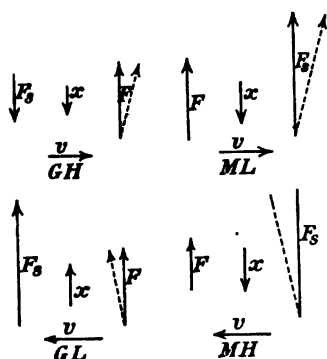


FIG. 106.

of the vectors of disturbing force F (whether due to the prime mover if the machine is used as a generator or to the tool driven if the machine is used as a motor), the surging force F_s , both at one metre radius, the linear deviation x from the mean position of the machine vector and the difference in speed v corresponding to this deviation may be found and represented in a simple chart as shown in Fig. 106. In this diagram the

inscription *GH* means that the machine is used as a generator and that its rotating masses are heavy. *ML* means a motor with light rotating masses; *GL* a generator with light and *MH* a motor with heavy rotating masses. All this refers to an absolutely undamped machine. This is an ideal case, for even if no special damping arrangements are provided, there must be some slight interference with the free swing by reason of friction and of eddy currents in the pole shoes. Any damping force acts in opposition to the

direction of motion, that is, to the v vector and in a generator the disturbing vector F must have a component in the direction of v . It can therefore not be absolutely parallel to the surging vector, but must slightly lag in the case of GH and slightly lead in the case of GL as shown by the dotted lines. Lag and lead are however so small that practically the chart Fig. 106 as drawn in full lines may be taken as a correct representation of the relative position of the vectors concerned in parallel operation of machines not provided with special damping arrangements.

Theory of Dampers. When these are provided the conditions of parallel operation are materially altered. All damping arrangements consist of some system of heavy conductors laid parallel to the shaft close to the outer circumference of the field magnets and short-circuited on themselves.



FIG. 107.

Those conductors form a grid encircling the magnetic system. Generally the bars of this grid are laid in tunnels in the pole shoes and short-circuited on either side by stout bars, but sometimes they are also placed within the neutral space, and the ends are connected to two heavy copper rings as shown in Fig. 107. The three phase current in the armature conductors produces a flux which travels through space, that is round the interior of the stator, with a speed corresponding to the frequency. The poles also travel with the same speed, provided there is no swinging. If there is, then the slight changes in the speed of travel of poles relatively to the armature flux (the value v in the chart Fig. 106) will induce currents in the grid of the damper which by Lenz's law must oppose the relative change of position, that is to say, the force due to these self-induced currents must always be opposed to the v vector and in magnitude proportional to it. To perceive this action more clearly let us suppose the rotor non-excited and not driven by an engine and the armature currents supplied from some outside source. The flux produced by the armature currents in sweeping through the damping grid will drag it along in the same way as the rotor of a "squirrel cage" induction motor is dragged along by the stator field. This action is more fully considered in the chapter on induction motors; for the present it is sufficient to point out that the damper acts in the same way as the rotor of

a squirrel cage motor and that the torque exerted is subject to the same law, namely that it is proportional to the difference in speed between flux and conductors, the so-called "slip." The flux travels round with a speed corresponding to synchronism, the rotor travels with a slightly decreased speed. The slip for full torque is of the order of 1 to 5 per cent. according to the resistance of this squirrel cage winding. If such a winding is used as a damping grid we may characterise its effectiveness by reference to what the slip would be if the machine were working as an induction motor. We may call it a one per cent. or a two per cent. or a five per cent. damper, meaning thereby that the damping torque or force at one metre radius F_b is equal to full load force F_0 if the speed variation v is respectively one, two or five per cent. of the average speed V . The latter taken at one metre radius equals the mechanical angular speed Ω_m of the rotor and this is related to the angular speed Ω of the vector representing the disturbing force F by the equation $n\Omega_m = \Omega$ where n is the number of cycles of disturbing force in one revolution.

The damping force being proportional to speed may be expressed by

$$F_b = b \frac{dx}{dt}$$

and its crest value is $b\Omega x_1$ where x_1 is the deviation and b the damping factor (dimensions MT^{-1}).

The differential equation for the oscillatory motion of the rotor if provided with dampers may now be established.

$$F \sin \Omega t - m \frac{d^2 x}{dt^2} - b \frac{dx}{dt} - cx = 0 \quad . \quad . \quad (79)$$

This is a well-known equation in theoretical mechanics and the analytical solution can be found in some text-books, but as this method of solution is rather complicated and laborious we shall here adopt a much simpler graphic method based on the following reasoning. Assume two identical machines, one undamped and the other damped. The undamped machine has a certain deviation x_1 and surging power P_s resulting from the disturbing power P_d . Since power = force \times speed, we may introduce forces instead of powers and using (77) write

$$x_1 = \frac{F_d}{c - m\Omega^2}$$

We also have $F_s = cx_1$.

What must be the disturbing force applied to the damped machine in order that it shall have the same deviation x_1 , and consequently also the same surging force F_s , as the undamped machine? This question can easily be answered by drawing a vector diagram such as Fig. 108 which represents the most usual case of a generator whose mass effect is stronger than its control effect (case GH in the charts Fig. 106). In the diagram $OA = F_d$; $OC = x_1$; $OB = F_s$ and $OD = v$. The damping force must be supplied by the engine and must therefore be a component of the total disturbing force F , the other component, which produces deviation, being $F_d = OA$. The damping component is $bv = OG$ and the total disturbing force is $F = OH$. This produces in the damped machine exactly the same deviations, surging force and surging power as $F_d = F \cos \delta$ produces in the undamped machine. The problem is thus reduced to finding the value of $\cos \delta$. Since $v = x_1 \Omega$ we have

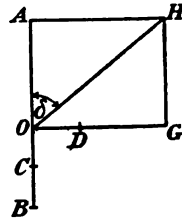


FIG. 108

$$\tan \delta = \frac{F_b}{F_d} = - \frac{x_1 \Omega b}{x_1 (m\Omega^2 - c)} = - \frac{\Omega b}{m\Omega^2 - c}$$

$$\cos \delta = \frac{1}{\sqrt{1 + \tan^2 \delta}} \quad \text{or}$$

$$\cos \delta = \frac{m\Omega^2 - c}{\sqrt{(m\Omega^2 - c)^2 + b^2 \Omega^2}}$$

But

$$x_1 = - \frac{F \cos \delta}{m\Omega^2 - c}$$

and inserting $\cos \delta$ we find

$$x_1 = - \frac{F}{\sqrt{(m\Omega^2 - c)^2 + b^2 \Omega^2}} \quad \dots \quad (80)$$

This equation shows that even for resonance, when $m\Omega^2 = c$, the deviation or amplitude of swing does not become infinite, but has the value $F/b\Omega$ and if the damping is strong enough the machine must keep in step.

Since $F_s = x_1 c$ and $\frac{F_s}{F} = \frac{P_s}{P_d}$ the relation between disturbing power and surging power is given by

$$P_s = P_d \frac{c}{\sqrt{(m\Omega^2 - c)^2 + b^2 \Omega^2}} \quad \dots \quad (81)$$

With an undamped machine we found

$$P_s = P_d \frac{c}{m\Omega^2 - c}$$

By providing dampers we diminish the surging current.

But the dampers waste power, namely $P_s = P_d \sin \delta$

$$P_b = P_d \frac{b\Omega}{\sqrt{(m\Omega^2 - c)^2 + b^2\Omega^2}} \quad \dots \quad (82)$$

The power wasted becomes a maximum for resonance, when $P_b = P_d$.

In a motor with very light masses and very strong control the power wasted is

$$P_b = P_d \frac{b\Omega}{\sqrt{c^2 + b^2\Omega^2}} \quad \dots \quad (83)$$

whilst the surging power is

$$P_s = P_d \frac{c}{\sqrt{c^2 + b^2\Omega^2}} \quad \dots \quad (84)$$

The stronger the control the less power is wasted in the damper. The stronger the damping and the greater the frequency of the cycle of disturbing force the smaller is the surging current.

Determination of Damping Factor b . We have still to find an expression for the damping factor b . Let σ per cent. be the characteristic of the damper as above explained and F_0 the full load torque if the machine is used as an induction motor running at U r.p.m. The linear speed at one metre radius is $V = 2\pi \frac{U}{60}$ and the speed of slip is $\frac{\sigma}{100} V$. By definition of the damping factor we have

$$F_0 = \frac{\sigma}{100} Vb$$

The power is

$$P_0 = \frac{\sigma}{100} V^2 b \times 9.81 \text{ watts}$$

Inserting the value for V we find by a simple transformation

$$b = \frac{930 P_0}{\sigma U^2} \quad \dots \quad (85)$$

where P_0 is the full load power expressed in watts.

Sectional Area of Damping Grid. The theory developed in the foregoing pages contains all that is necessary to determine the number of bars and their sectional area in a damping grid required for any specified duty, but in order to facilitate the application of the theory the results as far as damping is concerned are here summarised and the steps are indicated for the calculation of the sectional area of the damping grid. The problem as it occurs in practice is: Given total power, disturbing power and the surging power which may be permitted, find what damper will be required. Let the following symbols, identical with those previously used, represent:

P_0 power in watts at full load;

P_d disturbing power in watts at frequency $\Omega/2\pi$;

P_s surging power which may be permitted in watts;

σ percentage of slip in damper as defined on page 226;

m = mass; c = control factor; b = damping factor; all referring to a radius of one metre;

E voltage in one phase actually induced at load;

A stored energy in mkg.;

U = revs. p.m.; f = frequency; p = number of pairs of poles;

ωL = reactance at load as determined by the construction shown in Fig. 103.

$$c = 0.048 \left(\frac{E}{\omega L} \right) E \frac{p^2}{f} \quad . \quad . \quad . \quad . \quad . \quad (86)$$

$$b = \frac{930 P_0}{\sigma U^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

$$m = 182.5 \frac{A}{U^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

$$P_s = P_d \frac{c}{\sqrt{(m\Omega^2 - c)^2 + b^2\Omega^2}} \quad . \quad . \quad . \quad (81)$$

Since disturbing power and permissible surging power are given we may use formula (81) for finding b and then by (85) we find σ . In the theory of the induction motor it is shown that the total volume of current in stator and rotor is approximately the same; or the product of number of bars and current in each is about the same for stator and rotor. It is also shown that the percentage of slip is the percentage of power wasted in the secondary circuit, in our case the damping grid. The number of bars in the damping grid may be selected with regard to convenient housing within the

pole shoes. The greater their number the more evenly will be the distribution of the total volume of current, but care must be taken to leave sufficient area for the magnetic flux through the pole shoes. Let w be the total number of bars in the grid if counted all round and $3z$ the total number of conductors in the stator; then with a stator current of i_1 amperes the current i in each bar of the damping grid must be $3zi_1/w$ and if ρ denotes the ohmic resistance of one bar of the grid including connections the power lost is $w\rho i^2$. This should be σ per cent. of P_0 . We have therefore

$$w\rho \left(\frac{3zi_1}{w} \right)^2 = \frac{\sigma}{100} P_0$$

and from this we find

$$\rho = w \frac{\sigma}{100} \frac{P_0}{(3zi_1)^2} \quad \dots \quad (88)$$

Since the dimensions of the poles are known the sectional area of bars and end connections of the damping grid can be found from the resistance ρ .

Necessity of Dampers in Converters. The name of “converter” or “rotary converter” is given to a class of machines in which the armature is traversed by an A.C. and D.C. simultaneously. Such machines are more fully discussed in Chapter X, but for the present we may note that if the conversion is from A.C. to D.C. the A.C. part forms a motor and the mechanical load on this motor is represented by the D.C. output. The output may vary in quite an irregular manner and we are therefore faced with the problem of constructing a synchronous motor for a resisting torque subject to fluctuations which may be large and may be rapid or slow, but are certainly not regular. This means that F and Ω in (77) are not fixed quantities and therefore it is impossible to so determine the mass effect and the control factor as to make sure that the motor will keep in step. In such a case we must use a damper and then the deviation will be as given in (80). No flywheel should be used and the armature should be made as light as possible and the control as strong as possible. If that is done the surging power will be as given by (81) and provided the damping grid is very heavy the machine will keep in step however violent and irregular the fluctuation in the power demand on the D.C. side may be. A well-known instance of such irregular and violent fluctuations

in power demand is the electric tram service for which purpose converters are often used.

Bus Bars not Infinitely Strong. In the foregoing theory of parallel operation we have assumed that the machine is connected to a system of so large a power capacity that the e.m.f. vector of the bus bars may be considered to be practically constant in magnitude and speed. We have also considered the case of two equal machines coupled to the same bus bars and so controlling each other. These two are extreme conditions; the latter is often met in practice, the former never, but it is approached as regards each one of three, four or more machines connected to the same system. As regards each individual machine the bus bars, although not infinitely strong, are very much stronger than the machine itself. We have therefore an approach to the ideal case. With two machines we may have an equivalent of the ideal case if the switching together happens at such a phase position regarding the disturbing torque that it is positive in one engine at the same time that it is negative in the other. This is obviously the worst possible condition. In this case the bus bar vector must be that of an infinitely strong system because positive and negative impulses cancel out. Even in this case, and therefore in all other cases, parallel operation even without dampers is possible, provided there is no resonance between the period of the disturbing force of the engine and that of the natural swing. Nevertheless the provision of dampers is advisable because a disturbing force may be introduced into the system by one of the machines which take power from it. It is evidently immaterial whether a disturbing force is introduced by the engine or by some other apparatus connected to the system and, as it is impossible to know beforehand what consuming devices will eventually have to be supplied from the system of mains and what the periodicity of any possible disturbing force will be, it is good policy to fit all generators with dampers.

V Curves. Let an alternator be coupled to infinitely strong bus bars and let it be driven as a generator by a prime mover giving constant torque. In such a case the power is constant and apart from a small correction on account of losses we may say that $I \cos \phi$ is a constant, the magnitude of which depends on the power of the engine and the voltage at the bus bars. A change of excitation

within reasonable limits will not throw the alternator out of step; it will simply change the power factor and the current I and it is obvious that there must be one particular value of the exciting current for which the power factor becomes unity and the current a minimum, namely equal to the watt component corresponding to the given power. At any larger or smaller excitation the current must be larger and if we plot from tests the exciting current as abscissae and the armature current as ordinates we obtain a V shaped curve. At a small power the V is more pointed and the ordinate of its lowest point is smaller since the watt component is smaller. With larger powers the V becomes more open and rounded and the excitation corresponding to unity power factor is a little greater, but the difference is not very marked. We thus find that a generator so excited as to work with unity power factor at normal load will very nearly have unity power factor if it is worked with the same excitation, but at a reduced or increased load.

If the machine is used as a motor driving some appliance which offers a constant resisting torque the output of mechanical power and (apart from losses) the electric power drawn from the bus bars is constant and independent of excitation. If we again plot exciting and armature amperes we find a similar V curve and it will be obvious that to work with unity power factor and therefore with a minimum of copper loss and maximum efficiency that excitation should be used which corresponds to the lowest point of the V. If the excitation is increased the total armature current contains a wattless component which is leading over the bus bar voltage and the motor may therefore be used to improve the power factor of the whole system. Whether we consider the motor mainly as a power

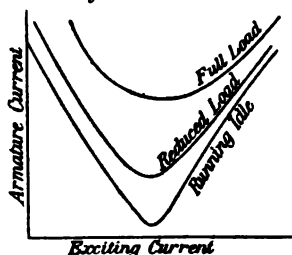


FIG. 109.

producing machine or at the same time as a means for improving the power factor of the whole system a knowledge of its V curve is important and for this reason it is desirable to be able to predetermine the V curve from the design.

Fig. 109 shows the character of these V curves as found on test. With an excitation corresponding to the lowest point the motor has unity power factor. With a larger excitation it delivers to the line a leading component of current and with a smaller excitation it takes from the line a lagging component. If the

excitation is reduced beyond the point where the V curve turns sharply upward the motor falls out of step. With a very light load this critical point lies near the axis corresponding to zero excitation and may even lie beyond it. The explanation of the fact that the machine can work even if its exciting current is cut off is that it excites itself by armature reaction. This property is utilised in starting small converters as is shown in the next chapter.

The current flowing through the armature (whether the machine is generating or motoring) is due to the resultant between the bus bar e.m.f. and the e.m.f. actually induced in the armature. The latter is determined not only by the excitation, but also by the magnetising or demagnetising effect of the armature current. It has already been mentioned in Chapter VIII that in a generator a lagging current weakens the field whilst in a motor it strengthens it. For convenient reference these relations are shown in the little key diagram Fig. 109*a*. The open arrow represents the e.m.f. vector and the closed arrow the current vector. When drawing a diagram to represent the working condition of a motor coupled to bus bars

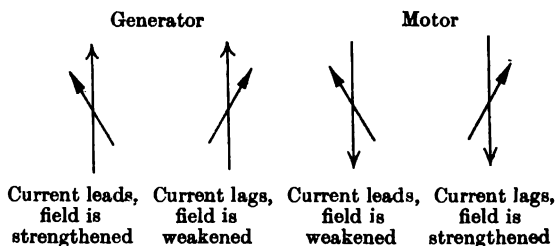


FIG. 109*a*.

we start by assuming a working point and determine the current vector corresponding to a particular load. By referring to the key diagram we can then see whether some additional excitation is required to keep the working point in the assumed position. The construction is shown in Fig. 110. Of the two characteristics the upper refers to gap and armature, the lower to the whole machine. On the right is a vector diagram drawn for a definite terminal voltage OT . For a load of P watts the watt component of the motoring current is $\frac{P}{OT\sqrt{3}} = I_0$ and this is also equal to the total current $I \times \cos \phi$. The total current is produced by the resultant of the fixed terminal e.m.f. OT and the e.m.f. OS actually induced in the armature. This resultant is ST . Let ωL be the reactance

corresponding to the working point C and taken from the reactance curve in the diagram. We now have

$$I_0 = \cos \varphi \frac{ST}{\omega L} = \frac{QS}{\omega L} \text{ and } QS = \omega L I_0$$

Since the load is constant I_0 is also constant and we find that the distance of the point S from the vector of terminal e.m.f. is a constant, that is to say, the locus of S is a vertical whatever may be the position of the working point C . The voltage triangle OST may now be drawn. Make to any convenient scale $OG = I_0$ and draw from O a line at right angles to the backward prolongation of ST . Where this line cuts the horizontal through G is the outer end of the current vector and the magnitude of the current I is given by the length OA . AG is the wattless leading component injected into the line.

Make Og in the scale used for abscissae in the characteristic equal to $\frac{b}{n} I_0$, then Oa represents in the same scale the current flowing through the armature and ab is its wattless component. By reference to the key diagram Fig. 109 *a* we find that this current

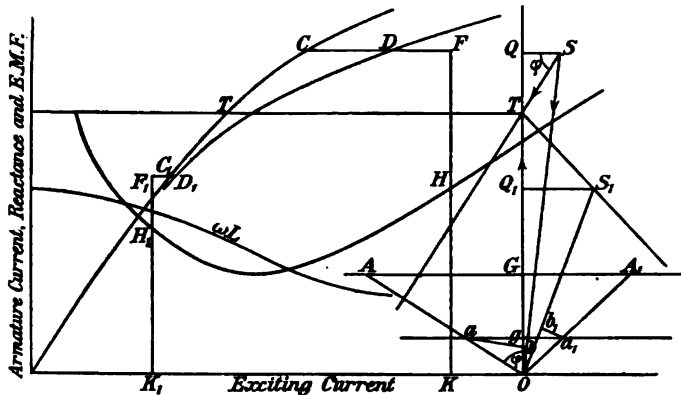


FIG. 110. Construction of V curve.

demagnetises and has therefore a tendency to shift the working point *C* lower down on its curve. In order to counteract this tendency we must increase the excitation by an equal amount. If we wish to get the V curve in the usual way, that is in relation to the whole machine, we must also add to the excitation corresponding to *C* the horizontal distance between the two characteristics, namely *CD*. Make *DF = ab* then the point *K* gives the total excitation

required to produce at the given load the armature current OA . We thus obtain H as one point on the V curve. The construction is repeated for a working point C_1 lower than T , the same lettering being used, but distinguished by the index 1. By repeating the construction for other working points the whole curve may be drawn. It will be noticed that the excitation may be varied within fairly wide limits, but it is not advisable to work a motor under-excited because with an increase of load the left branch of the V curve is drawn to the right and the critical point where this curve turns sharply upward may be reached with even a small overload. To work a motor under-excited has the further disadvantage of worsening the power factor of the system. It is therefore advisable to work motors rather over-, than under-excited. This is indeed necessary where the motor is required for power factor improvement. Fig. 110 is drawn to scale from test data obtained with an ordinary commercial motor. In this case the excitation has to be about doubled to obtain a strong leading current. A motor of ordinary proportions is therefore not very suitable for this purpose. To obtain a large leading current without over-heating the field-winding it is necessary to decrease the ratio b/n which means a motor with small armature reaction or in other words a weak armature and a strong field. Then we obtain a narrower V .

Synchronisers. Before machines can be put safely into parallel it is necessary that they be in step and have the same voltage. These conditions are indicated by some special apparatus called a synchroniser. A great variety of such apparatus are available, some merely of the indicating kind, others also combined with some interlocking device which prevents a machine being switched in if not in step*. All synchronisers are based on the principle of indicating the phase position between bus bars and incoming machine on the low tension side of little transformers. In the original type of synchroniser the secondaries were connected to a voltmeter and in parallel with this to one or two glow lamps, generally so as to oppose each other when the machine was in step with the bus bars†. This condition is indicated by the lamps becoming dark and the voltmeter

* A description of Besag's safety synchroniser will be found in the *Elektrotechnische Zeitschrift*, 1912, p. 135.

† This is called dark synchronising and is preferable to light synchronising because smaller angular differences can more easily be detected, especially with metallic filament lamps.

showing zero. If the frequency of the incoming machine is not right there will be beats in the P.D., the lamps lighting up and going out periodically. The nearer the frequency comes to the correct value the longer become the beats. When the lamps are dark the machine may be switched in.

The defect of this otherwise efficient device is that one cannot tell from the beats whether the incoming machine is running too

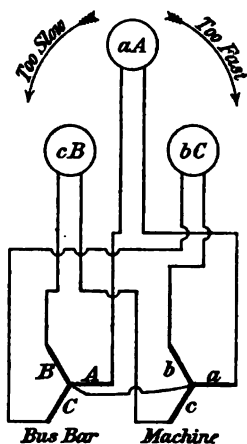


FIG. 111.

fast or too slow. To find which is the case one has to speed up and slow down and at the same time watch the beats. The operation takes therefore longer than may sometimes be desirable when it is important to bring in a new machine quickly. This drawback is overcome in the three phase synchroniser the principle of which is illustrated in Fig. 111. The synchronising transformers are three phase, their secondaries being marked *A*, *B*, *C* for the transformer receiving current from the bus bars and *a*, *b*, *c* for that receiving current from the incoming machine. The secondaries supply current to three lamps *aA*, *bC*, *cB* set in a dial. The diagram shows correct

phase position for synchronism, when the lamp *aA* receives no current and is therefore dark. The two other lamps each receive current under about 86 per cent. of full voltage and are therefore alight, but not bright. A voltmeter may be put across the *aA* leads to show whether the voltage of the incoming machine is right. If this voltmeter shows zero the machine may be switched in. If the speed is not right there will be beats in the light given by all the lamps as in a single phase synchroniser, but at the same time the light travels round the dial. With the connections as shown and the sequence *abc* the light travels clockwise if the machine runs too fast. Only one lamp dial is required for all the machines in the station and if this is put in a conspicuous position the driver of any incoming machine need only watch the light so as to bring the engine to the correct speed. The attendant at the switchboard meanwhile regulates the excitation and may switch in when the voltmeter shows zero.

In another type of synchroniser the travelling lights are replaced by a travelling pointer. This is attached to the rotor of a little

induction motor. The rotor has a three phase winding which receives current from the machine transformer, whilst the stator which also has a three phase winding receives current from the bus bar transformer. Thus there is generated both in the stator and rotor a rotating field. If the speed of the incoming machine is right both fields travel at the same speed and the rotor remains at rest and if the phase condition is right the pointer in its mid position. If there is a difference in frequency the pointer rotates, one way if the incoming machine runs too fast and the other way if too slow. This type of synchroniser is generally placed near the switchboard so that the attendant and not the engine driver may by "remote control" regulate the speed of the engine.

CHAPTER X

CONVERTERS

The principle of the rotary converter—Ratio of voltages—Output of a converter
—Voltage regulation on the D.C. side—Starting converters.

The Principle of the Rotary Converter. Let in a lap wound D.C. armature points π electrical radians apart be tapped and let all tapplings corresponding to an even multiple of π be connected to one slip ring and the tapplings corresponding to the intermediate odd multiples of π be connected to another slip ring. We obtain thus an alternator with coil sides equal to the pole pitch and single phase current will be obtained from the two slip rings. Conversely if single phase current is supplied to the slip rings the machine will work as a single phase synchronous motor. At the same time D.C. may be taken from the commutator and brushes in the usual way. We have here a machine for the conversion of current from A.C. to D.C. or vice versa. Since both currents are obtained from the same armature winding revolving in the same field system it is obvious that apart from the very slight influence of resistance and armature reaction the ratio of the A.C. and D.C. terminal voltage is constant, that is to say, cannot be altered by a change in excitation.

Ratio of Voltages. Let a converter fed by A.C. be running light, then the disturbing influence of resistance and armature reaction will be negligible and there will be no sensible difference between induced and terminal voltage. The e.m.f. between the two slip rings being alternating, changes continuously. It goes through zero at the moment that the tapplings are midway between brushes and it has crest value at the moment that the tapplings coincide with the brushes. This crest value is also the sustained value of the brush e.m.f., so that the problem of finding the ratio between the A.C. and D.C. e.m.f. is the same as the problem of finding the ratio between crest value and effective (or virtual) value of the A.C. wave. If the e.m.f. wave is known the problem may be solved by any of the methods given in Vol. I, pp. 303–307. To find the wave form we map out as in Fig. 112 the form of the

induction curve taking account of the bevelling of the polar edges and estimating the fringe; then add up the e.m.f.'s of the different wires in a coil side, which e.m.f.'s are obviously proportional to the ordinates of the flux curve. Repeat the process for a large number of positions of the coil side and finally calculate the R.M.S. value. This method is quite accurate, but rather laborious. A simpler way giving the A.C. e.m.f. within a few per cent. correctly is as follows. Replace the true flux curve by the dotted rectangle. This would be the flux curve if the poles were not bevelled and there were no fringe.

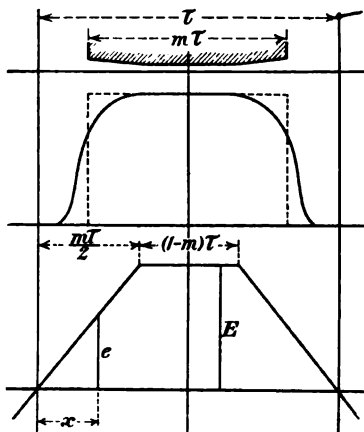


FIG. 112. Flux and e.m.f. in converter.

In this case the e.m.f. wave has a very simple form as shown in the lower part of the figure. The coil side of width τ may move to either side of its central position by the amount $(1 - m)\tau/2$ without any wires coming outside of the influence of the field. Within the space $(1 - m)\tau$ the crest value E (and that is also the D.C. e.m.f.) is maintained. Beyond that position to either side wires come under the influence of the opposite poles and the e.m.f. is reduced. The reduction must be down to zero if the coil side has been shifted by $\tau/2$ to either side. Since with a rectangular field form the change in flux is a linear function of the position, the graph showing momentary e.m.f. as a function of position must be a straight line to either side of the short distance over which the e.m.f. has crest value. We thus obtain a trapezium as the graph connecting position and instantaneous value of e.m.f. The instantaneous value within the limits $x = 0$ and $x = m\tau/2$ is

$$e = E \frac{2x}{m\tau}$$

and within the limits $x = m\tau/2$ and $x = \tau/2$ it is $e = E$. The effective value or R.M.S.

$$= \sqrt{\frac{2}{\tau} \int_0^{\frac{\tau}{2}} e^2 dx}$$

The integration is so simple that it need not be given in detail. The result is

$$e = E \sqrt{1 - \frac{2}{3}m} \quad . \quad . \quad . \quad . \quad . \quad (89)$$

The mean ordinate of the e.m.f. curve is

$$e_{\text{mean}} = E \left(1 - \frac{m}{2} \right)$$

Since the form factor (Vol. I, p. 300) is $\frac{\text{effective e.m.f.}}{\text{mean e.m.f.}}$ and K in (55) is twice the form factor, we have

$$K = \frac{2\sqrt{1 - \frac{3}{4}m}}{1 - \frac{m}{2}} \quad \dots \quad (90)$$

The second line in the following table gives the ratio e/E calculated by the approximate formula (89) and the third line gives the correct value calculated by the more laborious method previously explained. It will be seen that the agreement is fairly close

$m = \frac{\text{Pole width}}{\text{Pole pitch}} =$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
$\frac{e}{E} = \sqrt{1 - \frac{3}{4}m}$	$\cdot 814$	$\cdot 745$	$\cdot 705$
$\frac{\text{Effective Slip Ring e.m.f.}}{\text{Brush e.m.f.}} =$	$\cdot 77$	$\cdot 72$	$\cdot 695$
Percentage error =	5	$3\frac{1}{2}$	$1\frac{1}{2}$

The form factor in all these cases is not very different from 1.11, namely that of a sine curve. This is borne out by experiment. Ondo-graph records taken from converters show a wave form very closely approaching a sine curve.

The theory here developed refers to a single phase converter. Its extension to a multiphase converter is quite simple. Since the wave form closely approaches a sine curve we may assume the A.C. part of the real converter replaced by an ideal alternator (Vol. I, Fig. 148) with all the winding concentrated in a diametral coil. The crest value of the e.m.f. curve is then proportional to the flux passing through the diametral section or to the chord spanning 180 electrical degrees of the armature circle. Now let us shorten the chord so as to span not 180, but only 120 degrees. The maximum of flux which can go through the coil is now reduced in the ratio of the diameter to the chord of 120°. The crest value of the e.m.f. in this shorter coil will therefore also be reduced in the same proportion, namely as 1 to $\sin 60^\circ$. Effective and crest values being always in the same proportion, the ratio in which effective value is reduced must also be 1 to $\sin 60^\circ$. If we use a coil which spans only a quarter of the circle we have a reduction in the ratio of

1 to $\sin 45^\circ$ and so on. We thus get the following relation between angular distance of tapping points and effective voltage between them.

Electrical degrees	180	120	90	60
Effective e.m.f.	e	$e \frac{\sqrt{3}}{2}$	$e \frac{1}{\sqrt{2}}$	$e \frac{1}{2}$

If we now go back from the ideal machine to the real converter we find that with the angular distance of tapping points above assumed we get a single phase, three phase, two and four phase and a six phase converter respectively. Two slip rings are required in the single phase, three in the three phase, four in the two and four phase and six in the six phase converter, the tapping points being distant from each other τ , $\frac{2}{3}\tau$, $\frac{1}{2}\tau$ and $\frac{1}{3}\tau$.

The percentage of alternating effective e.m.f. between slip rings to the D.C. e.m.f. is shown in the following table.

SLIP RING E.M.F. EXPRESSED AS A PERCENTAGE OF D.C. E.M.F.

$\frac{\text{Pole arc}}{\text{Pole pitch}} = m =$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
Single Phase	77	72	69.5
Two Phase	77	72	69.5
Four Phase	54.5	51	49.2
Three Phase	66.5	62.2	60
Six Phase (according to slip rings chosen)	38.5 66.5 77	36 62.2 72	34.8 60 69.5

The figures in this table refer to a converter running light. If under load and at unity power factor the power lost must be supplied on the primary side. If the power factor is not unity then the ratio of A.C. and D.C. voltage is changed by armature reaction as shown in Fig. 115. Taking 94 per cent. as a fair average for the efficiency at full load, then some 6 per cent. more must be put in at the primary than is taken out from the secondary side. If we assume 2 per cent. on account of armature resistance it follows that if the conversion is from A.C. to D.C. the above figures will have to be increased by 2 per cent.; if the conversion is from D.C. to A.C. they must be lowered by 2 per cent. The ratio between the values of alternating and direct current can now be found for each case. By way of example the calculation is here given for $m = \frac{2}{3}$ and a three phase

converter. Let E be the terminal voltage and I_e the direct current, then the power which has to be supplied on the primary side is $1.06 EI_e$. Let i be the current in each of the three circuits, then for unity power factor we have

$$3ie = 1.06 EI_e \text{ or } \sqrt{3} Ie = 1.06 EI_e$$

where I is the line current on the primary side. Allowing 2 per cent. for armature resistance we have

$$e/E = 0.634 \text{ and } \sqrt{3} \times 0.634 I = 1.06 I_e$$

from which we find $I = 0.96 I_e$.

Output of a Converter. It is important to compare the amount of power which may be converted in a particular armature with the power which may be obtained out of the same armature if working as an ordinary D.C. generator with the same internal loss, that is to say, with the same heating. The current in each armature conductor has two components; one is the direct

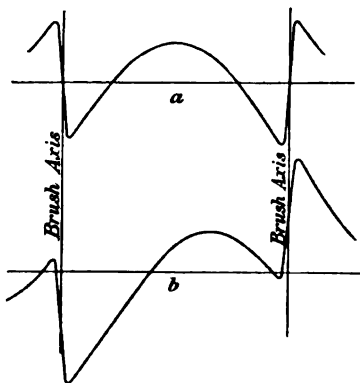


FIG. 113. Resultant current in one armature conductor: (a) Midway between two tapping points; (b) Close to one tapping point.

current which changes sign each time the conductor passes under a brush and the other is the alternating current which follows a sine law. With unity power factor the A.C. in any coil side has crest value when the centre of the coil side is in a polar axis and zero value when it coincides with a brush axis. A conductor lying in the centre of the coil side will therefore carry crest value of A.C. if midway between the brushes, but a particular conductor on either side of the centre of the coil side is nearer to one brush than to

the other brush when carrying crest value of A.C. The R.M.S. of the resultant current must therefore be different in the different conductors of one and the same coil side and the heating of individual conductors must also be different. This will be seen from Fig. 113 which shows the graph of the resultant current whilst a particular conductor travels through a distance equal to the pole pitch. The top figure a refers to a conductor in the middle of the coil side and

the bottom figure *b* to one near a tapping point. The graph is obtained by the superposition of the A.C. and D.C. To get the copper heat in the winding as a whole we must draw similar graphs for different positions of the conductor in the coil side, determine the effective value for each graph and take the mean of all these effective values. This squared and multiplied by the resistance between two tappings gives the power wasted in copper heat. A glance at the figure shows that the effective value of the current in a conductor close to the tapping point is greater than that in a wire between tapping points and this explains the well-known fact that the armature conductors near a tapping point get hotter than those midway between these points.

The problem of finding the total copper heat can also be solved analytically but this involves a rather complicated integration. The subject is treated in specialist works*, but as we are mainly concerned with the practical use of this investigation it will suffice to quote the results obtained by the analytical method without reproducing it here. It is as follows. The copper heat actually developed in a three, four and six phase converter at unity power factor is respectively 55, 37 and 26 per cent. of the copper heat which would be developed if the armature were used to generate the same D.C. These figures are obtained on the assumption of absolute parity between input and output and require therefore a slight correction to account for losses. In determining the voltage ratio we have assumed an ohmic drop of 2 per cent. so that when converting from A.C. to D.C. the figures in the table on p. 241 must be increased by 2 per cent. The total efficiency of the converter may be taken as 0.94. Out of a total loss of 6 per cent. copper heat in the armature accounts for 2 per cent. and there remain 4 per cent. for excitation, iron heat and mechanical losses. These 4 per cent. must be covered by a corresponding increase of 4 per cent. in the A.C. input and this raises the copper heat by 8 per cent. The heating of the armature when working as a converter at unity power factor as compared with the heating when working as a generator will therefore have the following relation:

Three phase	0.595
Four phase	0.43
Six phase	0.28

* See *Electrical Engineering*, by Berg and Upson, p. 386.

Since the heating is less than that permissible in a generator we may increase the load. The amount of increase is found from the consideration that the loss is proportional to the square of the current and to get the same copper heat in all cases we may increase the output inversely as the square root of the above figures. This gives for three, four and six phase converters respectively 1.3, 1.52 and 1.88 times the output of the same machine worked as an ordinary D.C. generator.

This is at unity power factor. At a smaller power factor the output is necessarily reduced since the watt-component of the A.C. current is smaller. Although by suitably adjusting the excitation it is always possible to work at unity power factor it is sometimes necessary to depart from this practice as will be shown presently; we must therefore investigate also the case when the

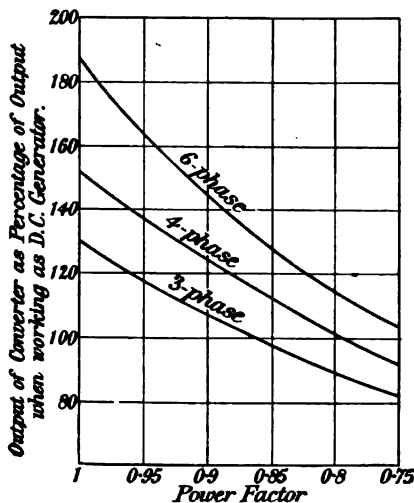


FIG. 114.

converter works at a power factor less than unity. Also this problem can be treated analytically, but this method is still more complicated than the calculation for unity power factor. We can avoid the analytical treatment by the following reasoning. A departure from unity power factor does not alter the watt component of the input current, but it adds a wattless component which is in quadrature. Since the total copper heat is proportional to the algebraic sum of the squares of the two components we need only add to

the copper heat at unity power factor the copper heat of the wattless current. The procedure can best be explained by an example. For this purpose we take a six phase converter working at 0.8 power factor. Let i be the watt component of the effective A.C. in each of the six circuits, then $i \tan \phi$ is the wattless component. If I is the D.C. then

$$6 \times 0.36Ei = EI \text{ and } i = 0.46I$$

If ρ is the resistance of the armature the ohmic loss as stated above

is $0.28 \times (\rho I^2)$ at unity power factor. The ohmic loss due to the wattless current in the whole armature is

$$6 \times \frac{2\rho}{3} i^2 \tan^2 \varphi = 4\rho i^2 \tan^2 \varphi$$

and since $i^2 = 0.214 I^2$ we have for the additional loss due to phase displacement $0.856 I^2 \rho \tan^2 \varphi$. The total loss is therefore

$$\rho I^2 (0.28 + 0.856 \tan^2 \varphi)$$

Since

$$\tan^2 \varphi = \frac{1 - 0.8^2}{0.8^2} = 0.56$$

the total loss is $\rho I^2 (0.28 + 0.856 \times 0.56)$ or $\rho I^2 \times 0.76$. The loss when the machine is working as generator is ρI^2 . By working it as a converter at 0.8 power factor the loss has decreased to 76 per cent. and the output for full loss may be increased in the ratio of

$$1 : \frac{1}{\sqrt{0.76}} = 1 : 1.15$$

Fig. 114 shows the output of three, four and six phase converters at different power factors.

Voltage Regulation on the D.C. side. It has been shown that the ratio of terminal voltages in a converter is nearly constant so that no substantial variation in the terminal e.m.f. on the secondary side can be made by change of excitation pure and simple. In most cases it is impracticable to vary the A.C. voltage at the source because the general system of supply over the whole district served must be given at as nearly a constant voltage as possible. We are therefore faced with the problem of how to adjust the D.C. voltage of the current given by a converter when it is fed by A.C. of constant voltage and frequency. Various solutions are possible, but all involve the principle of adjusting the A.C. voltage supplied to the converter proper; that is to say, of using some kind of booster.

An obvious and very satisfactory way of changing the A.C. voltage is to use as a booster an A.C. generator which is interposed between the supply terminals and the converter. To insure synchronism the machine is mounted in line with the converter and the shafts are rigidly coupled; or the booster is made part of the converter. By suitable adjustment of the exciting current a boost over a fairly large range either up or down can be given. The power factor

may be adjusted by the excitation of the converter itself and the voltage on the D.C. side is adjusted by the rheostat and reversing switch in the field circuit of the booster. A less expensive method consists in the use of a static booster, that is a transformer with a variable transforming ratio. The apparatus consists of a primary and secondary part, both having multi-phase windings so that in each a rotating field is produced. The primary is coupled across the mains and the rotating field produced in it has therefore constant magnitude proportional to the supply voltage. This field in sweeping through the coils of the secondary winding generates in them an alternating e.m.f. also of constant value, but the phase relation of the secondary e.m.f. to that of supply voltage can be varied by setting the secondary part of the apparatus into different angular positions to the primary part. We have here the same elements as in an induction motor with this difference that the rotor does not rotate, although it can be set into any angular position between two limits. This rotor is in series with the converter. If it is fixed in such a position that its coil sides are opposite to the coil sides of the stator, and if at the same time the excitation of the converter is so adjusted that it works with unity power factor, then the boosting e.m.f. is in phase with the current and the e.m.f. delivered at the terminals of the converter is augmented by the e.m.f. of the booster. If now the rotor is shifted by 180 electrical degrees, the boosting e.m.f. is out of phase by 180° and the e.m.f. delivered to the converter is reduced by the same amount. The apparatus can thus boost up or down and if the rotor is fixed in some intermediate position the amount of boost in either direction can be adjusted to any value between these extreme limits. Since the booster has itself an appreciable inductance (owing to the magnetic leakage between its two windings) the power factor of the whole system is not so good as that of a rotary booster and although this defect may be corrected by over excitation of the converter the additional attention required is a drawback. With a rotating booster, once the excitation of the converter has been adjusted for unity power factor it is right for all loads and any boost that may be required. The only regulation necessary is therefore that of the excitation of the booster. With a static booster to obtain unity power factor at all loads and at any boost we must regulate two things, the excitation of the converter and the position of the rotor of the booster. The latter regulation is generally by means of a worm and wheel by

hand in small boosters and by motor in large. The torque between stator and rotor is considerable and sometimes two boosters with mechanically coupled rotors are used. In this case the electrical connections of the windings are such that the torque of one rotor is opposed by that of the other so that the force required to shift the rotors is only small and setting by hand is possible.

An entirely automatic method of voltage regulation on the secondary side is often used in converters supplying D.C. current for electric traction. The method consists in the insertion of choking coils between the supply terminals and the converter and in using a compound winding on the field of the latter. The series turns are so connected that the field is strengthened with increasing load and the

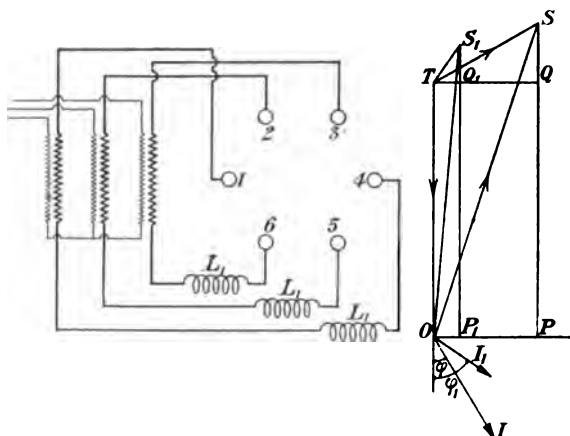


FIG. 115. Compounded converter for electric traction.

excitation as a whole is such as to make the induced e.m.f. rather larger than the supply e.m.f. Fig. 115 shows diagrammatically the connections. The little circles marked 1 to 6 represent the slip rings which are connected through inductances L_1 with the secondaries of a three phase transformer. A transformer is necessary because the supply voltage is that of the transmission line and may be thousands of volts, whereas the D.C. is required at a much lower voltage, of the order of 500 to 700 for traction and general power use. The inductances are necessary if an ordinary transformer is used having close regulation, but if the transformer is specially designed with large magnetic leakage (making its regulation bad) the choking coils may be omitted, the large inductance of the

transformer forming an equivalent. The vector diagram on the right shows the working condition for full load and a quarter load. OS is the e.m.f. actually induced in the armature and TS is the e.m.f. required to balance the reactance (see reactance characteristic in Fig. 103) of the machine itself and the reactance of the additional coils L_1 . We have therefore

$$TS = I (\omega L + \omega L_1)$$

By a suitable choice of scale TS may also represent the total current taken at full load. In the same way TS_1 is the total current I_1 taken at quarter load. TQ and TQ_1 are the watt components and SQ and S_1Q_1 are the wattless components respectively. The phase position of these currents are OI and OI_1 . It will be seen that at full load the current leads over the terminal, that is the line e.m.f. by the angle ϕ , and at quarter load it leads by the angle ϕ_1 . The converter therefore injects into the line a leading wattless component. This expressed as a percentage of the total supply current is about 50 per cent. in the case of full load and 70 per cent. in the case of quarter load. At the same time there is a difference of about 10 per cent. in the magnitude of the voltage actually induced in the armature winding. This requires an increase of excitation with increasing load and that is obtained by compounding. Thus a converter for traction service may be made to give a terminal voltage of 500 at light load and 550 at full load and at the same time improve the power factor of the whole supply system. If in a power house several converters are required to work in parallel and are compounded it is necessary to use equalising connections in the same way as is done with compounded D.C. generators.

Starting Converters. If direct current is available for starting the operation is quite simple. All we have to do is to run the machine up to speed by power supply on the D.C. side, synchronise in the way usual with alternators and then switch the machine on to the A.C. supply.

For starting a converter from the A.C. side different methods may be used, but in all cases it is advisable to open the D.C. circuit so that no power is given off on that side. The most obvious way is to provide a little starting motor of the induction type. If mounted on the converter shaft it must have one pair of poles less than the converter so that synchronous speed may be reached. Induction

motors run at a speed a little smaller than synchronous motors, the difference being called the "slip." If motor and converter had the same number of poles the latter could never be brought up to full speed, but by giving the motor two poles less its working speed at light load is higher than the synchronous speed of the converter and by using a squirrel cage motor with a suitable resistance in the rotor it is possible to bring down the light load speed to the correct value by putting a certain amount of load on. This is easily done by exciting the field of the converter. The iron losses are sufficient load for the induction motor to increase its slip to such an amount that its speed assumes the correct value. Another expedient is to use a slip ring induction motor and adjust its speed by inserting resistance in its rotor circuit. After full speed is reached the A.C. side of the converter is synchronised in the usual way and switched on to the supply. Before connecting to the D.C. side care must be taken that the voltage is in the right direction. If not, the field current must be reversed.

In small converters up to 300 or 350 kw. the expense of a special starting motor can be avoided by using the converter itself as an induction motor. It has already been pointed out that in order to ensure steadiness (that is, to avoid what is technically termed "hunting" due to the surging of power) converters must be provided with strong dampers. Such a damper has the same effect as the squirrel cage winding in an induction motor, that is to say, it will bring the speed up to nearly the synchronous value. During the starting the field must not be excited. To limit the A.C. taken from the line at starting the transformer is provided with secondary tapplings and a lower voltage than the normal is used. For this purpose a multiple contact switch in connection with these tapplings is used, so that the voltage may be gradually increased without at any time drawing an excessive current from the line. As the armature approaches synchronous speed the reaction of the armature current on the salient poles on the field magnetises the latter and the converter jumps into step (see V curves for light load, Fig. 109). Even if it should fail to synchronise itself completely it can be made to jump into step by exciting the field when near synchronism since the armature has little mass and the first rush of current suffices to put it into absolute synchronism. It is advisable to lift the D.C. brushes during starting because if this is not done the A.C. in the coils short-circuited by the brushes may damage the commutator.

If after having started it is found that the D.C. voltage is in the wrong direction the error may be rectified by a process technically termed "slipping a pole." This is done by reversing the exciting current momentarily, whilst watching a voltmeter with central zero. As the machine loses speed and slips a pole the needle comes to zero and swings out on the other side. The exciting switch is then closed and the polarity of the machine produced in such direction that connection with the D.C. system may be made.

For starting large converters the method just described is objectionable, not only on account of the expense of brush lifting

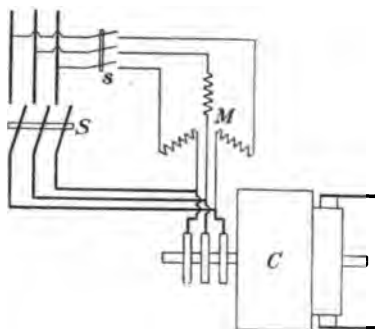


FIG 116. Rosenberg's method of starting converter.

gear, but more so because even with tappings the current taken from the line may exceed the normal current. To draw suddenly a current from the line representing 1000 kva. or more causes a considerable disturbance in the whole system. If a starting motor is used the current taken from the line at the first moment of starting need not exceed about 30 per cent. of the full load current and does therefore not materially disturb the supply system. There is, how-

ever, still the necessity to synchronise by hand. These difficulties are overcome in Dr Rosenberg's method of starting and self-synchronising diagrammatically represented in Fig. 116. *C* is the converter and *M* the stator of a small starting motor mounted on the same shaft. It has one pair of poles less than the converter and it must therefore take the converter at least up to synchronous speed. It cannot take it farther because as soon as this speed is approached the converter begins to excite itself and jumps into step even before *S* is closed. There is no danger of damaging the commutator because with the circuits of the starting motor interposed the voltage impressed on the slip rings is quite low so that no heavy short-circuit currents can be generated in coils under the brushes. A voltmeter with central zero is connected to the brushes to show whether the polarity is right. By this method a large converter may be started in about half a minute. First the switch *s* is closed and when synchronism is reached the switch *S* may be closed and *s*

opened. Synchronism is indicated by the central zero voltmeter showing a steady deflection. If it is to the wrong side the polarity must be changed by slipping a pole in the manner already mentioned, but generally the residual magnetism in the field system of the converter is sufficient to bring the machine into circuit with the correct polarity.

CHAPTER XI

TRANSFORMERS

Core and shell type—Magnetic reluctance of joints—Arrangement of coils—Subdivision of winding—Predetermination of inductive drop—Best proportions—Weight of active material—Some data from practice—Influence of frequency on output and regulation—Best division of losses—Choking coils—The autotransformer—Balancing transformer for three wire system—Cooling—Transformer oil—Temperature rise as a function of time—Temperature drop as a function of time—Heating and cooling with intermittent loads.

Core and Shell Type. Transformers are usually classified according to the arrangement of the magnetic relatively to the electric circuit. Both must be interlinked, but we may have one magnetic and two electric links or one electric and two magnetic links. In the former case we call the apparatus a core transformer, in the latter a shell transformer. In both types there must be a magnetic core within the coils and there must be a yoke joining the ends of the core or cores so as to close the magnetic circuit. In the core type there are two cores each surrounded by windings and the two cores are joined by two yokes. In the shell type there is only one core, but a double yoke which surrounds the coils, generally on two opposite sides. The usual form of core and shell transformers is shown in Fig. 117. In the core type the cores and yokes form a rectangle, the cores being the longer limbs. The joint with the yoke may be a butt joint as shown, or the plates may overlap at the corners so as to break joint in order to avoid the magnetic reluctance in the contact between core and yoke. In the illustration the core is shown as a square with the corners stepped, which is done by using slightly narrower outside plates when building up the core. The object is to reduce the inner diameter of the coils. Some makers grade the width of the plates more elaborately so as to obtain almost a circular cross section. Although in this way the perimeter of the coils for a given core section is reduced to a minimum, this construction has the disadvantage of leaving no channels between core and coils through which air or whatever cooling agent is used can flow. In the figure two such channels are provided for each core where the cooling fluid may sweep along the edges of the

plates. The heat conductivity parallel to the plates being much greater than that across the laminations, it is obvious that the other two sides of the square are not of much value for cooling the core. On the other hand the plates of the core must be firmly held together by cast metal end plates and the space between the core and the coil may be utilised for these end plates. The body of the core is clamped between these castings by insulated bolts with sunken heads. The laminations of the yokes are held together in a similar manner and in certain cases lugs are added on either side for the reception of longitudinal bolts by which the yokes are

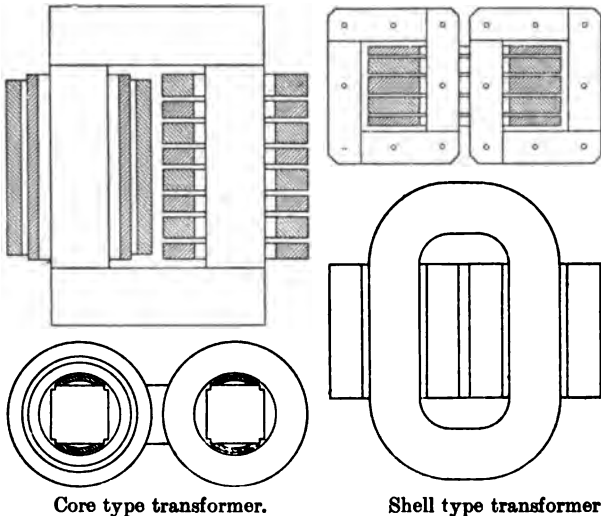


FIG. 117.

firmly pressed to the cores. Unless all the parts are firmly bolted together the transformer is noisy when at work.

In Fig. 117 the core is shown as a square; it may obviously be also of rectangular section. Transformers for different power can then be made out of the same stampings merely by varying the depth to which core and yoke are built up. The square section is, however, preferred in cases where the transformer is liable to an occasional short circuit as for instance in electric furnace work. Although a short circuit will not burn out a transformer if the breaker acts quickly, yet damage may be done to the coils by the enormous mechanical stress produced by the short circuit current. Since the circle is that shape which best resists mechanical deforma-

tion it is advisable to use circular coils and that leads to a square core.

In a shell transformer the iron part or carcass is generally built up of plates with overlapping joints as shown. To ensure correct position of the plates when they are inserted into the assembled coils, two holes are punched into each plate so that it may be threaded over wooden pins and thus brought to register exactly with the other plates. When the whole of the plates forming the carcass have thus been put into correct alignment with all joints overlapping at the eight corners, castings to hold the plates together are placed in position and bolted up by insulated bolts passing through the holes in the plates which have served for threading on to the wooden pins. This construction reduces the reluctance of the magnetic circuit to a minimum, but it has the disadvantage that in the event of any part of the winding having to be replaced the whole of the transformer has to be dismantled, the plates being taken out one by one and after the repair or replacement of the damaged winding again assembled one by one. This defect can be avoided by using butt joints as in a core transformer, but then another difficulty arises, namely that of a larger magnetising current.

Magnetic Reluctance of Joints. The influence of a joint whether of the overlapping or the butt type may conveniently be expressed by reference to a certain length of air gap, the relation between the actual magnetic reluctance and that of an equivalent air gap being a matter for experimental investigation. Dr H. Bohle has made such tests and published the results* from which the following table has been compiled. The experiments were made with overlapping joints and various kinds of butt joints, namely rough and machined, compressed and not compressed. As for silent working the joints must be under mechanical pressure, the figures referring to non-compressed joints are here omitted. The tests were made with inductions across the joint varying in steps of 1000 from 4000 to 14,000. The influence of the induction on the equivalent air gap is not very marked and for this reason steps of 3000 in the value of induction are taken in the table here reproduced from Dr Bohle's figures.

* "Magnetic reluctance of joints in transformer iron," *Journal Inst. El. Eng.*, No. 191, p. 527.

JOINTS IN THE MAGNETIC CIRCUIT OF A TRANSFORMER

$\frac{B}{1000}$	Equivalent air gap in mm.		
Induction in units of 1000 c.g.s. lines per sq. cm.	Overlapping joint	Butt joints compressed	
		Machined	Rough
4	0-0165	0-0290	0-0370
7	0-0270	0-0355	0-0460
10	0-0330	0-0395	0-0505
13	0-0360	0-0430	0-0530

According to these experiments the magnetic reluctance of a machined butt joint is not very much greater than that of a joint made by interleaving the plates. It should, however, be remembered that in these experiments the surfaces were in absolute metallic contact. This condition is hardly permissible in practice, for apart from the probability that in the machining (unless very carefully done with a square tool and very fine feed) edges are brought into actual contact, there is bound to be more or less metallic contact over the two surfaces of the joint when pressed together and as a consequence heating and loss of power by eddy currents. To avoid this it is necessary to insert a thin piece of insulating material. This occasions a certain increase in magnetic reluctance and for this reason the best modern practice is to use interleaved joints, both in shell and core transformers.

Arrangement of Coils. In core transformers the coils may be arranged either as concentric cylinders as shown on the left limb in Fig. 117, or as discs of equal diameter threaded over the core as shown on the right limb. The coils belonging to one circuit are sandwiched between those of the other circuit. With cylindrical coils it is immaterial which of the two cylinders is the primary and which the secondary coil; but as a matter of convenience it is advisable to make that coil which has the thinner wire and the greater number of turns the outer cylinder. This is done to facilitate a final and possibly very slight adjustment in the transforming ratio. To alter this ratio we must alter the number of turns in at least one coil and where the alteration desired is very slight that coil must be the one that has most turns. For this reason it is

better to make the fine wire coil the outer cylinder; then adding or taking off a few turns need not disturb the rest of the winding. The accurate adjustment of transforming ratio is always desirable, but it becomes essential in transformers intended for parallel operation.

In large transformers with cylindrical coils the winding on each limb may consist of one primary and one secondary cylinder as shown in Fig. 117, but sometimes there are more than two cylinders on each limb. If there are three, the innermost and outermost form the winding for the high voltage circuit and the middle cylinder that for the low voltage circuit. The cylinders should be a little shorter than the length of limb between yokes so as to leave sufficient space for the cooling fluid to enter and leave the channel between core and cylinder without being obstructed by the yokes.

Cylindrical coils may also be used in shell transformers, but it is generally more expedient to use disc winding, since flat coils can more easily be put into accurate shape than non-circular cylinders. All disc coils are wound in two flat spirals so as to have their inner ends joined and the other ends accessible on the outside. The two flat spirals are insulated from each other by a sheet of insulating material. The unit thus formed is termed a half coil; two such units insulated from each other are a full coil. In building up the nest of coils the first and last is generally a half coil, the intervening coils being full coils. In Fig. 117 the winding consists of two half and three full coils. All coils must be carefully insulated from each other by separating sheets and sometimes a space is left between them for the circulation of the cooling fluid. It is also good practice to leave a space between the two halves of the central iron core. This space is very efficient for cooling because the cooling fluid sweeps past the edge surface of the plates. Sometimes clearance spaces for cooling are also left between the flat faces of packets of plates, but these are not so efficient because the heat has to travel to them transversely to the laminations. Since the heat conductivity in the two directions* is approximately as 80 or 100 to 1 equal temperature gradient in both directions will be obtained if the thickness of the packet is about one-ninth of its width. This involves a great waste of core space and for this reason the packets are not made quite so thin, but such proportions as 1 to 3 or 4 are adopted as a compromise.

* T. M. Barlow, "The heat conductivity of iron stampings," *Journal Inst. El. Eng.*, No. 189, p. 601.

The general construction of three phase transformers is shown in Fig. 118. In the core type there is some saving of material in the combination of the three phases in one apparatus, but very little. For the shell type the minimum width of yoke and shell relatively to the width of the core c is shown in the figure and from this it will easily be seen that the saving in iron is almost insignificant, whilst there is no saving in copper as compared with three separate single phase transformers. In the core type there is some economy in copper, since we have only half the number of coils to provide and although these coils are larger than would be required in three single phase transformers, they do not collectively weigh as much as the twelve coils of three transformers. This is due to the larger flux

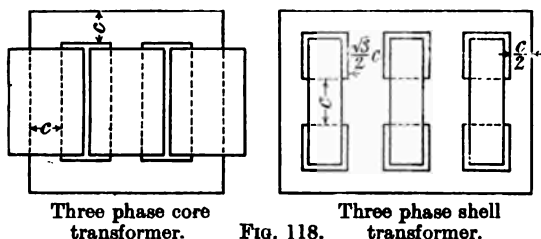


FIG. 118.

through each coil which allows the number of turns to be reduced. Consequently we find in practice that transformers are made three phase if of the core type, but very seldom if of the shell type. Large three phase units are, however, generally made up of three single phase transformers whereby the amount of spare plant is reduced to one-third. This plan has also the incidental advantage that if the three separate transformers are mesh coupled there need be no interruption of the service if one of them becomes disabled. It will be shown in the next chapter that a three phase service may be given with only two transformers which in this case will be overloaded. But the period of overload is very short, namely the time required to bring the spare transformer into service, so that no overheating need be feared.

Subdivision of Winding. In most cases transformers are intended to supply current under constant voltage at all loads. Such perfection cannot be reached in practice, but it may be approached very closely. If in a core transformer all the primary turns were put on one limb and the secondary on the other, a large

portion of the flux created by the primary under load would not pass through the secondary, but leak across the air outside of the secondary coils. Hence the e.m.f. induced in the secondary would decrease with the load. It is therefore necessary to put both windings on each limb and moreover to put the coils as close as considerations of insulation and efficient cooling will allow. The leakage is then confined to the narrow space between the coils and the difference of flux passing through the secondary at no load and full load need only be a few per cent. With cylindrical winding the reluctance of the leakage path is fairly high since its length is that of a core; with disc winding the length of the leakage path is much smaller, namely little more than the radial depth of the coils. To minimise the leakage it is therefore necessary to reduce the exciting force which drives the leakage lines through the space between two neighbouring coils. This leads to the necessity of splitting up the winding into a number of coils of small axial length. Besides the effect of a variation of induced e.m.f. in the secondary due to magnetic leakage there is also the influence of ohmic resistance which makes the induced e.m.f. in the primary slightly smaller than the e.m.f. impressed on the primary terminals and the secondary terminal e.m.f. again slightly smaller than the e.m.f. induced in the secondary. All these effects taken together are called the "regulation" of the transformer and this is expressed as a percentage. When we say that a transformer has at 0.75 power factor a regulation of 4 per cent. it means that with a terminal voltage of 100 at no load its terminal voltage will be only 96 at full load of 0.75 power factor. Part of the drop of 4 per cent. is on account of ohmic resistance and part on account of magnetic leakage. We speak therefore of a "resistance drop" and an "inductive drop." The former can easily be calculated by Ohm's law. The latter we proceed now to investigate.

Predetermination of Inductive Drop. The exciting force producing the leakage field is that due to one section of the subdivided winding. With cylindrical winding this means the ampereturns in one cylinder, with disc winding those in one disc or "full coil." It is impossible to calculate correctly the shape, distribution and linkage of the leakage field, but we can determine in a general way its influence as a function of ampereturns and the dimensions of the coil and then obtain a numerical value from experiment to be

used as a coefficient in the general formula expressing the ratio of inductive drop and open circuit voltage.

Fig. 119 shows roughly the induction in the space within and between the coils on the left for cylindrical coils and on the right for disc coils. The length

of primary and secondary cylinder is l and their radial depth a_1 and a_2 respectively. Let j be the number of turns per cm. radial depth of winding in the secondary cylinder, then the number of turns dn in an elementary layer da will be jda and these are interlinked

with the self-induced flux represented by the shaded area below the figure. Since primary and secondary currents are very nearly in opposition, the flow of leakage lines will be clockwise round one coil and counter-clockwise round the other, the boundary between the two fluxes being somewhere in the annular space b between the two coils. The space b_2 belongs to the flux of II and the space b_1 to that produced by I. The flux interlinked with the elementary layer is proportional to

$$\left[Bb_2 + \left(B + \frac{a}{a_2} B \right) \left(\frac{a_2^2 - a^2}{2} \right) \right] p = B \left(b_2 + \frac{a_2^2 - a^2}{2a_2} \right) p$$

where B is the induction produced by the X ampereturns of coil II and p the perimeter of the annular air space, which we assume to be also that of the two coils themselves. This assumption is not correct; the perimeter of II is smaller and that of I larger than p , but, as we are only interested in the sum of the inductive e.m.f.'s in both circuits, the error committed by taking p the same for both may be considered to cancel out.

The crest value of the self-induced e.m.f. of the jda turns of the elementary layer is

$$de_{s2} = \omega B \left(b_2 + \frac{a_2^2 - a^2}{2a_2} \right) jda$$

and this integrated over the whole coil gives

$$e_{s2} = \omega B j a_2 \left(b_2 + \frac{a_2}{3} \right) p$$

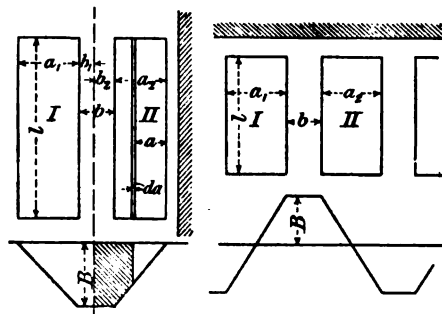


FIG. 119.

We do not know the exact value of B , but we know that it is proportional to X/l . Since $ja_2 = n_2$ we can write

$$e_{s2} \equiv \omega X n_2 \left(b_2 + \frac{a_2}{3} \right) \frac{p}{l}$$

In the same way we find for the self-induced e.m.f. of the primary

$$e_{s1} \equiv \omega X n_1 \left(b_1 + \frac{a_1}{3} \right) \frac{p}{l}$$

Let e_1 and e_2 be the crest value of the induced e.m.f. in primary and secondary, then

$$e_1 = \omega \Phi n_1 \quad \text{and} \quad e_2 = \omega \Phi n_2$$

Put

$$\frac{e_{s1}}{e_1} + \frac{e_{s2}}{e_2} = \frac{e_s}{e}$$

then $\frac{e_s}{e}$ is the ratio of inductive drop to e.m.f.

The percentage of inductive drop is

$$100 \frac{e_s}{e} = K \cdot \frac{X}{\Phi} \left(b + \frac{a_1 + a_2}{3} \right) \frac{p}{l} \quad \dots \quad (91)$$

K being a coefficient which must be experimentally determined. Although K is not the same for all makes of core transformers with cylindrical coils I have found that $K = 0.1$ gives the regulation in fair agreement with the results of tests, if dimensions in (91) are inserted in cm., the flux in megalines and X in units of 1000 effective ampereturns.

The same reasoning may be applied if the winding of a core transformer consists of sandwiched disc coils, but since the length of the magnetic circuit is $2l$ the value of K in (91) is not 0.1, but 0.05, or possibly a little more on account of the end discs which are close to the yokes. X is the exciting force of one disc in units of 1000 effective ampereturns.

The value of K in shell transformers varies slightly with the arrangement of the coils. The following figures are derived from experiments.

Coils				Value of K
One full primary and two half secondaries	0.067
Two full primary, one full and two half secondaries	0.060
Three full primary, two full and two half secondaries	0.058

Best proportions. The output of a transformer depends on the core section, number of turns and size of wire. The weight of

the active material depends on section and length of magnetic circuit, number of turns and length of one turn and size of wire. If we wish to economise in copper we must have few turns and a large flux. But a large flux means a large section of core and therefore a great perimeter of coil. This may to some extent neutralise the saving in copper due to the reduced number of turns. If, on the other hand, we wish to economise in transformer plates we must make up for the smaller flux by an increased number of turns and to get these on to the core the length of the magnetic circuit must be increased, thus again neutralising to some extent by the increased length of the magnetic circuit the saving due to its lessened cross section. The matter is still further complicated by questions of insulation and cooling. The space required for insulation is not a fixed proportion of the available winding space through the window between the two cores or between core and shell, but depends on the size of wire and the voltage. The space required for cooling channels depends on the cooling agent adopted (air or oil) on the manner of its application, whether by natural circulation or forced flow and on the temperature rise permitted. An increase in the section of the magnetic circuit does not necessarily imply a proportional increase of flux, since the ability to dissipate heat increases with the square and the generation of heat with the same induction with the third power of the linear dimensions. It will thus be seen that the design of a transformer is largely a matter of trial and compromise and although several authors have developed lengthy mathematical formulae for the determination of the best proportions these attempts have not much practical value and cannot save the designer the labour of making comparative designs in order to determine the best proportions. As a starting point for the design the following figures may be taken. They refer to the size of window in relation to the width of core c and are averages of good modern practice. Let a be the width and b the height of the window, then with all dimensions in mm. we may put

For core transformers with the coils closely surrounding the core

$$a = 10 + 1.2c$$

For circular coils leaving cooling channels between core and coil

$$a = 30 + 1.3c$$

In both cases

$$b = 80 + 2.6c$$

This is under the supposition that the material of the plates is the best alloyed transformer steel the thickness being not less than 0.3 or more than 0.6 mm. To use inferior material for transformer plates is the worst possible economy. Not only is the efficiency lowered and the cost of provisions for cooling increased, but owing to the larger core section any saving in the cost of iron is more than balanced by the increased cost of copper.

In shell transformers we have

$$a = c \text{ and } b = 1.5 \text{ to } 2c$$

Weight of Active Material. On purely theoretical grounds we should expect a large reduction in the weight of active material per kva. with an increase of linear dimensions. Take two transformers, one twice the linear dimensions of the other, but containing the same number of turns. With equal electric and magnetic loading the large transformer would have four times the flux, that is, four times the e.m.f. and four times the current. Its output would therefore be 16 times that of the small transformer, whilst its weight would only be eight times that of the small transformer. On the assumption of equal flux and current density the weight of active material per kva. would therefore have been reduced by half. This conclusion is not borne out in practice. The large transformer does give a greater output per unit weight than the small one, but not in the ratio of the increase in linear dimensions. The reason is that our assumption of equal specific loading is an impossible condition because of the difficulty of cooling. On analysing modern transformers all designed for $f = 50$, but differing in output between such wide limits as 50 kva. and 2000 kva., the weight of active material per kva. does not vary between wider limits than six to two kilogrammes, of which 70 per cent. may be taken for iron and 30 per cent. for copper.

Some data from practice. As examples of the best modern practice I give here a table (see p. 263) of the weight of materials used in transformers of 100, 500 and 2000 kva. with different methods of cooling. I am indebted for this table to Mr A. P. M. Fleming, the transformer expert of the British Westinghouse Company.

Influence of Frequency on Output and Regulation. The output of a transformer is limited by the rate at which the heat generated in iron and copper can be dissipated without exceeding

**WEIGHTS OF MATERIAL FOR 3 PHASE CORE TYPE TRANSFORMERS FOR 50 CYCLES.
10 TO 20 KILOVOLTS PRIMARY TERMINAL VOLTAGE**

O.I.S.C. = Oil immersed self cooled.

O.I.W.C. = " " water "

O.I.F.C. = " " forced " (oil circulation with water cooling).

A.I.A.B. = air insulated air blast cooled.

Kva. ...	O.I.S.C.			O.I.W.C. O.I.F.C.	A.I.A.B.
	100	500	2000	2000	2000
Copper ... lbs.	350	1250	3000	2500	2600
Sheet Iron ... "	800	3000	7300	5500	6000
Tank ... "	560	2850	7000	3000	—
Details and Insulation ... "	290	800	1500	1300	1400
Total excluding oil lbs.	2000	7900	18800	12300	10000
Oil ... gals.	110	390	860	650	—
" ... lbs.	1000	3500	7700	5850	—

a certain temperature rise. Assuming then the same cooling condition the question what maximum output can be obtained when the transformer is worked at different frequency can be answered by investigating what electric and magnetic loading will produce the same internal losses as at the normal frequency. As regards copper losses it is obvious that the same current may be allowed, so that only the e.m.f. is a variable in this problem. The e.m.f. is proportional to the product of induction and frequency. The induction for equal heating at different frequencies is found from tests; it may be expressed as the power lost per kg. of iron at these frequencies. This information is given in Vol. I, p. 256, for the Stalloy brand of transformer plates and for 25, 50 and 100 frequency. To get the same heating in all cases we need only read off in the diagram the value of B for the same loss per kg. at the three frequencies. These values of B multiplied with the respective frequencies give figures which are proportional to the output. Let P_1 be the maximum output at the frequency f_1 and P the output at frequency f , then it will be found that the result of this calculation may be expressed with fair approximation by the formula

$$\frac{P}{P_1} = \left(\frac{f}{f_1} \right)^{0.38} \dots \dots \dots (92)$$

To find the influence of frequency on inductive drop we use formula (91). Since X is constant and the flux Φ is proportional to B we find that the percentage drop D varies inversely as the induction. Since the induced e.m.f. and for equal heating the power is proportional to the induction, we have $DP = D_1P_1$ and combining with (92) we obtain

$$D = D_1 \left(\frac{f}{f_1} \right)^{0.62} \dots \dots \dots (93)$$

If one and the same transformer is used at different frequencies, but always loaded up to its heating limit, the inductive drop is reduced at a lower frequency. In other words good regulation is more easily obtained at the lower frequency.

Best Division of Losses. The iron loss in a transformer intended to work at constant voltage is independent of the current; the copper loss is proportional to the square of the current. If with a view to decrease iron loss we increase the core section we must use finer wire so as to accommodate the same number of turns in the reduced winding space. If on the other hand we wish to reduce the resistance of the windings so as to have less copper loss we must increase the size of the window and make the core smaller. This means a higher induction and more iron losses. The problem is how to divide the losses between iron and copper so as to get a minimum of total loss. This problem can also be put into another form; namely a transformer made for a particular voltage and frequency being given, what should be its load in amperes so as to obtain maximum efficiency? Let P_i be the iron losses at the given voltage e and Ri^2 the sum of the copper losses, then the efficiency will be

$$\eta = \frac{e_i}{e_i + P_i + Ri^2}$$

This is to be a maximum. Since i is the only variable we find the condition for maximum efficiency by differentiating to i and equating to zero. This gives

$$P_i = Ri^2 \dots \dots \dots (94)$$

This equation means that maximum efficiency will be obtained if the transformer is so designed that iron and copper losses are equal at that load at which the transformer is principally worked. In the above reasoning we have represented the ohmic losses in the two circuits by a single expression, namely Ri^2 and this is

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$$\frac{i_1^2 n_1^2}{a_1 s_1} + \frac{i^2 n^2}{as}$$

and since

$$i_1^2 = \frac{i^2 n^2}{n_1^2}$$

the sum of the losses is also proportional to

$$i^2 n^2 \left(\frac{1}{as} + \frac{1}{a_1 s_1} \right)$$

To make this a minimum we must so divide the available space $a + a_1$ for all the coils into the two parts a and a_1 that the quantity in brackets becomes a minimum. Since $da_1 = -da$ we have

$$\frac{d}{da} \left(\frac{1}{as} \right) = -\frac{1}{a^2 s} \quad \text{and} \quad \frac{d}{da} \left(\frac{1}{a_1 s_1} \right) = +\frac{1}{a_1^2 s_1}$$

The condition for minimum loss is therefore $\frac{1}{a^2 s} = \frac{1}{a_1^2 s_1} = 0$; or

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$$D = D_1 \left(\frac{f}{f_1} \right)^{0.62} \dots \dots \dots (93)$$

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SCALED MILLER FOR MOVING CRADLE → SCALE = CM = 2.346

Adjust Cradle Gap
 Measure Binding

TRANSFORMERS

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and secondary current are proportional to the smallest. Now arises the question how the winding space (its sum) shall be a minimum. We consider a transformer with sandwiched windings of the same perimeter and the resistance per turn divided by the section of the winding space left after that required for the primary has been deducted must be divided between the two coils. Let a be the total axial length of the winding space and a_1 that allowed for the primary winding. If we give more winding space to the primary, that left available for the secondary, the resistance must be such that the sum of the losses is a minimum. Let s be the space factor of the secondary and let n and n_1 be the number of turns of the primary and secondary respectively. The sectional area of the wires will be proportional to n^2/as and n_1^2/a_1s_1 respectively. The resistance of the windings will be proportional to

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we must so divide the available space between the two parts a and a_1 that the quantity in the bracket is a minimum. Since $da_1 = -da$ we have

$$\text{and } \frac{d}{da} \left(\frac{1}{a_1 s_1} \right) = + \frac{1}{a_1^2 s_1}$$

the loss is therefore $\frac{1}{a^2 s} = \frac{1}{a_1^2 s_1} = 0$; or

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To make this a minimum we must so divide the available space $a + a_1$ for all the coils into the two parts a and a_1 that the quantity in brackets becomes a minimum. Since $da_1 = -da$ we have

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This equation means that the larger share of the available winding space must be assigned to that circuit in which the smaller wire

is used because its space factor is smaller. The losses in the two circuits must not be equal, but that in the low pressure circuit must be a little larger.

Choking Coils. In external appearance a choking or reactive coil may resemble a transformer, but it differs from the transformer in two essentials. It has only one winding and the magnetic circuit must contain an air gap. That an air gap is necessary will be seen on consideration of the function of a reactive coil. It must let a current pass which is as nearly as possible displaced a quarter phase in relation to the e.m.f. over the terminals of the coil. There must be a flow of energy from the electric circuit to the magnetic circuit during a quarter period and back again from the magnetic to the electric circuit during the next quarter period. The larger the amount of energy thus surging forward and backward the greater the reactive action of the coil. Now the amount of energy which can be magnetically stored in iron is very small in comparison with the amount that can be magnetically stored in air (compare the small shaded surface in Fig. 98, p. 189, Vol. I with the much larger surface of the triangle) and if we require a strong reactive action the magnetic circuit must contain an air gap. The most convenient way of making a choking coil of small kva. capacity is to use the carcass of a transformer with butt joints and insert plates of fibre or hard wood in the joints, thus forming air spaces of definite length. If n is the number of turns of winding, δ the sum of the air gaps in the magnetic circuit and i the effective current, then the crest value of the induction across the air gaps is

$$B = \frac{0.4\pi ni\sqrt{2}}{\delta} = \frac{1.75ni}{\delta}$$

the reluctance of the iron being neglected. Let A be the cross sectional area of the magnetic circuit through air and f the frequency, then the reactance e.m.f. is

$$e = 4.44fABn10^{-8}$$

and the apparent power is ei voltamperes.

Some loss of real power both in copper and iron is of course unavoidable, but it should be as little as possible in comparison with the apparent power. This is only another way of saying that the angle of lag between e and i should be as near 90° as possible. Using the previous notation we have: real power lost = $P_i + Ri^2$ and apparent power = ei . Greatest reactive effect with minimum

loss will be obtained if $\frac{e i}{P_i + Ri^2}$ is a maximum, or $\frac{P_i}{i} + Ri$ is a minimum. This condition is fulfilled if $\frac{P_i}{i} = Ri$. We get the greatest reactive effect at the smallest expenditure of power if the coil is so designed that the copper loss equals the iron loss. The power factor is then

$$\cos \varphi = \frac{2Ri}{e}$$

The use of iron in a choking coil of only a few kva. choking effect presents no difficulty, but when a coil is required to deal with large currents and voltages a considerable difficulty arises on account of magnetic leakage. The e.m.f. across the air gap becomes very large with the result that leakage lines are forced out of the iron not only in the plane of the plates, but also transversely to them, producing in the large surface of the plates eddy currents and heat with loss of real power. For large choking coils it is therefore better to abandon iron altogether and to construct the coil on the lines of a toroid of the most economical proportions as stated in Vol. I, p. 212. With a conductor of large cross section it is hardly practicable to adhere to the regular form of the toroid, namely a circular cross section, but there is no difficulty in adopting a rectangular or square section and to subdividing the winding into a number of flat discs with air spaces between them for cooling. I have found by direct experiment that the choking effect of a toroid with square section is within a few per cent. the same as that of a toroid with an equal circular section so that the formulae given in Vol. I, p. 212, may be used. With a circular section its best diameter is 0.311 of the mean diameter of the coil and if the circular is replaced by a square section the side of the square should be 0.27 of the mean diameter. A slight departure from the square does not materially alter the inductance as long as the cross section is that of a square of side $0.27D$, where D is the mean diameter of the toroid. For a regular toroid we found the formula for the inductance in henrys

$$L = 9.35n^2D10^{-7}$$

Experiment shows that for a rectangular section the coefficient 9.35 is a little too large. Substituting for this figure 8.7 gives better agreement with observed results. The number of turns n which can be accommodated in the given section depends on the sectional area of the conductor and on the space factor. Since all coils are

to be of the same geometric proportions we can substitute for n an expression containing D , the diameter in metres, q the cross section of the conductor in sq. cm. and s the space factor. The calculation is so simple that it need not be given in detail. The result is $L = 8.7n^2D10^{-7}$ and for the coil of best proportions

$$L = 0.5 \left(\frac{s}{q} \right)^2 D^5$$

In a similar way we can establish a formula for the weight of copper as a function of space factor and diameter. The weight is in kg.

$$W = 2.17sD^310^3$$

The power of the coil depends on current density and frequency. Let Δ be the current density in amperes per sq. cm. and f the frequency, then the power of the coil is in volt-amperes

$$VA = \pi f (s\Delta)^2 D^5$$

By combining the expressions for weight and power we get the plant efficiency in volt-ampere per kilogram of copper

$$\frac{VA}{W} = 14.5fs \left(\frac{\Delta}{100} \right)^2 D^2$$

Thus a coil of 0.8m diameter having 288 turns, 0.316 space factor and worked at a current density of 192 amperes per sq. cm. will give a choking effect of 192 kva. The weight of copper is 350 kg.; its plant efficiency is 540 va. per kg. Its loss when hot is about 3 kw. so that the phase angle is about 89° or practically quadrature.

The Autotransformer. When there is no objection to a metallic connection between primary and secondary, one and the

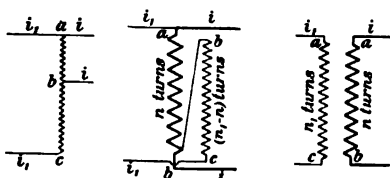


FIG. 120. Autotransformer.

same winding on a transformer can be made to serve both circuits. That of higher e.m.f. includes all the turns of this single winding; that of lower e.m.f. only a part of them. In

Fig. 120 the conventional symbol of an autotransformer is shown on the left; ac represents the primary winding and the part ab of it serves as the secondary winding. Only the primary current flows through bc , but the coils in ab carry both the primary and secondary current. As these currents have 180° phase displacement the coil ab has only to carry

the algebraic difference between the secondary and primary current. Thus a saving in copper is effected, or if the same amount of copper is used as in an ordinary transformer (shown on the right in the figure) the output may be increased with the same temperature rise. The terminals of the coils are similarly lettered. The arrangement of coils as shown in the conventional diagram on the left is impracticable because of the very large inductive drop which would result from crowding all secondary turns on one end of the core. The winding must, as in an ordinary transformer, be interleaved so as to ensure good regulation. This is diagrammatically shown in the middle figure. The economic advantage of the autotransformer is the better utilisation of the material and whether this advantage is important or not depends on the ratio of transformation. Let

m be this ratio so that $i = mi_1$ and $e_1 = me$ and $n = \frac{n_1}{m}$. In the

autotransformer the thin wire coil has $n_1 - n = n(m - 1)$ turns and is therefore shorter than the corresponding coil of the ordinary transformer. If the same amount of copper may be used in its construction then the sectional area of the wire may be increased in the same ratio as the length has been reduced, so that the resistance is decreased in the square of that ratio. The loss of power by resistance is proportional to the square of the current and since the resistance has been decreased in the ratio of

$$1 : \left(\frac{m-1}{m} \right)^2$$

the current may for equal heating be increased inversely to the square root of this ratio. It may therefore be $\frac{m}{m-1}$ times the current in the thin wire coil of the ordinary transformer. Sectional area of wire and its length in the thick wire coil are not altered, but the current is the difference between primary and secondary current. The secondary current is therefore also increased in the ratio of $1 : \frac{m}{m-1}$, and since the flux remains the same we find that for equal weights of material and equal heating the output P_a of an autotransformer is related to the output P of an ordinary transformer according to the equation

$$\frac{P_a}{P} = \frac{m}{m-1} \quad \dots \quad (96)$$

where $m = e_1/e$ is the transforming ratio.

Transforming ratio	=	1.5	2	3	4	10
Percentage gain in output	=	200	100	50	33	11

These figures show that the better utilisation of the material in an autotransformer is considerable if the transforming ratio is low, but inappreciable at a high transforming ratio.

***Balancing Transformer for Three Wire System.** Let in a D.C. armature the winding be tapped at points 180 electrical degrees apart and connected to two slip rings. The alternating current collected from these slip rings may be sent through an autotransformer with transforming ratio 1:1. The middle point of the winding will at all times have a potential midway between the potentials of the slip rings and therefore divide the D.C. voltage into equal parts. The centre of the autotransformer may therefore be connected to the zero wire of a D.C. three wire system, the outer wires being connected to the brushes. The autotransformer thus takes the place of the balancing set Fig. 47, p. 77, Vol. I, but it cannot perform the function of dividing the voltage between outers so perfectly, because the drop in the transformer will slightly lower the voltage on the heavily loaded side instead of slightly raising it as the rotary machines do automatically. Where a small voltage variation may be permitted, this system has the advantage of low cost and simplicity and is often used if only a few lamps are required to be supplied from a machine of double the lamp voltage. With an out-of-balance demand of, say, even as much as 40 per cent. each winding must take 20 per cent. of maximum lamp current, the induced e.m.f. being equal to the lamp voltage. The transformer may therefore be designed for only one-tenth of the total power required for all the lamps.

If the D.C. armature is part of a three phase converter (Chapter X), then no special apparatus is needed for splitting the D.C. brush voltage into two equal parts; all that is required is to connect the zero wire of the lighting system to the star point of the secondary of the transformer which is necessary anyhow to supply A.C. to the three slip rings of the converter. If the converter is compounded for constant voltage it is thus possible to supply a small three wire D.C. lighting system directly from the D.C. brushes and the star point of the transformer.

* For a detailed investigation, see "The Theory of the Static Balancer" by C. C. Hawkins. *Journal Inst. El. Eng.*, No. 204, page 704, vol. 45.

Cooling. Since the output of a transformer is mainly limited by temperature rise, the problem of abstracting the heat generated by magnetic and electric losses is of paramount importance. The iron loss depending on the constant e.m.f. is itself a constant; the copper loss varies as the square of the load. For any given load the total loss is therefore known and the temperature rise can be calculated if we know the laws connecting power dissipated per unit of cooling surface, and temperature drop from the surface to the cooling medium. Questions of this kind arise in many engineering problems and although formulae for heat transmission and heat dissipation may be found in any engineering text-book their application is neither obvious nor easy*. There are mainly two difficulties; one is the influence of the surface of the body to be cooled and the other the uncertainty as to the speed at which the cooling medium sweeps past this surface. The usual assumption, that with a quiescent gas about five calories are transferred per hour through a square metre of metallic surface per degree temperature drop, if translated into electrical measure gives the temperature drop as 2000 divided by the specific cooling surface, the latter meaning the number of sq. cm. divided by the number of watts to be dissipated. Introducing the symbol σ for this value and T for the temperature drop between quiescent gas and surface we have $T = 2000/\sigma$. Generally we may write as the formula for temperature rise of the surface over the temperature of the cooling medium

$$T = \frac{c}{\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

In this formula c is 2000 if deduced from figures found in engineering hand-books for a metal surface, but if we determine c experimentally for a transformer core it comes out only about half this figure. The reason is that our assumption of a quiescent body of air in contact with the surface is incorrect. As the air gets heated it rises, making way for new particles of cooler air; in other words there is not only radiation and conduction of heat, but also convection by the movement of air, even if the air is not forcibly blown past the surface.

The nature of the surface of the body to be cooled also affects the rate of dissipation of heat. It is greater for the iron carcass than for the coils if these are of cotton covered wire and taped and

* For an experimental investigation see Gifford, "The influence of various cooling media upon the rise in temperature of soft iron stampings," *Journal Inst. El. Eng.*, No. 201, p. 753.

varnished. Where oil is the cooling medium the rate is also considerably greater than for air and the rate may be further increased by facilitating the circulation of the oil. Thus the problem is very complicated and the predetermination of temperature rise cannot be solved on purely theoretical lines, but only by experiment and then only approximately.

Roughly speaking the cooling surface of a transformer is proportional to the square and the heat generated to the third power of the linear dimensions; hence in small transformers the specific cooling surface is greater than in large transformers and up to a certain size (of the order of 10 kw.) no special provision for cooling is required. The transformer can be put into a case for mechanical protection and if the case consists mainly of a perforated metal sheet the access of air thus provided will keep it cool. The temperature rise over the surrounding air may in this case be calculated as follows:

$$\text{For the carcass} \quad \dots \quad T = \frac{940}{\sigma} \quad . \quad . \quad (98)$$

$$\text{For the surface of the winding} \quad T = \frac{1500}{\sigma} \quad . \quad . \quad (99)$$

Where the case is of solid metal so that no exchange of air between outside and inside is possible, the same formulae may be used, but the temperature rise refers to the inside air. The heat must then flow from the inside air to the case and from the case to the outside air, and this involves a further temperature difference of about $T = 1200/\sigma$, where σ refers to the surface of the case.

In larger air cooled transformers it is necessary to provide an air blast. In this case the value of σ will be the lower the stronger the blast. As a rough approximation we may put

$$\text{For the carcass} \quad T = \frac{450}{\sigma} \quad . \quad . \quad . \quad (100)$$

$$\text{For the coils} \quad T = \frac{750}{\sigma} \quad . \quad . \quad . \quad (101)$$

A convenient method for cooling by air blast is to place the transformers over a culvert into which air is blown by a fan. It enters the case at the bottom and issues at the top through a hole over which is placed a little windmill to show that the air actually is passing. Internally there are adjustable slides to direct the stream of air through the channels in carcass and winding. It is customary to supply about 5 cub. m. of air per minute per kw. of lost power. The temperature of the air at the exit is then about

10° C. higher than at the inlet and the temperature rise is then 5° C. in excess of the mean temperature difference between the body and the entering blast. Where the air is pure, as for instance in a hydro-electric station in the mountains, this method of cooling is quite satisfactory, but it should not be used in a station situated in the middle of an industrial area because the air passing through the transformer would deposit soot and other impurities which in time must destroy the insulation.

As a general rule oil cooling is preferred to air cooling, not only because the dielectric strength of oil is from three to four times that of air, but also because the capacity of oil for carrying the heat away from the body is also much greater. This is primarily due to the greater specific heat and also to the large coefficient of expansion. The oil that has become heated by contact with the body is appreciably reduced in specific weight and rises, thus producing natural circulation which assists the cooling effect. The heat taken up by the oil must eventually find its way to the outside. Where no special means are provided for abstracting the heat from the oil we speak of a "self-cooling transformer." Various means may, however, be used for abstracting the heat from the oil. One of these consists in the provision of a coiled pipe in the upper part of the tank through which cold water is circulated. The hot oil rising is cooled by contact with the pipe and returns along the walls to the bottom. Where pure water is available this is the usual system. But if the water is impure a deposit may be formed within the pipe and the cooling become insufficient. Another way is to circulate the oil within the tank by means of a pump. In this case the pipe circuit includes a coil which is immersed in the water of a pond, brook or artificial cooling tank. A third way is to spray cold water over the exterior of the case. Any of these methods are comprised under the name "water cooled transformer."

The following values of c in $T = \frac{c}{\sigma}$ are averages as found in practice:

Iron carcass to oil	110
Insulated and taped coil to oil	200
Oil to wall of tank	200
Oil to water through metal of pipe	220
Wall of tank to outside air	600

The difference in the value of c for the iron plates of the carcass and the iron walls of the tank is explained by the greater speed

with which the oil flows through the cooling channels, their area being necessarily smaller than the space between transformer and tank through which the oil descends.

The necessity to provide either a fan or a pump to assist cooling is a drawback which has led modern designers to develop the self-cooling principle to a greater extent than was customary formerly. This may be done in several ways. The most obvious is to cast the tank with external ribs or to use for its walls steel sheet with deep corrugations. This will increase the surface of a tank three or four times, but it will not improve the circulation materially. To obtain a vigorous circulation vertical pipes are placed all round the tank and connected with the interior at bottom and top so that the heated oil enters the external cooling pipes at their upper ends, descends through the pipes and re-enters the tank at the bottom. In very large transformers these pipes themselves are made of sheet steel with deep corrugations. In this way not only is the total cooling surface very much increased, but the circulation is also improved so that the self-cooling principle can be applied to transformers of 1000 kw. or more.

Transformer Oil. A variety of oils are on the market for use in transformers. For details of the physical, chemical, and thermic properties the reader is referred to the report of the Oil Research Committee of the Institute of Electrical Engineers published in the *Journal*, No. 239, p. 146. Here it must suffice to note a few characteristics. For self-cooling transformers a dark amber coloured oil of 0.868 s.g. and 188° C. flash point is generally used; for water cooled transformers a lighter coloured oil of 0.850 s.g. and 130° C. flash point. It is essential that the oil before filling into the tank should be thoroughly dried, since even a small amount of water lowers its specific resistance considerably. The bulk of the moisture is removed by blowing hot air through or by steam heating, the pressure in the coiled pipe being about 10 lbs. to the square inch. The last traces of moisture are removed by filtering through blotting paper under pressure. In the best equipped modern transformer stations filter presses form part of the equipment in order that the oil used may be permanently kept free from moisture. Since some room above the oil in the transformer tank must be kept free to allow for expansion, or as it is technically termed to allow the transformer to breathe, some moisture may be deposited in the

process of the air flowing into and out of this space. Hence the necessity of drying the oil afresh after it has been in use some time. For this purpose a draw off cock is fitted at the lowest point of the tank by which the oil is discharged into a pipe system connected with the filter presses. The dried oil as it comes from the filter is led by another pipe system to the top of the tank. It is thus possible to dry the oil of a transformer without having to take it out of service. The plan has also been tried to fill up the tank completely and allow the expansion to take place in an overhead vessel, but this arrangement involves an absolutely oil tight joint between cover and tank. It is very difficult to make such a joint even in the first instance and almost impossible to re-make if for any reason the transformer has to be opened.

All oils are liable to the formation of a more or less solid deposit, but some brands more than others. This deposit or sludge may clog up the cooling channels and must be removed from time to time. Where a filtering plant is provided and frequently used the transformer need not be opened for the purpose of cleaning since the sludge is drawn off with the oil on its way to the filter.

Temperature Rise as a Function of Time. When a transformer at room temperature is switched on to the supply circuit and its secondary circuit is loaded, heat is developed both in the carcase and the winding. The amount of heat if the load remains constant is simply proportional to time, but the temperature rise must follow some other law. After a certain temperature is reached the dissipation of heat keeps pace with its generation and the temperature cannot rise any more, however long the transformer may be kept working. According to the Engineering Standards Committee's rules the final temperature rise (with an ambient temperature up to $40^{\circ}\text{C}.$) shall not exceed $55^{\circ}\text{C}.$ for iron and copper and $50^{\circ}\text{C}.$ for oil. Since the heat flows outwards there must be a slight difference in temperature between active material and oil. To take this into account would make the mathematical treatment of the problem rather complicated, so we shall assume that all materials inside the tank (iron, copper and oil) are at the same temperature. In doing so we slightly overestimate the heat required for raising the temperature of the contents of the tank; on the other hand by neglecting the heat absorbed by the metal of the tank we underestimate the total heat slightly. The two errors may

be assumed approximately to cancel, so that in calculating the number of calories necessary to raise the active material and the oil to the same temperature, namely that of the active material, the error involved will not be serious. If W_i , W_c and W_o represent the weight of iron, copper and oil respectively, and 0.114, 0.093 and 0.42 the values of the specific heat of these materials, the heat in calories stored per degree centigrade is

$$0.114W_i + 0.093W_c + 0.42W_o.$$

and since one calorie is represented by the energy of 4160 watt seconds the energy stored at a temperature rise of y degrees is yA joules, where

$$A = 4160 (0.114W_i + 0.093W_c + 0.42W_o) \quad . \quad (102)$$

Let P = watts lost in copper and iron,

Pdx = joules lost in time dx ,

$yDdx$ = joules dissipated in time dx at temperature y° C. over the temperature of the ambient air in a self-cooling transformer. Let D be a constant denoting the ability to dissipate heat. The numerical value of this constant is found by considering the stable condition when the final temperature rise T has been reached. In this case all the power lost is dissipated so that we have $P = D \times T$ and $D = P/T$. In a transformer with forced blast W_o is zero and the temperature rise y must be reckoned in reference to the mean temperature of the blast. In an oil filled transformer with water cooling the oil must be considered to be the ambient medium and the temperature rise reckoned in relation to it. In this case also W_o must be omitted from the expression for A . In all cases we have during the heating period the relation: energy put into the systems equals the energy required to raise its temperature plus that dissipated, or in symbols

$$Pdx = Dydx + Ady \quad . \quad . \quad . \quad (103)$$

$$dx = \frac{Ady}{P - Dy} = - \frac{A}{D} \frac{d(P - Dy)}{P - Dy}$$

Let x_1y_1 and x_2y_2 be two conditions during the heating period, then by integration we find

$$x_2 - x_1 = x = \frac{A}{D} \log_e \left(\frac{P - Dy_1}{P - Dy_2} \right)$$

and since $D = P/T$ we can also write

$$x = \frac{AT}{P} \log_e \left(\frac{T - y_1}{T - y_2} \right)$$

If time is counted from the moment that the transformer is switched on, then $y_1 = 0$ and we have

$$x = \frac{AT}{P} \log_e \frac{T}{T - y_2} = \frac{AT}{P} \log_e \frac{T}{z}$$

if by z we denote the difference between the final temperature and that which has been reached in the time of x seconds from the start, when the transformer had the temperature of the ambient. For a particular value of z the ratio T/z is equal to e , the basis of natural logarithms, and since $\log_e e = 1$ we have $x = T \frac{A}{P}$. In this case $T/z = 2.718$ and $y_2 = 0.63T$. The time corresponding to this specified temperature is called the *heating-time-constant*. If we give it the symbol t we have

$$t = T \frac{A}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

The heating-time-constant may be defined as the time required to produce a temperature rise of 63 per cent. of the final value. It will be seen by (104) that t is independent of the load since by (97)

$$T = \frac{cP}{S} \quad \text{and} \quad \frac{T}{P} = \frac{c}{S}$$

which is a constant. By introducing t into the above formula for x we have

$$x = t \log_e \left(\frac{T - y_1}{T - y_2} \right) \quad . \quad . \quad . \quad . \quad . \quad (105)$$

This is a logarithmic function and its graph is a logarithmic curve as shown in Fig. 121.

In (105) time is expressed as a function of temperature rise; to get temperature rise as a function of time we write

$$\frac{T - y_1}{T - y_2} = e^{\frac{x_2 - x_1}{t}} \quad . \quad . \quad . \quad . \quad . \quad (106)$$

For the origin $x_1 = 0$ and $y_1 = 0$. Inserting we find

$$\frac{T}{T - y} = e^{\frac{x}{t}} \quad \text{and} \quad y = T \left(1 - e^{-\frac{x}{t}} \right) \quad . \quad . \quad (107)$$

The slope of the tangent at any point is $\frac{dy}{dx} = \frac{T}{te^{\frac{x}{t}}}$ and at the origin, where $x = 0$, it is

$$\tan \alpha = \frac{T}{t} \quad \dots \dots \dots (108)$$

This expression gives a convenient method of finding the heating-time-constant experimentally. It can, of course, be found by

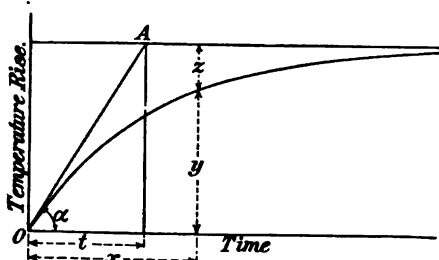


FIG. 121. Heating curve.

working the transformer at any definite load until 63 per cent. of the final temperature has been reached, but with a large transformer it takes some hours until $y = 0.63 T$ is reached. The test can be shortened by restricting it to the lower part of the logarithmic curve so that the

slope of the tangent is found. This prolonged, until it cuts the horizontal whose ordinate is T , gives t . There is no quick way of finding the final temperature rise, but the time during which observations have to be taken may be considerably shortened by a preliminary heating up of the transformer. This may be done by alternately heating up the iron by A.C. in one circuit, the other being open, and then heating up the copper by D.C. This has also the advantage of economising energy. When in this way the transformer has been heated up to what may be supposed to be its final temperature it is put to work in the normal way and observations are taken at intervals of about half hours. By plotting the results it is possible to judge the final temperature which would be reached after an infinite time.

Formula (108) provides a second definition for the heating-time-constant. If there were absolutely no dissipation of heat at any temperature, then the graph connecting temperature and time would be the straight line OA in Fig. 121. If the cooling arrangements were absolutely inefficient so that none of the heat generated in iron and copper is dissipated, then the time taken to reach final temperature rise is the heating-time-constant.

In (104) TA means joules or watt seconds and P means watts;

this gives t in seconds. It is convenient to express t in hours when the value of the heating-time-constant is

$$t_h = \frac{TA}{3600P} \quad \dots \quad (109)$$

and x in (105) is then also expressed in hours. Using common logarithms we have

$$x = 2.3t_h \log \frac{T}{T - y} \quad \dots \quad (110)$$

The time required to bring the temperature rise y up to a certain percentage of the final temperature rise T may now be expressed as follows:

63 per cent. is reached in time $t_h = \frac{TA}{3600P}$ hours,	
90 per cent. is reached in	$2.3t_h$ hours,
99 per cent. is reached in	$4.6t_h$ hours,
99.5 per cent. is reached in	$5.3t_h$ hours,
99.9 per cent. is reached in	$6.9t_h$ hours.

Temperature Drop as a Function of Time. Let the temperature of a transformer exceed that of the ambient by T_1 and let its load be reduced so that the loss is P_0 to which corresponds the final temperature of T_0 over that of the ambient. The temperature rise which at the moment the load was reduced was T_1 will now drop to the final value T_0 and at time x it will have some intermediate value y . The energy dissipated during the time dx must not only include that put in, namely $P_0 dx$, but also that represented by the drop in temperature dy . We have therefore as the differential equation of the cooling process

$$Dydx = P_0 dx - Ady$$

the negative sign indicating that the temperature decreases with increase of time. The solution of this equation being similar to that representing the heating process need not be given in detail. The result is

$$x = t \log_e \left(\frac{T_1 - T_0}{y - T_0} \right) \quad \dots \quad (111)$$

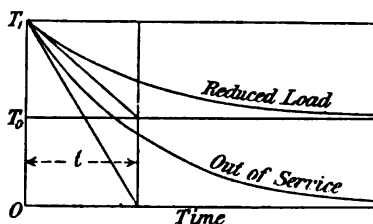


FIG. 122. Cooling curve.

where T_1 is the initial temperature rise and y is the excess of temperature over that of the ambient x seconds after the load was reduced. If the load is reduced to zero by the transformer being taken out of service altogether, $T_0 = 0$ and

$$x = t \log_e \frac{T_1}{y} \quad . \quad . \quad . \quad . \quad . \quad (112)$$

The curves corresponding to (111) and (112) are shown in Fig. 122.

Heating and Cooling with Intermittent Loads. Let a transformer be worked for a hours at heavy load when the loss is P and b hours at a much lighter load when the loss is P_0 . Or let it be taken out of service altogether for b consecutive hours when $P_0 = 0$. During the time of heavy load the transformer will heat up and during the time of reduced load it will cool down again. With regular alternations of heavy and light load the temperature and temperature rise will vary between certain limits, the maxima occurring at the end of the a hours of heavy load and the minima

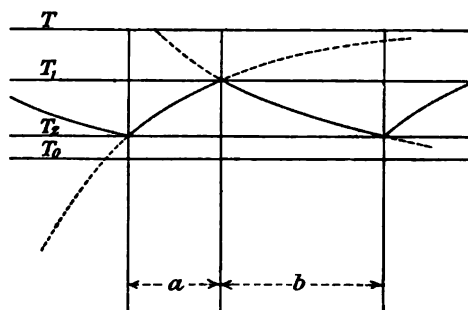


FIG. 123. Intermittent service.

at the end of the b hours of light load, which we assume to coincide with the beginning of the heavy load period. The question now arises as to the extent the transformer may be overloaded in this intermittent service. If the heavy load period is less than the heating-time-

constant, then the transformer would at normal rating never get up to the final temperature permitted and a transformer of a somewhat smaller rating might be used. Thus in a lighting installation peak time only lasts for a few hours every evening; all the rest of the time the output is sensibly less and since the copper loss goes by the square of the current we have practically only half the loss during the rest of the 24 hours. Let in Fig. 123 a and b represent the periods of heavy and light load and T_1 and T_2 the corresponding temperatures at the end of each period. T_1 is the permissible temperature rise. Let P_1 be the loss which would in continuous service occasion the final rise T_1 and P a greater

loss (corresponding to overload) which in the intermittent service causes the same rise to T_1 . Let further T be the rise which the excessive loss P would occasion in continuous service and T_0 the rise corresponding in continuous service to the loss P_0 at reduced load. We have now for the heating and cooling periods the two expressions

$$\frac{a}{t} = \log_e \left(\frac{T - T_2}{T - T_1} \right) \text{ and } \frac{b}{t} = \log_e \left(\frac{T_1 - T_0}{T_2 - T_0} \right)$$

The two unknown quantities are T and T_2 . Of these only T interests us as it determines to what extent we may overload the transformer.

We may write

$$\frac{T - T_2}{T - T_1} = e^{\frac{a}{t}} \text{ and } \frac{T_1 - T_0}{T_2 - T_0} = e^{\frac{b}{t}}$$

and if for simplicity we put $e^{\frac{a}{t}} = \alpha$ and $e^{\frac{b}{t}} = \beta$ the problem is reduced to the solution of the two simple equations

$$\alpha (T - T_1) = T - T_2 \text{ and } \beta (T_2 - T_0) = T_1 - T_0$$

Eliminating T_2 we find

$$T_1 \alpha - T (\alpha - 1) = \frac{T_1 - T_0}{\beta} + T_0$$

Solving for T and remembering that loss and final temperature rise are proportional we obtain

$$P = \frac{P_1 (\alpha \beta - 1) - P_0 (\beta - 1)}{\alpha \beta - \beta} \quad \dots \quad (113)$$

In this formula P_1 is the loss corresponding to the normal rating of the transformer for continuous service and P the loss which occurs during the time of overload in the intermittent service. P_0 is the loss during the period of greatly reduced load. If the transformer is designed for equal iron and copper loss at normal rating P_0 will be only very little more than $P_1/2$. If the transformer is taken out of service altogether during b hours a day, then

$$P = P_1 \frac{\alpha \beta - 1}{\alpha \beta - \beta} \quad \dots \quad (114)$$

In certain cases it may be financially advantageous to use two transformers, one just large enough to take at normal rating the light load during the b hours and the other larger to take the peak load. This transformer may be sensibly overloaded since it will be taken out of service for b hours a day and has therefore time

to cool down in the interval between its periods of overload. The theory of heating and cooling here developed is not restricted to transformers, but applies equally to motors and in fact to any electrical apparatus which is internally heated and whose output is limited by temperature rise. In many cases the periods of heavy and light or no load are exceedingly short as compared to the heating-time-constant. The latter is generally a matter of hours and the alternation in service conditions may count by minutes. Then the exponents in the terms α and β are very small and we may, when developing in a logarithmic series, neglect all terms after the second. This gives

$$\alpha = 1 + \frac{a}{t}, \quad \beta = 1 + \frac{b}{t}, \quad \alpha\beta = 1 + \frac{a+b}{t}$$

By inserting in (113) we find

$$P = \frac{P_1(a+b) - P_0b}{a} \quad . \quad . \quad . \quad (115)$$

and

$$P_1 = \frac{Pa + P_0b}{a+b} \quad . \quad . \quad . \quad (116)$$

which means that the average between overload and light load loss must not exceed the permissible loss in continuous service at normal rating.

CHAPTER XII

GRAPHIC TREATMENT OF TRANSFORMER PROBLEMS

Magnetising current of a transformer—Frequency changer—Vector diagram of a transformer—Simplified vector diagram—Coupled transformers—Predetermination of terminal voltage under varying load and power factor—Testing large transformers—Two-phase to three-phase transformer—Advantage of mesh coupling for three-phase work.

Magnetising Current of a Transformer. If the hysteric loop of the iron used in the transformer is known it can be so drawn that abscissae represent current (instead of H) and ordinates flux (instead of B). In redrawing the loop account must be taken of the length of the magnetic circuit and the reluctance of joints. To find the wave form of the magnetising current we reason as follows. The impressed e.m.f. having sine form it follows that the flux wave must also have sine form. In Fig. 124 these two curves are shown

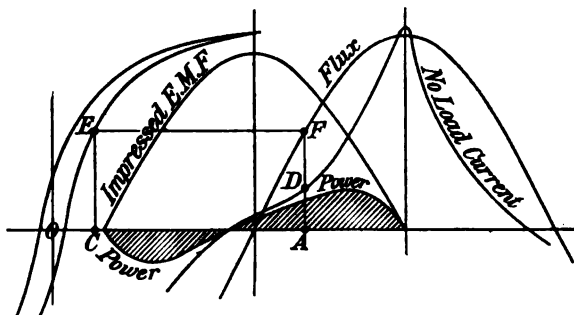


FIG. 124. Primary current at no load.

in their correct phase position, namely in quadrature, the impressed e.m.f. leading. If the reluctance of the magnetic circuit were constant then the current curve would also be a sine curve in phase with the flux curve, but on account of hysteresis the reluctance is not constant; it is greater on the ascending part of the loop than on the descending and for zero current when there is some residual induction it may be considered to be zero. We must therefore use the hysteric loop to find the relation between flux and current and we must use the sine curve representing flux as a function of

time to draw the current curve as a function of time. Thus let E be a point on the ascending positive branch of the loop. At that moment the flux has the value CE and the corresponding time is found by drawing a horizontal through E and marking the point F on the flux curve. Counting time from the moment that the flux passes through zero we find that A is the abscissa of the corresponding point on the current curve. OC is the magnitude of the current at that moment. Make $AD = OC$. This gives the point D on the current curve. Repeating the construction for various points both on the ascending and descending branches of the loop we get the curve marked in the diagram "no load current." It will be noticed that it has a strong third harmonic. The shaded curve represents power, the ordinates being the instantaneous values, ei . The shaded areas are energy input and output during a half cycle and the difference between the two areas is the energy lost in hysteresis.

Frequency Changer. The strong third harmonic is not due to hysteresis, but to saturation. Hysteresis produces only the

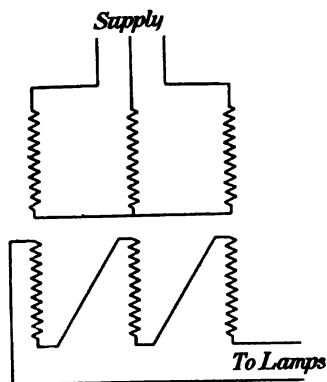


FIG. 125. Spinelli's method for trebling frequency.

dissymmetry in the curve and if it were possible to obtain iron which has no hysteresis we should still have a peaky curve, but symmetrical to the flux curve. Iron without hysteretic loss cannot be obtained, but by driving saturation much further (which can be done by increasing the impressed e.m.f.) the effect of saturation will more or less swamp the hysteretic effect so that the two sides of the peak will be nearly symmetrical to the centre line of the flux curve. This effect of saturation to produce a peaky current curve has been used for the purpose of increasing the frequency. Various methods may be employed*; here we shall only consider two, namely an earlier method due to Spinelli and an improvement

* See Maurice Joly, *Lumière Electrique*, Vol. xiv, p. 105, 1911; Vallauri, *Atti Associazione Elettrotecnica Italiana*, Vol. xv, p. 391, 1911; Spinelli, *Elettricista*, 1913, p. 215, and *The Electrician*, 1912, p. 97; A. M. Taylor, *Journal Inst. El. Eng.*, No. 235, p. 700.

on it due to Mr A. M. Taylor. Spinelli uses three transformers, which have their primaries star coupled whilst the secondaries are in mesh connection. Core section and number of turns are such that with the supply voltage at normal frequency the iron is well saturated with the result that a third harmonic is developed in the primary current and this gives rise to a third harmonic in the coils of the secondary. In Fig. 125 the primary and secondary coils are drawn as if they were end on in line; this is merely done to avoid wires crossing; in reality the coils are wound on the same core as in any ordinary transformer. The mesh formed by the secondaries is opened at one point for the insertion of the lamp circuit. It has been shown in Chapter VII that in a three phase mesh the fundamental e.m.f. cannot produce a circulating current since the sum of the e.m.f.'s is zero at all times, but that the third harmonic does produce circulating current. In the present case this current passes through the lamp circuit and is of treble frequency.

Mr Taylor's improvement consists in the separation of the element which produces the third harmonic from the transformer proper. He uses three highly saturated choking coils marked *A*, *B*, *C*, in Fig. 126, and three primary windings *a*, *b*, *c*, on the same core of a transformer of usual proportions. Since the core is not saturated and has therefore approximately constant reluctance the fundamental of the currents *a*, *b*, *c* produce no magnetisation; but the third harmonic produces magnetisation and consequently an e.m.f. of treble frequency in the secondary windings of this transformer. This action will be more clearly understood by reference to Fig. 127

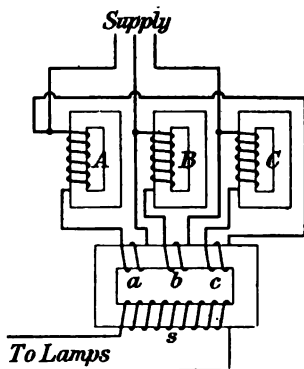


FIG. 126. Taylor's method for trebling frequency.

where the curves *A*, *B*, *C* represent the three currents and the curve *R* their resultant. The flux in the transformer being strictly proportional to *R* (by which it is produced) an e.m.f. of treble frequency is therefore produced in the lamp circuit. The object of these frequency changers is to make low frequency power current available for lighting. The great simplicity and the absence of any moving parts is a point in their favour, but the power factor is necessarily low because the fundamental must be wattless. It is

only the third harmonic which carries power, the fundamental although present both in the current and the e.m.f. waves must not

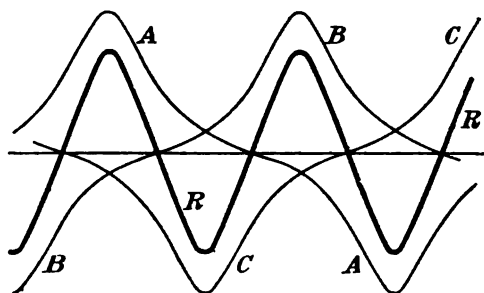


FIG. 127.

carry power, for if it did the lamps would flicker. The frequency changer must therefore be so designed as to obtain approximately quadrature between the fundamental components of current and e.m.f. and as these must exceed in magnitude any third harmonics

which could be filtered out by the effect of saturation in a choking coil, the power factor must necessarily be low. It is probably of the order of 0.2. This drawback becomes insignificant if the lighting load is only a small fraction of the power load and it is for such cases that the static frequency changer is intended.

Vector Diagram of a Transformer. In drawing a vector diagram of flux, current and e.m.f. in the windings of a transformer it is convenient to assume a transforming ratio of 1:1; which means that primary and secondary coils have the same number of turns. We shall also assume constant reluctance so that the same vector merely by a change of scale may represent either amperes, ampere-turns or megalines. This assumption is justified since the induction in transformers is never carried anywhere near saturation. We shall also assume for the present that the transformer has neither iron nor copper losses. This assumption is, of course, not justified, but it is convenient to draw the vector diagram first by neglecting these small disturbing influences and then to correct the result in the completed diagram. To assume the same winding in both circuits is no restriction to the general validity of the argument, since the law connecting resistance and inductance with the number of turns is the same. Let R , L and n be resistance, inductance and number of turns in the secondary and R_1 , L_1 and n_1 the corresponding values in the primary, then if we wish to reduce the primary winding to the same number of turns as the secondary, the resistance and

to account for this disturbing influence we make use of the conception of the translation coefficient as explained in Vol. I, p. 219. The correct value of M is $\eta \sqrt{L_1 L}$ where η is the translation coefficient. This means that to produce the secondary voltage the primary current will have to be larger, say, OD instead of OD_1 , which would be the value in a perfect transformer having no leakage. To make up for the translation coefficient falling short of unity we must increase the primary current correspondingly; we have therefore

$$OD = \frac{OD_1}{\eta}$$

To find the e.m.f. which must be impressed on the primary to produce the current OD we must first of all find the flux which passes through the primary. This is also a resultant of two excitations, namely that produced by the whole of the primary ampereturns and that produced by the ampereturns of the secondary, but reduced to the proportion corresponding to the translation coefficient. It has been proved in Vol. I that this is the same for both circuits. We therefore make in Fig. 128 $OC_1 = \eta OC$ and find the primary flux as the resultant of OD and OC_1 . This gives OE . Since the impressed e.m.f. must lead over the flux by 90 degrees we can now draw the vector of the impressed e.m.f. e_1 . For equal number of turns in both circuits the e.m.f.'s are proportional to the fluxes and we have

$$\frac{e_1}{e} = \frac{OE}{OA}$$

The power factor of the primary supply current is also found from the diagram since the phase angle is ψ . It should be mentioned that the diagram is out of scale; in order to make the construction clear I have assumed a much smaller translation coefficient than corresponds to a good design of transformer. The result is that the primary current is excessive and the power factor very low. In reality the primary current is only very little greater than the secondary and the primary power factor only very little smaller than that of the load.

The construction above explained enables us to find what primary e.m.f. must be impressed to obtain a certain terminal voltage, but it is not a solution (except by a trial and error method) of the practical problem to find the secondary terminal voltage if the impressed primary voltage is given. To find a solution by vector diagram we must obtain a graph showing primary current and

power factor as a function of the load on the secondary. The obtuse angle at A is $90 + \phi$, since by construction AL is parallel to CO and AO is at right angles to Oe . As long as ϕ remains the same, the angle at A must also remain the same whatever the value OC of the secondary current may be. With constant primary impressed e.m.f. OE must be a constant at all loads. Since in the triangles OD_1L and ODE

$$\frac{OL}{OE} = \frac{OD_1}{OD} = \eta$$

OL is also constant and the point A must lie on a circle having OL for a chord. To find the position of the centre a of this circle we make use of the well-known theorem that the central angle of a circular arc is twice the peripheral angle, or in symbols

$$180 + 2(\angle aLO) = 2(90 + \phi)$$

from which we find $\angle aLO = \phi$.

Prolong OE and draw DM parallel to OA . This gives triangle MDE which is similar to OAL . The obtuse angle at D is therefore also constant, namely $90 + \phi$, and point D that is the end of the vector representing primary current has its locus on the circle with centre b described over the chord EM . The power is $OD \cos \psi \times e_1$ and as e_1 is constant we find that the power is given by the height of D over the base line OM . If the impedance in the secondary load circuit is infinite, that is to say, if the secondary is open, the primary current taken by the transformer is the no load current and since by hypothesis there is no loss in the transformer the power input must be zero. The end of the primary current vector must therefore lie on the axis OM ; and as it also must lie on the circle we find OE as the vector of the no load primary current. Now imagine the impedance of the load to be infinitely small, that is the terminals of the secondary to be short-circuited. Again the power is zero and the working point is at M . We have therefore OE = no load primary current and OM = primary current when the secondary is short-circuited. The relation between these extreme values is easily found.

$$\begin{aligned} D_1L &= \eta DE, \quad DE = \eta OC, \quad D_1L = \eta^2 OC \\ AL &= D_1A - D_1L, \quad AL = OC - D_1L, \quad AL = OC(1 - \eta^2) \\ \frac{AL}{OL} &= \frac{AL}{\eta OE}, \quad \frac{AL}{OL} = \frac{DE}{EM}, \quad \frac{AL}{\eta OE} = \frac{\eta OC}{EM} \\ AL &= \eta^2 \frac{OC \times OE}{EM} = OC(1 - \eta^2) \quad \text{therefore} \quad \frac{EM}{OE} = \frac{\eta^2}{1 - \eta^2} \end{aligned}$$

This value is called the circle ratio; denoting it by the symbol θ we write

$$\theta = \frac{\eta^2}{1 - \eta^2} \quad \dots \quad (117)$$

$$OM = OE + EM, \quad OM = OE (\theta + 1), \quad \frac{OM}{OE} = \frac{1}{1 - \eta^2}$$

Simplified Vector Diagram. Fig. 128 has been drawn for a translation coefficient of about 0.77 so as to separate the lines sufficiently to make the construction clear.

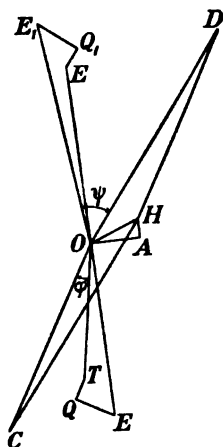


FIG. 129. Simplified vector diagram of a transformer.

In a modern transformer with an inductive drop of a few per cent. the translation coefficient is much nearer unity; it is of the order of 0.996 so that the circle ratio becomes very large, something between 200 and 300. This means that E lies very close to O and the radius of the circle through D is so large that near E the circle may be replaced by a straight line including with the horizontal the angle $90 - \phi$. The working point D then lies somewhere near E on this straight line. It is in this form that the vector diagram of a transformer is usually drawn. It is not absolutely correct, but sufficiently near to the actual condition for all practical purposes. The physical meaning of this alteration in representing the working condition of a transformer

is that we suppose the inductive drop not due to leakage between the coils, but to an inductance placed outside of the transformer. We replace the actual transformer which has internal magnetic leakage by an ideal or perfect transformer whose translation coefficient is unity and then worsen this ideal transformer by inserting between its windings and the terminals inductances, which affect the terminal voltage in about the same way as the internal leakage does in the practically possible transformer. A vector diagram developed on this basis is shown in Fig. 129. Here again we cannot draw the diagram to scale since the lines would fall too close together. We are therefore assuming the external inductances to be much greater than corresponds to the actual internal leakage fields. In drawing the diagram we take account of the iron loss and the resistance drop, assuming that with the transformation ratio 1:1 it is equal

in both circuits. Let in Fig. 129 OT and OC represent terminal voltage and current in the secondary. The induced e.m.f. must have a component TQ to balance resistance drop and another QE leading over the current by 90° to balance inductive drop. The induced e.m.f. must therefore be OE and to obtain this there must be a flux OA leading over OE by 90° . AH is the watt component of the magnetising current to cover iron losses. The flux produced by OA is also interlinked with the primary and the primary impressed e.m.f. must have a component OE equal and opposite to the induced e.m.f. For this reason we draw OE for the primary from O upwards. The primary current OD is found from the parallelogram $CODHC$.

The impressed e.m.f. must have a component EQ_1 in phase with the current and another Q_1E_1 leading by 90° . We thus get OE_1 as the vector of the primary e.m.f., OD as that of the primary current, OC as that of the secondary current and OT as that of the secondary terminal e.m.f. The phase angle of the load is ϕ , that of the supply is ψ . It will be noticed that OC and OD are nearly, but not quite, in line. If the diagram had been drawn to scale OH would be only a few per cent. of OC and then the two currents would be very nearly in exact opposition and of nearly equal magnitude. Also the impedance triangles TQE and EQ_1E_1 are in reality much smaller

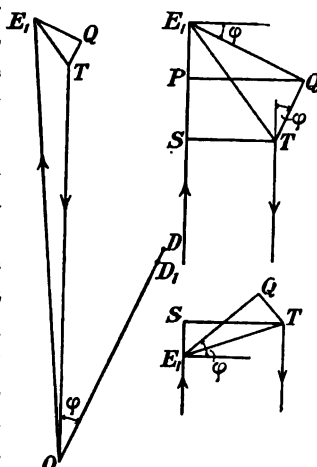


FIG. 130. Graphic determination of drop.

than shown. We can now draw the vector diagram in a simpler manner by turning the secondary OE through 180° and throwing the two impedance triangles into one. This gives Fig. 130. TO is the secondary terminal e.m.f. as indicated by the arrow, OE_1 is the primary terminal e.m.f., TQ is the total resistance drop and QE_1 the e.m.f. due to inductance in both circuits. OD is the primary and D_1O the secondary current. The drop between no load and load is the difference between the length of the vectors OT and OE_1 . These are more nearly parallel than shown in the figure because the hypotenuse TE_1 of the impedance triangle is only a few per cent. of the length of the e.m.f. vector OT . The top of this diagram is shown to a larger scale on the right. In this diagram the two

e.m.f. vectors are drawn parallel. QE_1 is the reactive and QT the resistance drop. Both are proportional to the current. Let $QE_1 = e_s$, as defined in the previous chapter, page 258, then we may put $e_s = \omega Li$. If by R we denote the equivalent resistance of both windings then $QT = Ri$. The total drop is $E_1S = E_1P + PS$. It will be seen from the diagram that the total drop at the given power factor $\cos \phi$ and current i is

$$\text{Drop} = i(\omega L \sin \phi + R \cos \phi) \quad . \quad . \quad . \quad (118)$$

We have defined the inductive drop as a percentage of the terminal e.m.f. In the same way we can define the total drop and write

$$\text{Percentage Drop} = \frac{100i}{e} (\omega L \sin \phi + R \cos \phi) \quad . \quad (119)$$

or Total drop = Inductive Drop $\times \sin \phi$ + Resistance Drop $\times \cos \phi$.

If the load is of such a character as to cause the current to lead ϕ becomes negative and if the reactance ωL is large as compared with the resistance R the drop may become negative. This is shown in the lower part of Fig. 130. In this case the secondary terminal voltage at load is larger than at no load; the voltage may rise as the load is increased. Since the power factor has a considerable influence on the drop it is necessary when specifying the regulation of a transformer to state at what load and power factor it shall be tested. The drop becomes a maximum when TE_1 is vertical and this condition obtains if

$$\cos \phi = \frac{R}{\sqrt{(\omega L)^2 + R^2}}$$

that is to say, if the power factor equals resistance of the transformer divided by its impedance.

Coupled Transformers. The conception that the practically possible transformer may be replaced by an ideal or perfect transformer and the addition of "equivalent coils" to represent leakage and losses is useful in determining the working condition of transformers coupled in parallel. In Fig. 131 T represents the ideal transformer and L, R are the equivalent coils. The transforming ratio is 1:1. Conductors at the same potential have the same lettering. Since there is no change of voltage the ideal transformer becomes unnecessary and we may connect the source directly with

the load, provided we leave the equivalent coils in circuit. This is represented by the middle diagram in Fig. 131. The vector diagram Fig. 130 is still applicable; OE_1 is the voltage of the source, OT that delivered to the load and TE_1 that measured over the terminals of the equivalent coils. If there are two transformers in parallel (last line of Fig. 131) these values apply to each transformer and this condition enables us to determine current and phase angle in each transformer as a function of the total current and power factor of the load. To do this it is necessary to first find what single reactance $X = \omega L$ and single resistance R is the resultant of the two reactances X_1 , X_2 , and the two resistances R_1 and R_2 .

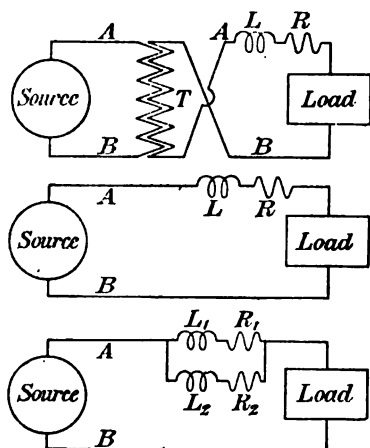


FIG. 131. Equivalent coils.

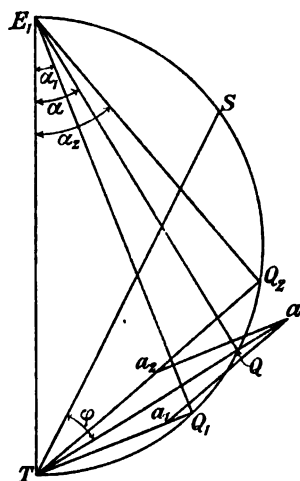


FIG. 132. Transformers in parallel.

This is most easily done graphically as shown in Fig. 132. Assume any convenient value for the impedance drop $TE_1 = Zi = Z_1i_1 = Z_2i_2$. The values of Z are $\sqrt{X^2 + R^2}$. Since reactance and resistance for each transformer are known, we can draw the vectors E_1Q_1 and E_1Q_2 under the corresponding angles α from the relation $\sin \alpha = R/Z$. We thus find $TQ_1 = R_1i_1$ and $TQ_2 = R_2i_2$ and can mark off to any convenient scale the values of the two currents. Let these be represented by Ta_1 and Ta_2 . Their vectorial sum is $Ta = i$. Let I be the prescribed load current then the corresponding components passing through the transformers are

$$I_1 = i_1 \frac{I}{i} \quad \text{and} \quad I_2 = i_2 \frac{I}{i}$$

The drop is found by drawing a line under the phase angle ϕ of the load relatively to the current vector Ta and measuring its length to the point S where it cuts the circle. The scale is that to which TE_1 has been originally marked off, but as this scale has been altered in the ratio i to I we have

$$\text{Drop} = TS \frac{I}{i}$$

The object of this investigation is to see whether in the parallel coupling of transformers there is a danger that one may be overloaded. The percentage of reactive and resistance drop in modern transformers is so small (1 to 5 per cent.) that neither of these can act as equalisers if there is any sensible difference between them or in the terminal voltage on open circuit. It is therefore important that transformers intended for parallel operation should not only have the same transforming ratio, but also the same percentage drop, both as regards resistance and reactance.

Predetermination of Terminal Voltage under Varying Load and Power Factor. The direct measurement of the drop under varying load conditions in the manufacturer's test room, so as to see whether the specification has been filled, is easy enough with

small transformers, but not very accurate because we must compare two voltages which differ only by a few per cent. An instrumental error of $\frac{1}{4}$ per cent. and an equal personal error in taking a reading may easily occur, and if the errors in the two readings happen to be additive we may have an error of 1 per cent. on a guarantee of some 3 to 5 per cent. No great accuracy can therefore be expected from the direct method and if the problem is to test a large transformer

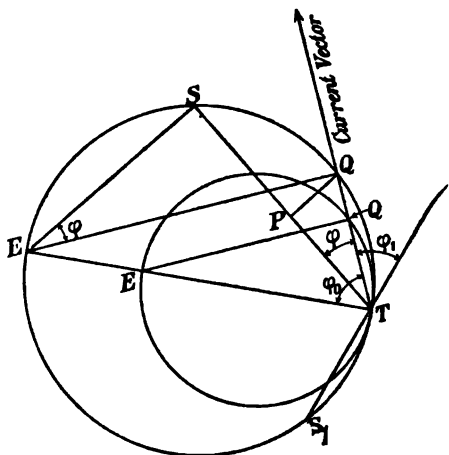


FIG. 133. Drop as affected by load and power factor.

it is further complicated by the difficulty and expense of providing a load of known power factor and the power to fully load up the

transformer. The indirect method is not only far more accurate, but quite easily applied since the power required for the test is only a very small fraction of the power for which the transformer is intended. To test the transforming ratio the test is made with the secondary open and therefore only sufficient power to cover iron losses need be available. Moreover the test may be applied to the winding of lower voltage thus reducing the difficulty of insulation in the testing circuit and increasing the accuracy with which the no load current can be measured. To test for "drop" we apply the small amount of power required to cover copper losses to the high voltage winding whilst the low voltage winding is short-circuited. The test current need then be no greater than the full load current in the high voltage winding, whilst the voltage required is quite small. The only instruments required for this test are a wattmeter, a voltmeter and an amperemeter in the supply circuit. The applied voltage is regulated so that the current has about normal full load value. Let P represent the power in watts, i the current and e the voltage applied to the transformer terminals. From the readings we can draw the impedance triangle TQE (Fig. 133) for the particular current used in this test. The phase angle at short circuit is φ_0 and may be determined from the relation

$$\cos \varphi_0 = \frac{P}{ei}$$

Since the impedance triangle varies in size with the current, but does not alter its shape, it is not necessary to make the test with the full current; all that is required is to use a current large enough to get a sufficient degree of accuracy in the three readings of i , e and P . The point Q lies on a circle whose diameter is proportional to the current and we may draw a number of such circles each representing a definite load. Since $QE = \omega Li$ and $TQ = Ri$ we have by reference to (118)

$$\text{Drop} = TQ \cos \varphi + QE \sin \varphi$$

which with reference to Fig. 133 may be also written

$$\text{Drop} = TE \cos (\varphi_c - \varphi) \quad . \quad . \quad . \quad . \quad (120)$$

The drop becomes a maximum for a lagging phase angle of $\varphi = \varphi_0$; it is zero for a leading phase angle $\varphi = - (90 - \varphi_0)$; and it becomes negative, that is to say, a rise of voltage TS_1 for a leading phase angle φ_1 greater than $90 - \varphi_0$. In all cases the drop is proportional

to the load. By using a suitable scale for the load marked off on the impedance line TE we can draw a number of circles each corresponding to a definite kva. load and then find the drop for any load and any power factor by measuring the length TS with the volt scale to which the diagram has been drawn from the readings of the short circuit test. The percentage drop at full current is found from

$$\text{Percentage Drop} = \frac{100e \cos (\varphi_0 - \varphi)}{E}$$

where e is the impedance voltage at full current and E the terminal voltage on open circuit.

Testing Large Transformers. The properties to be tested are transforming ratio, primary current on open circuit, drop of terminal voltage on load, heating and efficiency. The tests for the two first named properties are so obvious that no description is required. The test for drop has been dealt with in the foregoing section. We need therefore only consider the question how heating and efficiency may be tested in a convenient and inexpensive manner. It has already been pointed out that any direct method of testing is hardly practicable with a transformer of hundreds or thousands of kva. capacity since so large a power merely for testing is seldom available and the provision of a large load having a definite power factor involves considerable expense. These difficulties can be overcome by arranging transformer tests on some indirect method. The efficiency may be tested by determining the copper losses by wattmeter on short circuit and the iron losses also by wattmeter on open circuit. The sum of the two wattmeter readings is the total loss at load. The efficiency at load can thus be computed. To determine the heating time constant and final temperature rise if only one transformer is available is not so easy nor can it be done accurately. A method sometimes used is to heat up the copper and iron alternately by supplying just sufficient e.m.f. to the primary so as to produce full current in the short circuited secondary and then open the secondary and apply full e.m.f. to the primary. By changing over from one condition to the other every half hour or so the total energy wasted in say 24 hours is just half of what would be wasted if the transformer had been working under full load condition all the time. Since the heating time constant is independent of the load this test would give it correctly if the final temperature rise

were strictly proportional to the loss, but as this is not certain such a test can only be considered as a rough approximation. Greater accuracy may be obtained by raising the voltage both on the short circuit and the open circuit tests so that the wattmeter readings are doubled. Then the total energy lost in, say, 24 hours (or whatever time may be necessary to reach final temperature) will be approximately the same as if the transformer had been working at full load during that time.

When two transformers equal in all respects are available the test for heating and efficiency can be made more accurately and more simply by circulating power between them and measuring the volt-amperes of circulating power and the power wasted. This test is called Sumpner's test. In such a test the power required is only

that necessary to cover iron and copper losses with a small margin in regulating apparatus for accurate adjustment to the correct voltage and working current. No artificial load is required and the total power to be provided is very small. Thus to test a pair of 1000 kva. transformers having an efficiency of the order of 98 per cent. testing plant

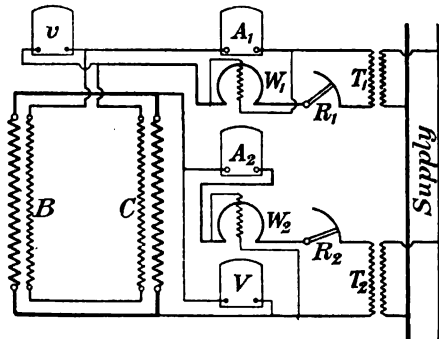


FIG. 134. Heating and efficiency test for large transformers

absorbing about 50 or 60 kw. would suffice. Fig. 134 shows the arrangement of this test. B and C are the two equal transformers. They are so connected that full voltage applied to one pair of circuits will produce full magnetisation, but no circulating current in the other pair of circuits. To produce a circulating current through all four circuits a current must be injected into one pair of circuits, preferably the high voltage circuits because these require the smaller current. The magnetising current should be supplied to the other pair because these coils require the smaller voltage. The circulating current is measured on the amperemeter A_1 and the power lost in copper is indicated by the wattmeter W_1 . The exact adjustment to a definite current loading may be made by the rheostat R_1 . The magnetising or no load current for both transformers is indicated on A_2 and the iron losses are given by the wattmeter W_2 . The terminal

e.m.f. on the low pressure coils is measured on the voltmeter V and a rheostat R_2 serves for accurate adjustment to any definite voltage. The circulating current as well as the magnetising current could be taken direct from a supply main if its voltage is sufficiently high, but as the testing plant must serve for different transformers whose voltage may greatly vary it is more convenient to insert small transformers T_1 and T_2 in order to be able to use the same supply circuit in all tests. To find no load current and iron losses open R_1 so that no circulating current can flow, adjust R_2 so that V shows the correct voltage E and measure the current and power. Let these be i and P_2 . The magnetising current of one transformer is then $i/2$ and the iron loss is $P_2/2$. The reactance of the low pressure winding is $\omega L = 2E/i$ and that of the high pressure winding is $\omega L_1 = n^2 2E/i$, n being the transforming ratio.

Now close R_1 and adjust for the full value of the high pressure current I_1 which will be shown on A_1 . The corresponding current in the low pressure winding is nI_1 , but this does not affect the reading on A_2 which remains constant. Let P_1 be the power indicated on W_1 then $P_1/2$ is the copper loss in one transformer. The output in va. is nEI_1 and the efficiency at unity power factor is

$$\eta = \frac{nEI_1}{nEI_1 + \frac{P_1 + P_2}{2}} \quad \dots \quad (121)$$

Incidentally this test also gives the impedance triangle and $\cos \phi_0$ of Fig. 133. Let e be the e.m.f. indicated on v when the current is I_1 , then the impedance of one transformer on short circuit is $Z = e/2I_1$ and the equivalent resistance is $R = P_1/2I_1^2$. The power factor on short circuit is

$$\cos \phi_0 = \frac{P_1}{eI_1} \quad \dots \quad (122)$$

The test here described can also be made in a slightly different manner by introducing both the magnetising and the circulating current into the same circuit, the other circuit being simply closed on itself and containing no instrument. To insure in this case perfect symmetry and equal iron loss in both transformers the e.m.f. producing magnetising current should be injected into a tapping midway between the terminals of the transformer producing the circulating current.

Two-phase to Three-phase Transformer. A transformer may be used not only to change the voltage, but also the number of phases. An example of this use has already been mentioned in connection with converters where a three phase primary supply is transformed to a six phase slip ring supply. By using a system of coupling two single phase transformers devised by Mr C. F. Scott it is also possible to convert from two to three phases or vice versa. It has been shown in Vol. I that there is an appreciable saving in conducting material by effecting transmission with three phases at high voltage. This is generally much higher than that for which motors or other consuming devices can be built, so that transformation is necessary in any case. If the motor is two phase then the transformation may be arranged as shown in Fig. 135. Two transformers are required, one having the primary winding BC adapted for the full line voltage and the other having the primary winding

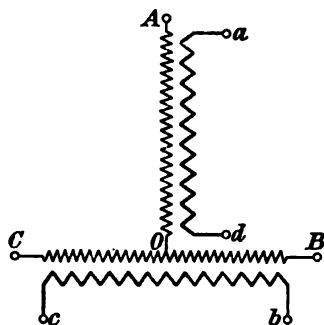


FIG. 135. Scott's three-phase to two-phase transformer.

OA adapted for $\frac{\sqrt{3}}{2}$ of the line voltage, the number of turns being reduced in the same proportion. It will be obvious that the P.D. between the points AB , BC and CA is then equal to the line voltage and as far as the supply is concerned this combination of two primary coils is equivalent to the three primaries of an ordinary three phase transformer. The secondary coils are da and bc and have the same number of turns. They generate equal e.m.f., but in quadrature and a two phase supply may be taken from the four terminals d , a and b , c . Conversely if the generator is a two phase machine the voltage may be stepped up and the power delivered to a three phase line by this system.

Advantage of Mesh Coupling for Three-Phase Work.

It has already been stated that by the use of three single transformers instead of a three phase transformer capital outlay may be reduced because the amount of spare plant required is reduced to one-third. This applies both to star and mesh coupling, but in the former the supply must be interrupted during the time that the

spare transformer is brought into service, whilst with mesh coupling no interruption need take place if one transformer becomes defective and its circuit breakers come out. This removes one link in the full mesh or "delta connection" and the two remaining transformers are then in "open mesh" or "V connection." This alteration does not affect the incoming and outgoing line currents, but the two V-connected transformers having now to do the work normally done by three transformers must obviously be overloaded. If working in full mesh the phase current in each transformer is $\frac{1}{\sqrt{3}} \times$ line current; if working in open mesh it is equal to the line current. Since the copper heat is proportional to the square of the current the copper loss has now been trebled. It has been shown that in a good design copper and iron losses should be about equal under normal conditions. The total loss in each transformer when working in open mesh will therefore be about doubled as compared with full mesh working. To work permanently in this condition at full load would also about double the temperature rise and is therefore inadmissible, but as this condition need only last for the very short time required to connect up the spare transformer and bring it into service there is no danger of an excessive temperature rise.

Where immunity from interruption of service is not of primary importance the open mesh is sometimes adopted as the normal working condition because of it requiring only two transformers instead of three. The question now arises whether this advantage of simplicity involves a reduction in plant efficiency, that is to say, an increase in weight and cost of material per kva. of transformed power. The transformers intended for open mesh must have the same terminal voltage as those intended for the full or closed mesh, but their current capacity must be $\sqrt{3}$ times that for closed mesh. Hence the output of each is 73 per cent. greater. But as only two are required the total rating of them as compared with the total rating of the three smaller transformers will be as

$$2\sqrt{3}/3 = 3.47/3 = 1.16$$

We require about 16 per cent. more transformer capacity for open mesh than for closed mesh working.

CHAPTER XIII

THE INDUCTION MOTOR

Principle of action—Relation between current, flux, e.m.f., speed and torque—The translation coefficient—Graphic theory of induction motor—How to draw the circle diagram—A simplified circle diagram—Starting resistance—Power factor—Influence of saturation—Graphic representation of efficiency—Relation between translation coefficient and leakage factor—Cascade working—Induction generators—The single phase motor—Some practical hints.

Principle of Action. Let in Fig. 136 an excited field system $NSNS$ be rotated in the direction of the arrow and let ab represent two conductors on the circumference of the rotor which form a closed coil with a span equal to the pole pitch. In the wire a there will be induced an e.m.f. downwards and in wire b an e.m.f. upwards producing a current as shown by the arrow. A downward current in a combined with the flux coming from the north pole produces

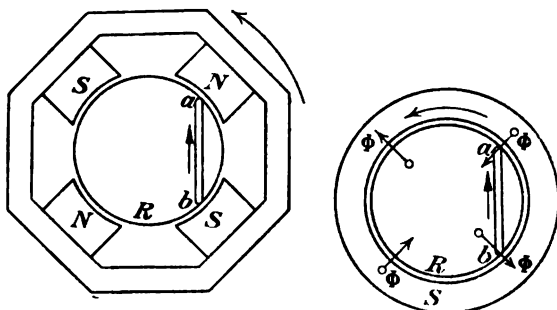


FIG. 136. Illustrating the principle of the induction motor.

a force acting towards the left, that is to say, in the direction in which the physical field system is rotated by power applied externally. In b the current is upwards and this combined with the flux that enters S produces a force also acting in the sense in which the field system rotates. The rotor then exerts a torque in the direction in which the field rotates. If instead of one coil only we suppose the rotor surface covered completely by active wires all formed into such coils as ab , the torque will be correspondingly greater and obviously the same for all relative positions between rotor and field. A machine

of this kind would act as a motor, but would have no practical value since it receives mechanical power on the primary side and gives out mechanical power on the secondary side. The kind of motor required is one which receives electrical power on the primary side and gives out mechanical power on the secondary side. The essential condition of the machine shown on the left of Fig. 136 is that there are groups of lines of force sweeping past the wires a, b . It is not essential that these fluxes shall emanate from salient physical poles and we may therefore replace the salient poles by a continuous iron ring provided with a multiphase winding which will produce the same fluxes. This arrangement is shown on the right. A stator S is provided on its inner periphery with a two or three phase winding so arranged that alternating current passing through its coils will produce groups of lines of force progressing along the circumference with a speed corresponding to the frequency. In the sketch we have a four pole field rotating counter-clockwise with the speed of $f_1/2$ revs. per second, where f_1 is the frequency. Generally, if the winding is such as to produce a field of $2p$ poles its speed of rotation is f_1/p revs. per second.

When the rotor is standing still the field sweeps past all its wires with the frequency of the primary supply current. If the rotor is running this frequency is diminished and it becomes zero if the rotor runs synchronously with the field. In that condition there is no relative movement between field and rotor wires and in consequence no e.m.f. is induced in those wires and no current flows. There is therefore no torque. But short of synchronism there is relative movement and therefore torque. All this applies equally to the imaginary machine on the left and the real motor on the right of Fig. 136, but there is an important difference. In the imaginary machine the excitation of the poles is supposed to be constant. Currents in the rotor wires will therefore reduce the strength of the field since the ampereturns of D.C. in the exciting circuit are constant. In the real motor there is no reduction in the value of Φ whether the rotor is producing ampereturns by armature reaction or not. The primary field Φ is produced by an exciting current derived from a supply of constant voltage. If the rotor produces back turns the tendency of these is in the first place to weaken the primary field, but as soon as the field is weakened its counter e.m.f. drops and allows more exciting current to flow, thus automatically compensating for armature reaction; in other words Φ is a constant and

apart from the influence of saturation simply proportional to the e.m.f. impressed on the stator winding. The e.m.f. induced in the rotor wires is proportional to the relative movement, that is to the difference between synchronous and actual speed. The current is directly proportional to the e.m.f. and inversely proportional to the impedance which again is dependent on the difference of the two speeds. For a given speed not very far from synchronism the current is therefore proportional to the flux, that is to the impressed primary e.m.f. and since the torque is proportional to the product of flux and current we find that the torque is proportional to the square of the impressed primary e.m.f. There is thus a radical difference between the principle of action of an induction motor and that of a D.C. motor or an A.C. synchronous motor. A drop in applied e.m.f. in a D.C. shunt motor does affect the speed but not materially the torque; in an A.C. synchronous motor a drop in applied e.m.f. does affect the power factor, but not the speed and not necessarily the torque. In an induction motor a drop in applied e.m.f. does not materially affect the speed, but does very materially reduce the torque. Hence an important rule in the application of induction motors is that they should not be worked at a voltage less than that for which they have been designed. There is only one exception to this rule for small motors lightly loaded and that is dealt with in the chapter on phase advancing.

Relation between Current, Flux, e.m.f., Speed and Torque. From the foregoing it will be seen that the essential parts of the motor are two, namely a stator having a two or three phase winding to produce the rotating field and a rotor having also a multi-phase winding. It is not essential that stator and rotor should be wound for the same number of phases, but they must be wound for the same number of poles. In Fig. 136 each loop such as *ab* may be considered a one phase winding and a rotor may thus have a large number of phases. This winding may be simplified by joining the ends of the bars on either side to two copper rings and then the number of poles becomes indefinite and adjusts itself to the number of poles in the stator. This kind of winding is called a "squirrel cage winding," and a rotor so wound may be used in a stator whatever its number of poles. As it is the simplest kind of winding we shall start the theory with the squirrel cage motor. A rigorous theory of even this simplest kind of motor leads to great mathe-

mathematical difficulties. Although no great error is introduced by assuming the wave form of the impressed e.m.f. to be a sine curve, the effect of saturation in the stator teeth and the fact that the coils are grouped in bands introduces a strong third and some other higher harmonics in the form of the field and this influences the wave form of the e.m.f. induced in the rotor bars. The current in the rotor bars must therefore also vary according to a more complicated law than a sine function of time and the torque being proportional to the product of two rather complicated functions of time cannot be quite constant or exactly of that magnitude which corresponds to an ideal case where all values vary according to a sine function of time. An attempt to take all these disturbing influences into account would lead to so complicated expressions that their physical meaning would be obscured to the detriment of their practical value, and for this reason designers are generally satisfied with a more or less approximate theory the results of which, although not strictly accurate, are sufficiently transparent to retain the connection between a formula and its physical meaning.

We shall therefore assume the following: The waves of impressed e.m.f. and primary field are sine curves. The current in the rotor bars of the squirrel cage follows a sine law. The bars are so numerous and placed so close together that the general effect of the rotor currents may be represented by a continuous sheet of current. The current density of this sheet expressed in effective amperes per cm. of circumference varies according to a sine function of space.

Let f_1 be the primary frequency, u_1 the synchronous speed and p the number of pairs of poles. Then $u_1 = f_1/p$. Let τ be the pole pitch and r the radius of the rotor, so that $\tau p = \pi r$. Let Φ be the flux actually passing through the rotor, Δ_0 the crest value of current density, ρ the resistance and λ the inductance of one rotor bar and end connection, $u_2 = f_2/p$ the speed of the rotor and $f = f_1 - f_2$ the frequency with which the primary field cuts the rotor bars. The crest value of the e.m.f. induced in one bar is then for a rotor l cm.

long $B_0 v l$ where $B_0 = \frac{\pi \Phi}{2 \tau l}$ and $v = 2\pi f \frac{r}{p}$.

If η is the translation coefficient as defined in Vol. I, p. 219, we have $\Phi = \Phi_1 \eta$ where Φ_1 is the flux interlinked with the primary winding. Its value in megalines is by formula (55)

$$\Phi_1 = \frac{100e_1}{k_1 f_1 z_1} \dots \dots \dots (123)$$

It has been shown in Chapter VII how the value of k may be found for different types of winding. It will therefore suffice to give here the results

True sine wave		$k = 2.22$
Three phase star	$S = \frac{1}{3}\tau$	$k = 2.12$
Three phase mesh	$S = \frac{2}{3}\tau$	$k = 1.84$
Two phase	$S = \frac{1}{2}\tau$	$k = 2.00$

If I_0 is the crest value of the current in one rotor bar and Z the total number of bars, then the crest value of the current density is

$$\Delta_0 = \frac{ZI_0}{2\pi r} = \frac{ZI_0}{2p\tau} \quad \dots \quad (124)$$

The crest value of current density coincides in time and space with crest value of current.

Crest value of e.m.f. in any particular rotor bar is induced when that bar passes the point of maximum induction B_0 , but since the bar has reactance, maximum current is only induced later when the bar has moved a certain distance from that point. This is shown in Fig. 137 where the curve B represents the space distribution of the induction over a pole pitch and the curve Δ the space distribution of current density. Both curves move through space with the same speed which corresponds to the primary frequency. The B curve cuts through the primary winding with this frequency f_1 , but through the secondary or rotor winding with the much lower frequency $f = f_1 - f_2$. Since the B and Δ curves move through space with the same velocity, they retain their relative position and the lag of Δ behind B is a constant. The lag of I_0 behind B_0 is due to the reactance $2\pi f \lambda$ of the rotor bar. To find the force acting on the $Z/2p$ bars within a pole pitch we make use of the fundamental equation: Force = induction \times current \times length of wire. An elementary strip of the sheet of current at A exerts the force

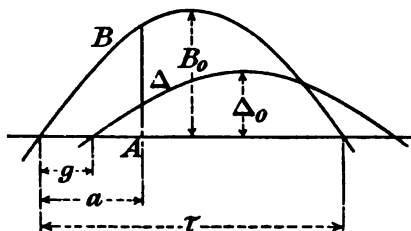


FIG. 137.

$B l \frac{\Delta}{10} da$ dynes

Integrating this between the limits $a = 0$ and $a = \tau$ we get the total force due to one pole; this multiplied by $2p$ and τ gives the torque.

Since in the figure

$$a = \frac{\tau}{\pi} \alpha \text{ and } g = \frac{\tau}{\pi} \psi$$

(where ψ is the phase angle) we have

$$dF = Bl \frac{\Delta}{10} \frac{\tau}{\pi} d\alpha; \quad B = B_0 \sin \alpha; \quad \Delta = \Delta_0 \sin (\alpha - \psi)$$

$$dF = B_0 l \frac{\Delta_0}{10} \frac{\tau}{\pi} \sin \alpha \sin (\alpha - \psi) d\alpha$$

The force due to one pole is in dynes

$$B_0 l \frac{\Delta_0}{10} \frac{\tau}{\pi} \int_0^\pi (\sin^2 \alpha \cos \psi - \sin \alpha \cos \alpha \sin \psi) d\alpha$$

The integral of the second term in the bracket is zero, that of the first term is $\frac{\pi}{2} \cos \psi$. Hence the total force due to all the $2p$ poles is in dynes

$$F = 2p B_0 l \frac{\Delta_0}{10} \frac{\tau}{2} \cos \psi$$

Since $B_0 l \tau = \frac{\pi}{2} \Phi$ and $\Delta_0 = \frac{ZI_0}{2\pi r}$ we have also

$$F = \frac{1}{40} \frac{p\Phi ZI_0}{r} \cos \psi$$

and the torque in dyne cm. is

$$\frac{1}{40} p\Phi ZI_0 \cos \psi$$

To get the torque in kgm. we divide by 98.1×10^6 and if we introduce the effective value i of the current instead of the crest value I_0 and express the flux in megalines we get for the torque in kgm.

$$T = 3.68 p\Phi \left(\frac{Z}{100} \right) \left(\frac{i}{100} \right) \cos \psi$$

The coefficient 3.68 in this formula is only valid for the special case here treated, namely a squirrel cage rotor. For a rotor wound in any other way the coefficient will have a different value, so that to comprise all cases we can write

$$T = pt\Phi \left(\frac{Z}{100} \right) \left(\frac{i}{100} \right) \cos \psi \quad \dots \quad (125)$$

where t is a torque coefficient depending on the particular way the rotor wires are grouped.

To find t for any particular winding we could again make use of the fundamental equation, force = Bli , and apply it to each phase separately and in different positions; then taking the mean of these results, obtain the average total torque. This process is rather laborious and may be avoided by the following reasoning. The torque is due to the attraction between two fluxes, namely that emanating from the stator and that which the rotor currents produce. It has been shown in Chapter VIII how the self-induced armature flux can be calculated for any type of winding. If then we calculate the self-induced flux with the particular winding and compare it with the flux, which the same current would produce in a squirrel cage winding, where the current density along the circumference follows strictly a sine law, the ratio of the two fluxes will give us a factor by which the coefficient $t = 3.68$ must be reduced. Thus for a D.C. armature tapped so as to form a mesh winding with $S = 2\tau/3$ (lower line in Fig. 93) the area is $qI\tau$; and if we denote by I_0 the crest value of the current in one wire of the equivalent squirrel cage, the highest ordinate of the sine curve is

$$X_0 = \frac{2}{\pi} 3qI_0$$

Equality of areas means $\tau x = \frac{2}{\pi} \tau X_0$, or $qI = \left(\frac{2}{\pi}\right)^2 3qI_0$, from which we find $I = 1.22 I_0$. The wound rotor requires 22 per cent. more current than a squirrel cage having the same number of active wires; or with the same current the wound rotor will give only $1/1.22 = 82$ per cent. of the torque. The torque coefficient of the squirrel cage rotor we found to be 3.68, that for the wound rotor is therefore $t = 3.03$, or say in a round figure 3. In the same way all other windings can be reduced to an equivalent squirrel cage. The result is in round figures

Values of t in (125)

Squirrel cage	3.7
Three phase, narrow coil sides $S = \frac{1}{3}\tau$	3.5
Two phase $S = \frac{1}{2}\tau$	3.3
Three phase, wide coil sides $S = \frac{2}{3}\tau$	3.0

In the formula for T there appears the factor $\cos \psi$ and this depends on resistance and inductance in the rotor winding and the frequency of the rotor currents. This frequency is due to the differ-

ence between the speed of the primary field and the rotor speed and is technically called the "slip." Mathematically the slip is expressed as a ratio thus

$$\sigma = \frac{f_1 - f_2}{f_1} = \frac{f}{f_1} = \frac{\omega}{\omega_1}$$

The e.m.f. induced in the z active wires of a phase and Φ_1 megallines is

$$e = \frac{k}{2\pi} \omega (\eta\Phi_1) z 10^{-2} \quad \dots \quad (126)$$

where $\eta\Phi_1 = \Phi$ and $\omega/2\pi = f$. The current is

$$i = \frac{e}{\sqrt{R^2 + (\omega L)^2}}$$

This inserted into (125) gives for the torque in kgm.

$$T = ptk (\eta\Phi_1)^2 Z \cos \psi \frac{\omega z}{2\pi \sqrt{R^2 + (\omega L)^2}} 10^{-6}$$

and since $\cos \psi = R/\sqrt{R^2 + (\omega L)^2}$ we have

$$T = ptk (\eta\Phi_1)^2 Z \frac{\omega z R}{2\pi (R^2 + \omega^2 L^2)} 10^{-6} \quad \dots \quad (127)$$

In this formula z is the number of active wires per phase and Z is the total number of active wires counted all round the rotor. This formula makes it possible to compare the torque obtainable with the same size rotor and the same total number of active bars, but grouped differently, namely as squirrel cage, two phase winding and three phase winding both with narrow and wide coils. In order to make the different types directly comparable we assume that the resistance and inductance vary as the number of active wires per phase. As regards the resistance this is strictly true; as regards inductance this is not strictly true because with the bunching of wires the inductance increases rather more than proportionately with the number of wires in a coil side. By assuming proportionality we therefore underestimate the inductance and overestimate the current, but the error is not serious because the frequency in the rotor circuit even at full load is so small that the reactance is almost negligible as compared with the resistance. Let then ρ be the resistance and λ the inductance of a single wire; then $R = z\rho$ and $L = z\lambda$. These values inserted in (127) give

$$T = ptk (\eta\Phi_1)^2 Z \frac{\omega \rho}{2\pi (\rho^2 + \omega^2 \lambda^2)} 10^{-6}$$

Since the different rotors have the same total number and size of wires and contain therefore the same amount of copper and since number of poles and flux are also the same in all, this formula enables us to judge the plant efficiency of the different types of rotor; that is to say, the output which is obtained with a given amount of material. For the same speed, output and torque are proportional; and since in the formula for T the only variables are t and k their product may be considered as a rough indication of the plant efficiency of the different designs.

This is shown in the following table; the last column representing the output referred to a basis of 100 for the squirrel cage rotor.

Rotor winding	t	k	tk	per cent.
Squirrel cage	3.7	2.2	8.1	100
Three phase, narrow coil sides	3.5	2.12	7.4	91
Two phase	3.3	2	6.6	82
Three phase, wide coil sides...	3	1.84	5.5	68

The squirrel cage is the simplest type and has also the advantage of requiring less copper than any wound rotor for the same output, but in motors which have to start on heavy loads, the squirrel cage is not suitable*. Next to the squirrel cage comes in point of merit the three phase rotor with coil sides occupying one-third of the pole pitch. Its output for equal copper weight and efficiency is only nine per cent. less than that of the squirrel cage and as no change over switch is required on the stator this type of motor is consequently the most economical in construction for medium and large sizes. The two phase rotor comes next in plant efficiency, but the flux curve due to rotor current departs more widely from the sine shape and this gives rise to rather strong upper harmonics, which cause some extra loss of power and some slight increase in the temperature rise. The smoothest flux curve is given by the rotor with wide coil sides, but then the plant efficiency is about 25 per cent. less than that of the motor with narrow coil sides.

Formula (127) shows that the torque is proportional to the square of the primary voltage. If this is constant the torque is a function of the speed. At stand-still where $\omega = \omega_1$, the denominator becomes large and the torque is small. At synchronous speed when $\omega = 0$ the torque is zero and it is a maximum for a particular value

* With a stator designed for running in mesh coupling the excessive starting current may be reduced in the ratio 3 to 1 by starting in star coupling, a special switch being provided for this purpose.

of ω which is found by differentiating $\omega R / (R^2 + \omega^2 L^2)$ in respect to ω and equating to zero. This gives $\omega = \frac{R}{L}$ and

$$T_{\max.} = ptk (\eta\Phi_1)^2 \frac{Zz}{4\pi L} 10^{-6} \quad . \quad . \quad . \quad (128)$$

Under normal conditions the speed of the motor is so near synchronism that ω is only a few per cent. of ω_1 and the second term in the bracket in the denominator of (127) may be neglected. We thus get for the torque in working condition the expression

$$T = ptk (\eta\Phi_1)^2 Z \frac{\omega z}{2\pi R} 10^{-6} \quad . \quad . \quad . \quad (129)$$

which means that the torque is directly proportional to the slip. The graph representing the relation between speed or slip and torque is of the general character shown in Fig. 138. The speed is counted

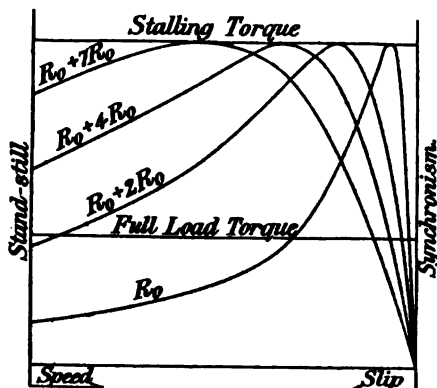


FIG. 138. Speed-torque curves of induction motor.

from the line on the left marked "stand-still" and the slip from the line on the right marked "synchronism." The inductance L in any given motor is a constant, but the resistance can be varied by inserting additional resistance into the phases. We thus get a series of speed-torque curves, each corresponding to a definite value of additional resistance. The phase windings are brought out to slip

rings which if short-circuited give the minimum resistance and this is the working condition of the motor represented by the curve marked R_0 . The maximum torque is a constant for all values of R not exceeding ωL . The different curves have then the same maxima. At the start when ω equals ω_1 formula (127) shows that the torque can only be small unless R is increased by inserting a starting resistance. Such a resistance is necessary if the motor is required to start under load. It may also be used to run at a reduced speed though this system of speed regulation is wasteful. During the start the working point is on the left branch of the speed-torque

curve and the condition is unstable because to an increase of speed corresponds an increase of torque. The motor therefore quickly runs up to the point of maximum torque and passes it, finally settling down on the right branch of the curve at a point corresponding to the load. A change of load has only an inappreciable influence on the speed since the slip is of the order of from 1 to 5 per cent. for full load according to the size of the motor. The maximum speed variation remains therefore within these limits so that practically the induction motor must be considered as a constant speed motor in the same sense as a D.C. shunt motor is a constant speed motor.

Fig. 138 shows that for all resistances not exceeding ωL the maximum torque is the same whatever may be the resistance added in the rotor circuit. The value of this maximum torque as given by (128) therefore applies also to the case when no resistance is added, that is to say, to the normal working of the motor. The flux is proportional to the ratio of primary voltage and frequency and may be considered a constant of the motor on which its rating is based. We thus find that the maximum torque a given motor can exert is also a constant of the design, that is a characteristic feature, and may be specified as regards its relation to normal full load torque. It is technically termed the "pull out" or "stalling" torque. The Rules of the Engineering Standards Committee require that a motor shall be capable of withstanding on test, without stalling, a torque 75 per cent. in excess of the normal. Since some margin on account of e.m.f. variation (which affects the torque with the square of its value) must be provided, it is advisable to design the motor for about double stalling torque. But in order to be able to predetermine the stalling torque from (128) we must know the flux interlinking with the rotor, that is to say, we must know the primary flux and the translation coefficient.

To get the primary flux we use the formula

$$e_1 = k_1 f_1 \Phi_1 z_1 10^{-2} \quad . \quad . \quad . \quad (126a)$$

where k_1 is the e.m.f. coefficient depending on the stator winding, Φ_1 is the flux in the stator in megalines and e_1 is the impressed voltage. Apart from the almost insignificant influence of resistance and other losses Φ_1 is a constant whatever the working condition may be. If the rotor circuit is open a certain magnetising current is required to produce this primary flux. To find the magnetising current we

again use the conception of an equivalent squirrel cage winding, but this time applied to the stator. In a star wound stator (upper line in Fig. 93) we have $qI1.16 = \frac{2}{\pi} X_0$ where X_0 is crest value of ampereturns in the equivalent winding. Let δ be air gap augmented by an appropriate amount to include reluctance of iron parts. Then the crest value of gap induction is $B_0 = 1.25X_0/2\delta$ and the flux is for a stator l cm. long $\Phi_1 = \frac{2}{\pi} B_0 \tau l$. Inserting B_0 we get $\Phi_1 = \frac{2}{\pi} \tau l 1.25X_0/2\delta$ and inserting $X_0 = \frac{\pi}{2} qI1.16$ we get

$$\Phi_1 = 1.25 \times 1.16 \times qI\tau l/2\delta$$

If instead of crest value we insert effective value of the magnetising current and call it i_μ the formula becomes

$$\Phi_1 = 2.02qi_\mu\tau l/2\delta$$

Let Z_1 be the total number of active stator turns, then for a machine of $2p$ poles we have $q = \frac{Z_1}{6p}$ and the primary flux Φ_1 produced by the magnetising current i_1 is expressed by

$$\Phi_1 = 0.34 \frac{Z_1 i_\mu \tau l}{p 2\delta}$$

The coefficient 0.34 is only valid for the particular case here exemplified. To cover all cases we may write

$$\Phi_1 = y \frac{Z_1 i_\mu \tau l}{p 2\delta} 10^{-8} \text{ megalines} \quad \dots \quad (130)$$

where y is a coefficient depending on the type of winding. For open or semi-open slots τ must be inserted not with its full value, but with a reduced value appropriate to the reduction of area by the slot opening.

Type of winding	Values of y
Three Phase $S = \frac{1}{3}\tau$	0.34
Two Phase $S = \frac{1}{2}\tau$	0.32
Three Phase $S = \frac{2}{3}\tau$	0.29

By using (130) we can find the magnetising component of the current for any given value Φ_1 of the stator field and this is very nearly the no load current when the rotor winding is open. The value of Φ_1 corresponding to the primary voltage e_1 is found from (126a) and thus the ratio between impressed e.m.f. and magnetising

current i_μ is found. Incidentally we also obtain the inductance L_1 in henrys of the stator by itself (the rotor circuits being open), since

$$e_1 = 2\pi f_1 L_1 i_\mu$$

Having found from (126a) for the given e.m.f. the flux Φ_1 and from (130) the current which must pass to produce this flux, we next calculate the leakage flux which the corresponding ampereturns in stator and rotor produce across the slots, top of teeth and head connections. The leakage factor is

$$\lambda = \frac{\text{leakage flux}}{\text{total flux}}$$

Having thus found from the drawing and winding data of the motor the leakage factor λ the translation coefficient (as will be shown later) is given by

$$\eta = \sqrt{1 - \lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (131)$$

The Translation Coefficient. If not merely a drawing, but the finished motor is available, the translation coefficient can be found by a very simple test. In this test no knowledge of the constructive data of the motor is necessary; all that is required is access to the primary and secondary terminals. The only instruments required are two voltmeters, one across the primary and the other across the secondary terminals. Current is supplied to the primary, the secondary being open. The indications of the voltmeters are noted. The source of current is then disconnected from the primary and connected to the secondary. The e.m.f. of the supply should be so adjusted that in the first test it has approximately the normal value for which the primary is wound and in the second test approximately the value indicated by the voltmeter connected to the secondary during the first test. Let e_1 and e_2 be the voltages of supply in the two tests and e_2' the voltage of the secondary terminals in the first test and e_1' the voltage of the primary terminals in the second test, then with a translation coefficient η we have the following relations

$$\begin{aligned} e_1 &= k_1 f \Phi_1 z_1 10^{-2} & e_2 &= k_2 f \Phi_2 z_2 10^{-2} \\ e_2' &= k_2 f \eta \Phi_1 z_2 10^{-2} & e_1' &= k_1 f \eta \Phi_2 z_1 10^{-2} \end{aligned}$$

$k_1, k_2, z_1, z_2, \Phi_1$ and Φ_2 are all unknown, but we can eliminate these by forming $(e_1' e_2') / (e_1 e_2)$. This gives

$$\eta = \sqrt{\frac{e_1' e_2'}{e_1 e_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

Graphic Theory of Induction Motor. It is convenient to supplement the above analytical treatment of the induction motor by a graphic theory so as to make the relations between the essential features of the construction and the working conditions more transparent. The relation between load, current, slip, stalling torque, power factor, etc., can be represented in a simple way by a circle diagram*. Such a diagram has already been given in Fig. 128 in the chapter on graphic treatment of transformer problems, but there we assumed that the load has a power factor less than unity. The induction motor is a transformer, but one in which only a very small fraction of the power in the secondary is given off electrically, namely that part which is required to compensate for the ohmic loss in the rotor winding. By far the largest part of the power in the rotor is given out mechanically, namely as the product of speed and torque, and since this is not apparent, but true power, the current in the rotor must very nearly be in phase with the voltage induced in the rotor. It would be absolutely in phase if the reactance of the rotor were zero. Strictly speaking this is not the case, for the rotor must have some inductance, but since the frequency of the

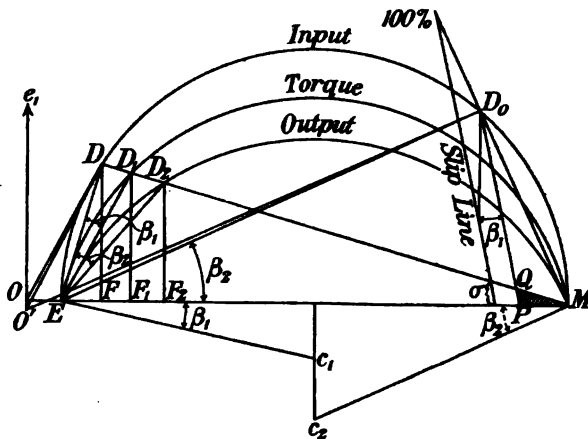


FIG. 139. Circle diagram of induction motor.

rotor currents is extremely small (only a few per cent. of the primary frequency) the reactance is practically negligible so that the angle ϕ in Fig. 128 may be taken as zero and the arcs over OL and EM

* First pointed out by Blondel and then further developed by Heyland, Behrend and others.

become semicircles. The diagram then assumes the form shown in Fig. 139. For simplicity the small semicircle OAL is omitted and the diagram is drawn more nearly to scale than Fig. 128. We shall at first assume equal winding in stator and rotor and no iron or copper losses. Then OD represents the primary current and ED represents $\eta \times$ secondary current as shown in Chapter XII. The height of D over the horizontal, that is the length DF , represents the power input and, since there are no losses, also the mechanical power given out by the motor. At no load the power is zero and D goes to E . At short circuit the power is also zero and D goes to M . OM represents therefore the primary short circuit current of an ideal motor and EM is the ideal short circuit current in the secondary multiplied by η . Since this current is proportional to the secondary e.m.f. EM may also be considered to represent the e.m.f. induced in the secondary and EDM as the impedance triangle pertaining to the secondary. At load ED measured with the volt scale is the wattless and DM the watt-component of the e.m.f. induced in the secondary. The primary impressed e.m.f. measured with the same scale is OM . Using the ampere-scale we find the no load or magnetising current in the primary OE , the total primary current at load OD and its wattless component OF . The secondary current is DE/η . All this refers to the ideal motor having no losses. To take the losses into account we reason as follows. Iron loss has to be compensated by an increase of primary current. Since this increase must represent true power the corresponding current vector must be in line with e_1 ; that is to say, the origin of the current diagram must be shifted downwards from O to, say, O' , and the primary current vector must be drawn from that point. The distance OO' is easily found if we know the ratio of iron loss to total input. If this is, say, two per cent. then OO' must be two per cent. of DF . In representing the copper losses diagrammatically we make the assumption that primary and secondary current are of the same magnitude and have the same phase position. This is not strictly true but sufficiently near the truth for practical purposes. The e.m.f. lost in copper resistance representing true power must therefore have in the volt diagram the same phase position as the power component of the total e.m.f. EM . The power component of the e.m.f. in the ideal motor is DM . If we deduct from this the e.m.f. lost in primary copper resistance we get the point D_1 and deducting again the e.m.f. lost in secondary copper we get the point D_2 .

Since ED is proportional to current and DM is the watt-component of a voltage, twice the area of the triangle EDM is proportional to power input. Expressing twice the area as the product of the constant base EM and the height DF we find that the height of D over the horizontal is, as already mentioned, a measure for the primary power input. From this power the loss in copper has to be deducted to arrive at the power given out by the stator and received by the rotor. The power input to the rotor is therefore proportional to D_1F_1 . This vector has also another significance. It was shown in (125) that the torque is proportional to the product of flux and current. D_1M is the e.m.f. impressed on the rotor and as the frequency is low the flux in the rotor must be proportional to D_1M and in phase with it. The product of ED and D_1M that is twice the area of triangle ED_1M represents therefore the product of current and flux, that is to say, the height D_1F_1 is a measure for the torque.

Of the e.m.f. D_1M received by the rotor the amount D_1D_2 is lost in copper resistance, leaving D_2M as the e.m.f. which multiplied with the current represents mechanical power transmitted to the shaft. From this has to be deducted the small amount of power lost in mechanical friction in order to arrive at the net available power given out by the motor.

Since in the volt scale ED , DD_1 and D_1D_2 are all proportional to current the triangles EDD_1 and EDD_2 retain their shape, though they alter in size with the current. The obtuse angles at D_1 and D_2 are therefore constant and the points D_1 and D_2 have their loci also on circles struck from the centres c_1 and c_2 respectively. We thus get three circles representing the power supplied electrically, the torque and the power given out mechanically. These circles are marked in Fig. 139 "Input," "Torque" and "Output." To get the values for torque and output at any working point D we draw the line DM and see where the respective circles are cut by this line. In the ideal motor, having no losses, the primary short circuit current measured with the ampere scale is given by OM . In the real motor the input at short circuit cannot be zero because power is required to compensate for losses. The output is however zero since the motor is at stand-still. The line DM must therefore cut the output circle at a point whose height is zero, that is to say, at M . This means that DM must be a tangent to the output circle at M and if we draw a line at right angles to c_2M and find its point

of intersection with the input circle D_0 the height of this point over the horizontal gives the copper losses and this height augmented by OO' represents the total losses at short circuit and stand-still. The primary short circuit current measured with the ampere scale is $O'D_0$ and the short circuit current in the secondary is ED_0/η .

How to Draw the Circle Diagram. In the foregoing section the significance of the different parts of this diagram has been explained in a general way, but before we can make practical use of such a diagram we must be able to draw it to a definite scale so as to fit a given motor or a given design of a motor; in other words we must determine the various scales for current, e.m.f., power and torque. If a finished motor is available the best way is to proceed by a practical test. It has been shown how the translation coefficient can be found. If we supply normal voltage to the primary with the secondary open we find the no load current. Its principal component is the magnetising current OE and since the other component required for iron losses is small the difference between magnetising and no load current is not great. By using a wattmeter it is, however, possible to determine the watt-component of the no load current and thus get OE accurately. In this connection it should be noted that the iron losses measured at stand-still with the rotor circuits open are greater than the iron losses, when the motor is at work, because the iron losses in the rotor are included. When the motor is working the frequency with which the flux sweeps through the rotor iron is so small that there are practically no iron losses in the rotor, but only those in the stator and these are covered by the additional primary current OO' in Fig. 139. The magnetising current having been found by a simple current and power measurement we know the value of OE and can plot this vector to any convenient ampere scale.

From (117) the circle ratio $\theta = \frac{\eta^2}{1 - \eta^2}$ can be calculated and the point M can be found from $EM = OE \times \theta$. We thus find OM and as this length represents the primary e.m.f. the volt scale is thereby determined and the input circle can be drawn. As a check we may make another test, namely the determination of the ratio between primary e.m.f. and short circuit current, the rotor being mechanically prevented from revolving. By measuring current, e.m.f., power and power factor we find the vector $O'D_0$. The point

D_0 should lie on the circle over EM . This check test should be made with a reduced primary voltage so that the short circuit current may not greatly exceed the normal working current. This for two reasons, first because a short-circuit test with full voltage may damage the motor by overheating and secondly an excessive current in both windings will saturate the iron and thus lower the inductance beyond the value corresponding to the normal working condition. The vector $O'D_0$ found by this test with a reduced e.m.f. must then be enlarged in proportion to the full e.m.f. before being inserted into the circle diagram.

The circle diagram may also be used to determine by watt, volt and amperemeter the inductance of the motor as a whole. When the secondary is open the inductance is high. It is $L_1 = e_1/\omega_1 i_\mu$, where $i_\mu = OE$. With the secondary closed the inductance is much lower, namely $L = e_1/\omega_1 i_{01}$, where $i_{01} = OM$. The inductance with the rotor open circuited is therefore OM/OE times as great as with the rotor open circuited. This means $L = (1 - \eta^2) L_1$. The larger the translation coefficient the smaller is the inductance of the motor taken as a whole and the better is its power factor. To pre-determine the circle diagram from the design of a motor we proceed as follows. Calculate, for a reduced voltage and primary current small enough not to produce saturation, the primary leakage flux and, for equal ampereturns in the secondary, the secondary leakage flux. Let F be the sum of the two fluxes, e the corresponding primary voltage and i the current for which F has been calculated. If instead of the reduced voltage e we apply the full primary voltage e_1 the short circuit current would (saturation neglected) come out at a much higher value. Let this be i_0 . Since voltage and current are proportional we have

$$i_0 = i \frac{e_1}{e} = i \frac{\Phi}{F}$$

F is a small fraction of Φ , namely that which corresponds to the leakage factor λ . By definition we have $\lambda = F/\Phi$ so that we can write $i_0 = i/\lambda$ and if we select for i the value of the magnetising current i_μ , then i_0 is the ideal short circuit current i_{01} and we obtain $i_{01} = i_\mu/\lambda$ or $i_{01} = i_\mu \frac{\Phi}{F}$. In the circle diagram we have

$$OM = OE \frac{\Phi}{F}$$

Another method of graphically representing slip devised by the author is shown in Fig. 140. Draw the line MP under angle β_1 and prolong backwards until it meets at S the line D_0E , also prolonged backwards. Drop from E a perpendicular and mark point P . Then divide the distance PS into 100 parts and call point S 100 per cent. slip, then the slip corresponding to any working point such as D is found by prolonging the current vector ED backwards to Q and scaling off PQ on the percentage line PS . The proof that PQ represents slip follows from the similarity of the shaded triangle EPQ and the little blackened triangle at M . Corresponding sides of these two triangles are at right angles to each other; hence PQ in Fig. 139 is proportional to PQ in Fig. 140; that is to say PQ in both figures represents slip. To save space we invert the slip line and shift it into the dotted position.

A Simplified Circle Diagram. The above method of making a graph to represent the working condition of an induction motor involves the drawing of three circles and these with a motor having only a few per cent. of copper losses come much nearer together than shown in Fig. 139. To simplify the diagram a method may be used whereby only one circle need be drawn as

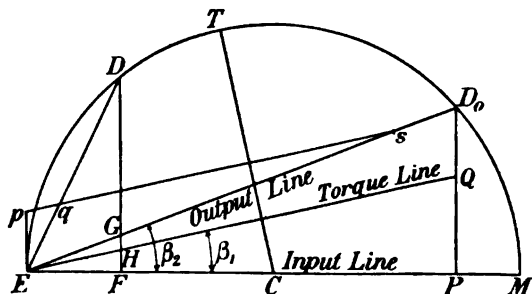


FIG. 141. Simplified circle diagram.

shown in Fig. 141. Also in this diagram to make the construction more easily understood the copper losses are assumed to be much greater than is generally the case. By a well-known theorem of the geometry of the circle we have $(ED)^2 = EF \times EM$ and as EM is constant and ED represents current we find that EF is proportional to the square of a current, that is to say EF is a measure of the copper losses in stator and rotor, whilst DF is a measure for

the power input. At short circuit, loss equals input, so that we may write

$$EP = k i_0^2 (R_1 + R_2)$$

$$D_0 P = c i_0^2 (R_1 + R_2)$$

where c and k are constants. From the similarity of the triangles EGF and ED_0P it follows that

$$GF = D_0 P \times \frac{EF}{EP}$$

which means that GF is the power lost in copper heat in stator and rotor if measured with the same scale as that which gives the power input as the length DF . The available mechanical power is therefore DG . As D travels on the circle G travels on the straight line ED_0 and the length of the ordinate between the circle and this line is the output. ED_0 is the "output line." By drawing EQ under the angle β_1 to the horizontal we divide the length PD_0 into two parts; PQ is proportional to the resistance of the primary and QD_0 is proportional to the resistance of the secondary. The current in both circuits being assumed equal (as in the ordinary circle diagram) the total loss is divided in the same proportion so that with the current ED , FH represents the loss in the primary and GH the loss in the secondary. The rotor therefore gets the power input DH and since power input is torque \times synchronous speed DH is a measure of the torque. EQ is therefore the "torque line." Maximum torque is obtained if the length of the ordinate between the circle and the torque line is a maximum, that is to say, if the working point is at T . This point is obtained by drawing a line from C , the centre of the circle, at right angles to the torque line. The slip may be represented in either of the two ways already explained. Actual torque and power are very slightly less than corresponds to the ordinates DH and DG respectively since the one or two per cent. for friction and windage has to be deducted. The actual input is slightly more than corresponds to DF since the watt component of the primary current (OO' in Fig. 139) has to be added. This is also of the order of a few per cent. of the input. To get the efficiency we scale DG and DF and deduct from their ratio the few per cent. lost in stator iron and friction and windage. No allowance for hysteresis need be made for the rotor since the frequency with which its magnetisation changes is very low. The slip may be represented by the method shown in Fig. 140. It is pq/ps .

Starting Resistance. The circle diagram may be used to find the amount of starting resistance required for keeping the primary current within some predetermined limit. In Fig. 142 D is the working point at full load and D_m lying a little higher is the

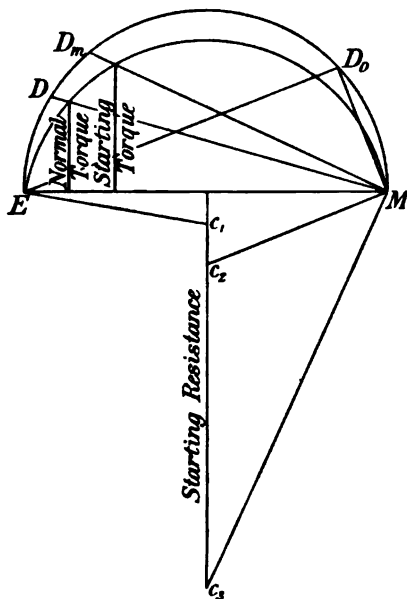


FIG. 142.

working point at the moment of starting. If the corresponding starting current should not exceed the value corresponding to the position of D_m sufficient resistance must be inserted into the rotor to bring D_0 to D_m . The natural resistance of the rotor is proportional to c_1c_3 and c_2M is at right angles to MD_0 . The additional starting resistance c_2c_3 is found by drawing Mc_3 at right angles to MD_m . The resistance c_2c_3 is that required in every rotor phase. To insert a starting resistance into the stator would be wrong and the diagram Fig. 139 shows this clearly. To increase stator resistance means an increase of DD_1 and a

lowering of the torque circle without affecting the starting current $O'D_0$. If the resistance is very large it might reduce the starting current somewhat, but then there would be hardly any torque left to make the start so that under all circumstances an increase of stator resistance is harmful.

Power Factor. The power factor is the cosine of the angle included between the vector of impressed primary e.m.f. and that of the primary current. In Fig. 139 this angle is included between the vertical e_1 and the current line $O'D$. For the sake of simplicity we shall in a general discussion of power factor neglect the small hysteretic component of the current $O'O$ and take OD as representing the primary current. The error is trifling and on the safe side as it makes the power factor appear a little worse than it really is. For a given magnetising current the power factor will be the better

the larger the diameter of the circle, that is to say, the greater the circle ratio. Now the diameter of the circle is the ideal short circuit current and that is greatly influenced by the reactance. The reactance depends on two things, namely inductance and primary frequency, hence to get a good power factor the frequency should be low and the magnetic leakage small. The latter condition means that the winding must be subdivided into as many slots as possible and the air gap should be as small as mechanical considerations permit. A low primary frequency is also of advantage and it is a common experience that a good power factor can more easily be obtained with a low than with a high frequency.

The power factor varies with the load. It is obviously a maximum if the working point D lies in such a position

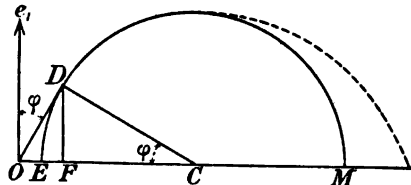


FIG. 143. Maximum power factor.

that OD is a tangent to the circle (Fig. 143). In this case

$$\cos \varphi = \frac{EC}{OC}. \quad \text{Now } EC = \frac{EM}{2} \text{ and } OC = OE + \frac{EM}{2} \text{ and since}$$

$$OE = \frac{ME}{\theta} \text{ we find}$$

$$\cos \varphi = \frac{\theta}{2 + \theta} \cdot \cdot \cdot \cdot \cdot \quad (133)$$

The circle ratio required to obtain a desired maximum power factor is

$$\theta = \frac{2 \cos \varphi}{1 - \cos \varphi} \cdot \cdot \cdot \cdot \cdot \quad (134)$$

and introducing the translation coefficient we find from (117)

$\eta = \sqrt{\frac{\theta}{1+\theta}}$ we can also write

$$\cos \varphi = \frac{\eta^2}{2 - \eta^2} \quad . \quad . \quad . \quad . \quad . \quad (135)$$

$$\eta = \sqrt{\frac{2 \cos \varphi}{1 + \cos \varphi}} \quad . \quad . \quad . \quad (136)$$

Some discrimination is necessary in the use of these formulae because they take no account of the stalling torque. If the circle ratio is low, point *D* will lie higher and the margin of torque to the stalling point may be too small; if the circle ratio is large the working point will lie lower and the stalling torque will be unnecessarily large, making the motor more expensive than it need be.

The correct way of relating circle ratio and power factor at full load must therefore take the stalling torque into account. If this is to be, say, twice the full load torque the ordinate DF should be about one-quarter the diameter of the circle and if we work out the trigonometrical relations on this basis we find for full load and double stalling torque

$$\cos \varphi = \frac{1}{\sqrt{1 + \left(0.27 + \frac{4}{\theta}\right)^2}} \quad \dots \quad (137)$$

This relation is represented in Fig. 144.

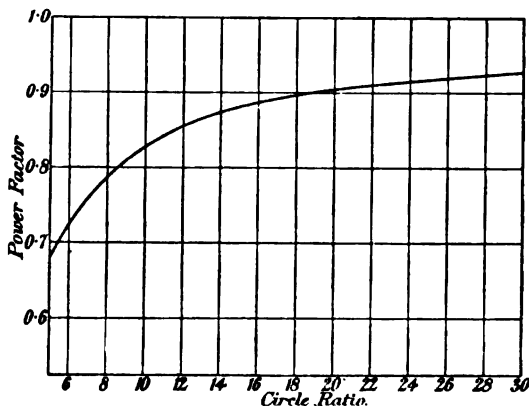


FIG. 144. Relation between circle ratio and power factor for double stalling torque.

Influence of Saturation. In deducing the circle diagram we have assumed that the inductance of the motor remains constant whatever the primary and secondary currents may be. This is not strictly correct. With excessive currents the teeth become highly saturated and the leakage flux is no longer proportional to the current and therefore the inductance is decreased. This means that the short circuit current, which is mainly limited by the inductance, is greater than corresponds to the circle diagram and the circle is distorted as shown by the dotted line in Fig. 143. If we made the short circuit test at the full primary voltage the point D_0 would lie somewhere on the dotted line and not on the circle. For this reason (quite apart from the danger of burning out the motor if short circuited under full voltage) the test should be made at such a voltage that the short circuit current has about the normal full

load value. Then there will be no excessive saturation in the teeth and no distortion of the circle. The diameter of the circle is reduced in proportion as the voltage is reduced and the actual circle diagram must be drawn to the full scale. As the working point must lie well to the left of the centre and as the distortion only occurs with excessive currents, the fact that the curve to the right of C is not a circle is of no importance.

Graphic Representation of Efficiency. In the circle diagram it is possible to represent iron and copper losses and therefore also the efficiency in a scalar way, that is by the length of a line. Friction and windage do not enter into the circle diagram and to that extent the scalar representation of efficiency must be inaccurate, but since both these losses are very small and can be estimated fairly closely it is easy to apply the necessary correction. Friction and windage together are of the order of one to two per cent. of full load.

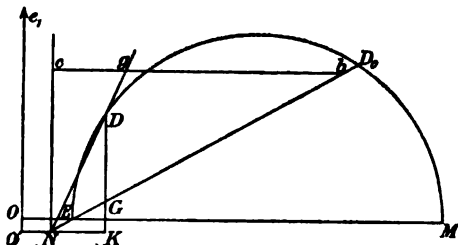


FIG. 145. Graphic representation of efficiency.

To find a line on which efficiency in per cent. can be scaled off we proceed as follows. Prolong as in Fig. 145 the rotor short circuit current ED_0 backward to the point N which lies below the axis by the amount OO' so that DK represents the total electrical input. To the same scale DG represents the mechanical output and the ratio $\eta = \frac{DG}{DK}$ is the efficiency. Through N draw a vertical and at a point c a horizontal cutting the ED_0 vector in b . The position of c should be so chosen that the length cb represents to any convenient scale 100 length units. From N draw a line to the working point D and note the point of intersection g with the horizontal cb . The triangle Ncg is similar to the triangle DKN and the triangles Ncb and KGN are also similar. We have therefore

$$\frac{GK}{KN} = \frac{cN}{cb} \quad \text{and} \quad \frac{KN}{KD} = \frac{cg}{cN}$$

By multiplication

$$\frac{GK}{KD} = \frac{cg}{cb}, \quad \text{or} \quad \frac{KD - GK}{KD} = \frac{cb - cg}{cb}, \quad \text{or} \quad \frac{DG}{DK} = \frac{gb}{cb}$$

But $\frac{DG}{DK}$ is the efficiency η and since cb has been made 100 length units we find the efficiency in per cent.

$$\eta = gb \dots \dots \dots (138)$$

Relation between Translation Coefficient and Leakage Factor. When the secondary circuit is open the application of the full primary e.m.f. produces the working flux Φ due to the magnetising current OE in Fig. 139. To obtain the same magnetising current with the secondary short-circuited we must apply a very much lower e.m.f. producing the leakage flux $\lambda\Phi$, where λ is the leakage factor. On short circuit the ampereturns in stator and rotor are very nearly alike, so that by applying the laws of the magnetic circuit we can calculate for any given ampereturns such as result from the application of the primary current OE the leakage flux produced in stator and rotor; their sum is $\lambda\Phi$ and this apart from saturation is proportional to the primary current. On short circuit full voltage produces the primary current OM and this corresponds to the full normal flux. We may therefore put the relations in this way: OE produces $\lambda\Phi$ and OM produces Φ from which follow

$$\lambda = \frac{OE}{OM}$$

It has been shown in Chapter XII that in the circle diagram of the "general transformer" $\frac{OE}{OM} = 1 - \eta^2$. We have therefore

$$\lambda = 1 - \eta^2 \text{ or } \eta = \sqrt{1 - \lambda} \dots \dots \dots (131)$$

as already stated on page 313. The circle ratio may now also be expressed by

$$\theta = \frac{1 - \lambda}{\lambda}$$

and the maximum power factor by $\cos \varphi = \frac{1 - \lambda}{1 + \lambda}$. To avoid any error due to the effect of saturation it is advisable to calculate λ for a fairly small current, say, the magnetising component of the primary current when the rotor is open. This gives $\lambda\Phi$ with short circuited rotor and Φ with open rotor.

Cascade Working. Let two motors be mechanically coupled and let the rotor currents of one be supplied to the primary circuit (which may be stator or rotor) of the second motor, then the two

motors are said to be in cascade or tandem connection. The motor which receives the current from the supply, so to say at first hand, is called the front motor, that which receives its supply from the rotor of the front motor is called the rear motor. If both rotors are mounted on the same shaft their speed must be the same. A common shaft is, however, not essential; any mechanical connection which ensures equality of speed serves equally well. Thus in the most important practical case of cascade working, namely in electric traction, the mechanical connection between front and rear motor is formed by the rails on which the locomotive runs. In the case of an electric locomotive for three phase working the stators of both motors are wound for the high voltage of the primary supply so that when working in cascade the rotor of the rear motor forms its primary circuit. This arrangement is adopted so that it may be possible to use either motor separately and at the same time avoid the difficulty of high insulation in the rotors. By constructing the motors with different numbers of poles three synchronous speeds may be obtained. Let $2p_1$ be the number of poles of the front motor, $2p_2$ that of the rear motor and u the synchronous speed; then with the front motor working alone and the rear motor running

idle we have $u_1 = \frac{f_1}{p_1}$ where f_1 is the primary frequency. With the

rear motor working alone $u_2 = \frac{f_1}{p_2}$ and when both motors are

working in cascade $u = \frac{f_1}{p_1 + p_2}$. This expression is obtained as

follows. The speed of the shaft is the synchronous speed of the rear motor, or $u = \frac{f}{p_2}$. The speed of the primary field is $u_1 = \frac{f_1}{p_1}$

and the frequency of the currents in the primary rotor is $f = p_1(u_1 - u)$.

We thus have $u = \frac{p_1 u_1 - p_1 u}{p_2}$ from which follows

$$u = \frac{u_1 p_1}{p_1 + p_2} \quad \text{or} \quad u = \frac{f_1}{p_1 + p_2} \quad \dots \quad (139)$$

For $p_1 = p_2$ the cascade combination runs at half speed. For an electric locomotive equipment in which the front motor has double the number of poles of the rear motor the cascade combination runs at $\frac{2}{3}$ of the speed of the front motor working alone. The speed of the rear motor is twice that of the front motor so that we obtain in all three speeds having the ratio 2:3:6. If the front motor is

16 pole and the rear motor 12 pole the speed ratio would be as 25, 43 and 57 miles per hour.

Fig. 146 shows diagrammatically a cascade combination in which

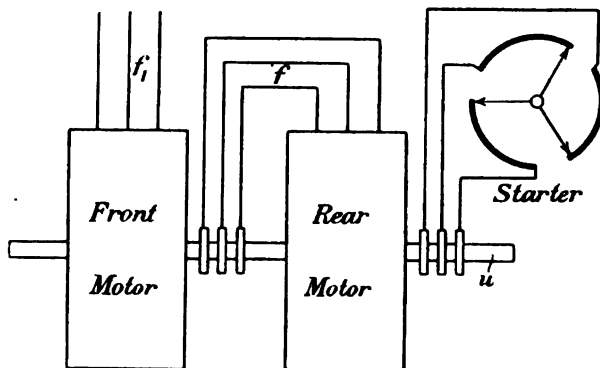


FIG. 146. Cascade combination.

the stator of the rear motor receives current from the rotor of the front motor. A rigorous theory of the cascade, even by the aid of

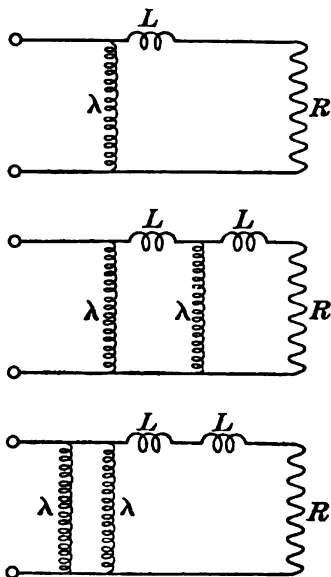


FIG. 147. Equivalent coils.

winding. L is the inductance of the motor as a whole and λ is the

the circle diagram, is very complicated and as this type of motor has only a limited field of application it will suffice to treat it in a general and approximate manner; and for this purpose we make use of the conception of equivalent coils. We assume that we have two precisely similar motors and that primary and secondary have the same kind of winding. The problem is to compare in a general way the performance of one motor working alone with the performance of the cascade. The upper diagram in Fig. 147 shows the single motor represented by equivalent coils. The mechanical load is represented by the resistance R which contains also the small amount of true ohmic resistance due to the

inductance of the primary alone. The exciting current is inversely proportional to the product of frequency and inductance λ . In the cascade combination shown in the middle diagram the exciting current of the front motor must be the same as that of the single motor since neither e.m.f., frequency nor inductance have changed, but the exciting current in the rear motor is a little smaller because the impressed e.m.f. has been reduced by a little more than half (namely to $\frac{1}{2} \frac{MD}{OM}$ in Fig. 139) whilst the frequency is halved. In the

diagram of equivalent coils, which takes no account of frequency, this reduction is due to the inductive drop in the first inductance L . If we shift the second λ to the left of L we shall overestimate the exciting current taken by the two motors in cascade but not materially since the reduction of exciting current in the rear motor is only of the order of 10 per cent. for a circle ratio of 30 and about 12 per cent. for a circle ratio of 20. By accepting the lower diagram as an approximate representation by equivalent coils we find that

as compared with the single motor the magnetising current has been doubled and the inductance has also been doubled. The ideal short circuit current (EM in Fig. 139) has therefore been halved whilst the exciting current (OE in Fig. 139) has

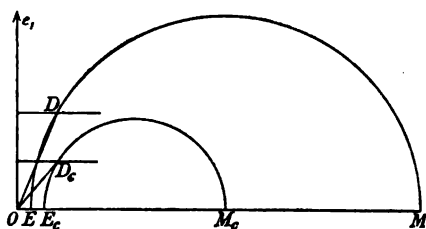


FIG. 148. Cascade working.

been doubled. This means that the circle ratio of the cascade is reduced to one-quarter of that of the single motor. These relations are shown in Fig. 148. The large circle pertains to the single motor with exciting current OE , the small circle of half the diameter pertains to the cascade with exciting current OE_c , also

$$E_c M_c = \frac{1}{2} EM \text{ and } OE_c = 2OE.$$

D and D_c are the working points. The watt components of the two rotor currents remain the same because both voltage and frequency have been simultaneously halved. The flux remains the same and therefore the torque, which is proportional to the product of flux and current, remains also the same in each motor. This means that the cascade gives double torque at half speed so that the output of the cascade is the same as that of the single motor. There is, however,

a sensible reduction of the power factor as will be clearly seen in Fig. 148. This figure has been drawn for the exceptionally large circle ratio of 30; with a smaller circle ratio the reduction in the value of the power factor would be still more marked. A single low speed motor can always be built for a better power factor than can be obtained by two high speed motors in cascade, and that is the reason why cascade working is only used if speed variation is an essential condition, as in electric railways. Most of the induction motors used for traction are designed for a circle ratio well over 30 and to obtain this a low primary frequency is essential. The standard frequency for traction is 15 cycles per second.

Induction Generators. If an induction motor is driven by mechanical power at a speed exceeding synchronism it must be

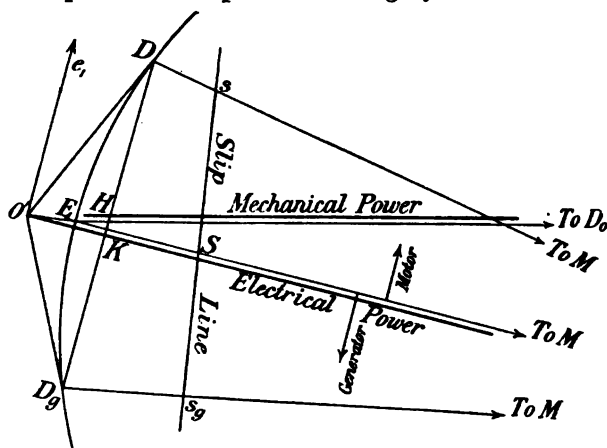


FIG. 149. Diagram of induction machine as motor and generator.

considered as working with a negative slip. In the circle diagram Fig. 139 the slip line is cut by the line DM below the axis and the primary current changes sign. It is no longer a current impressed on the machine, but a current produced by the machine which now works as a generator. An induction machine may therefore in the same way as a D.C. shunt motor be used either as a motor or as a generator, but there is this difference: that whereas the D.C. machine excites itself the A.C. machine requires the supply of an exciting current from some outside A.C. source. The wattless component of the current the A.C. generator produces must be supplied from outside. The transition from one working condition to the other

is shown in Fig. 149. The lines above the axis EM pertain to the motor, those below to the generator. The net mechanical power given out by the motor (after deducting that required to overcome friction and windage) is shown by the thick line lying a little above the ED_0 line. The electrical input to the motor is to be measured from the working point D to the thick line lying a little below the axis and passing through O' . In this way the loss of electrical input occasioned by the stator iron is accounted for. DK is the electrical input and DH the mechanical output of the motor. S_s is the slip which is counted positive.

Let D_0 be the working point if the machine is used as a generator, then D_0H represents the mechanical power impressed on the generator by its engine and D_0K the net electrical power obtained at its primary terminals. The slip is negative, namely S_s .

The induction generator has the advantage over a synchronous generator that it requires no synchronising; all that needs attention is to run it at or a little above synchronism. It may then be switched on to the bus bars and will automatically fall into step. Its disadvantage is that the power factor is necessarily below unity and thus an additional wattless and lagging component is thrown on the synchronous generators working on the bus bars. This disadvantage is eliminated if some sort of phase advancer is used in conjunction with the induction generator as will be shown in Chapter XV.

The Single Phase Motor. A single winding on the stator produces simply an alternating and not a rotating field, but, as was first pointed out by Galileo Ferraris*, an alternating field with a fixed axis in space may be considered the resultant of two fields of constant magnitude, the axes of which revolve in opposite directions with a speed corresponding to the frequency. The magnitude of each field is half the crest value of the alternating field. Imagine any type of polyphase rotor (squirrel cage or wound) placed into a stator with a single winding. The two components of the field will exert equal torque, but in opposite directions, and the resultant torque must be zero. The motor cannot start. In this condition the motor may be considered to have a slip of 100 per cent. with respect to either field. Now let the rotor be turned by hand so that

* Galileo Ferraris is the discoverer of the rotating field produced by two currents in quadrature. His first experiments date back to 1885, but his first publication on this subject was made in 1888. It is on his discovery that the A.C. induction motor is based.

the slip with respect to the field which rotates in the same direction becomes a little less than 100 per cent. At the same time the slip with respect to the other field becomes a little more than 100 per cent. There will now be a difference in the value of the two torques. That acting forward and assisting the motion is greater than that which opposes the motion and thus there is a resultant torque in the same direction in which the motor has been started. A single phase motor has no definite direction of rotation, but once started will run either way. When near synchronism the forward torque is equal to that of a multiphase motor having half the field strength and the backward torque is that of such a motor having nearly 200 per cent. slip. This relation is shown in Fig. 150. T represents the resultant or useful torque. It has been shown that the torque is proportional to the product of flux and rotor current and therefore also to the square of the voltage. In order that the same rotor may develop

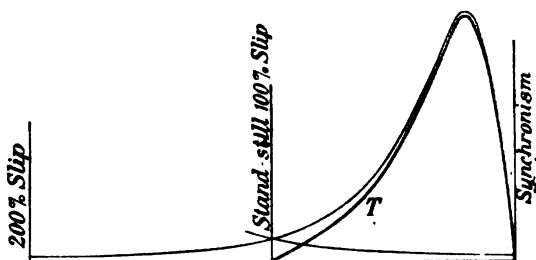


FIG. 150. Speed-torque diagram of single phase induction motor.

the same torque in a single phase stator the rotor current and flux must each be raised in the ratio of 1 to $\sqrt{2}$ and the ohmic loss of power in the rotor is therefore doubled as compared with, say, a three phase motor. Since this loss is proportional to the slip we find that the single phase motor has double the slip of a three phase motor and that the efficiency is lower because the copper loss in the rotor is doubled.

Only small motors can be started by hand; larger motors require some starting appliance such as an auxiliary winding on the stator through which a current displaced in phase from the main current is sent. The auxiliary winding is placed in space quadrature to the main winding and if it were possible to produce in the auxiliary current a quadrature time displacement we would have a two phase motor. In this case the stator would produce a circular rotating

field. So great a time displacement between the two currents as 90° cannot be produced, but some smaller displacement is possible by the use of a resistance or inductance in the auxiliary circuit. Both currents are obtained from the same supply. The field thereby produced will not be circular, but elliptical and although such a field will not produce so powerful a torque as a true circular field it is sufficient for starting the motor, even under some load, provided the usual starting resistance in the rotor is used. After the motor has been started the auxiliary winding may either be cut out altogether, or it may be retained and put in series with the main winding, but the resistance or inductance previously used to give a phase displacement must be put out of circuit. Some extra copper is thus required to produce a starting torque and in order that this may to some extent, at least, be utilised in the normal working condition the series arrangement between the two windings is preferable to the complete cutting out of the auxiliary winding.

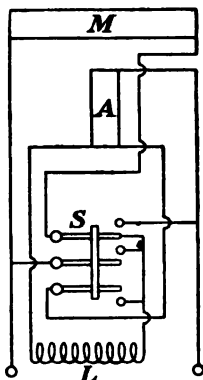


FIG. 151. Stator switch for single phase motor.

Fig. 151 shows how a switch for this purpose may be arranged. *M* is the main winding on the stator and *A* the auxiliary winding in space quadrature with it. *L* is a choking coil and *S* is the switch. If it is placed to the upper contacts the main and auxiliary windings are in parallel with the choking coil retarding the phase of the current through the auxiliary. If the switch is at its lower contacts the choking coil is short circuited and the main and auxiliary windings are put into series connection.

Some practical hints. It is convenient to be able to estimate the power of a motor from the dimensions of the rotor, namely length *L* and diameter *D*, both given in metres and the speed *U* in revs. per minute. For this purpose a formula similar to (26) may be used, namely

$$\text{H.P.} = CD^2LU \quad . \quad . \quad . \quad (140 \Delta)$$

where *C* is the output coefficient and varies according to the size of the motor between 0.8 and 2.0 or even more in very large motors. *C* is mainly influenced by the diameter, but also by other details such as H.P., frequency, voltage, power factor and the ratio of length to pole pitch. It is therefore not possible to establish a com-

prehensive formula for C , but as a very rough and first approximation we may take

$$C = 0.75 (1 + D) \dots \dots \dots (140 \text{ B})$$

and by a trial and error method find what value of D will simultaneously satisfy (140 A) and (140 B). The D thus found serves merely as a basis in starting the design and may have to be altered as the details are worked out.

The current density in effective ampere wires per cm. of circumference varies between about 150 for small and 250 for large motors; in exceptional cases for very large motors the density may even be as high as 300.

The crest value of the induction is about 6000 lines per sq. cm. of gap area.

The circumferential speed of the rotor is of the order of 25 metres per second and in the case of motors for driving turbo air compressors it may be as high as 80 metres per second.

The air gap should be as small as mechanical considerations allow. In small motors with ball bearings it is customary to make the air gap about 0.75 mm. wide. In larger motors the air gap is some function of the diameter and various formulae are used. Some of these are given hereunder. The width of air gap δ and diameter and length of rotor are in mm. and the circumferential speed v of the rotors is in metres per second.

$$\left. \begin{array}{l} \text{up to } D = 600 \quad \delta = \frac{D}{500} \\ \text{up to } D = 2000 \quad \delta = 0.8 + \frac{D}{1500} \\ \delta = 0.0006 \sqrt{LDv} \end{array} \right\} \dots \dots (141)$$

$$\delta = \frac{\sqrt{D}}{20} - 0.25 \dots \dots (142)$$

Formula (142) is only applicable to very large motors.

It has been shown how the circle ratio can be predetermined from a given design, but as this is a laborious process it is convenient to have a method by which the circle ratio may be estimated from the principal data even before the design is completed in all details. Such a method has been given by Mr B. Behrend*. His expression for the circle ratio is

$$\theta = \frac{\tau}{cc'\delta} \dots \dots \dots (143)$$

* *Elektrotechnische Zeitschrift*, 1904, p. 340.

where c and c' are coefficients depending on type and number of slots and number of phases. Let m be the number of phases and n the number of slots per phase per pole. Let the index 1 refer to the primary and the index 2 to the secondary, then for m phases and n slots per coil side the total number of wires is

$$Z_1 = 2pm_1n_1$$

$$Z_2 = 2pm_2n_2$$

Put

$$mn = \frac{m_1n_1 + m_2n_2}{2}$$

c' depends on $\delta \times mn$ and may be taken from the following table, δ being in mm.

$\delta \times mn =$	8	14	20	26	32	38	50
$c' =$	1.34	1.1	0.94	0.82	0.73	0.66	0.57

c depends on $\frac{L}{\tau}$ and the type of slot whether open or closed

$\frac{L}{\tau} =$	0.4	0.8	1.2	1.4	1.6	
$c =$	12.3	8.8	6.9	6.3	5.8	wide open slots
$c =$	17	15.8	13	12.7	12.5	nearly closed slots

CHAPTER XIV

ALTERNATING CURRENT COMMUTATOR MOTORS

Time and space diagrams—The single phase series motor—The circle diagram of a series motor—The repulsion motor—The Latour-Winter-Eichberg motor—The three phase series commutator motor.

Time and Space Diagrams. If the diagram of the working parts of a machine is so drawn that it represents not only their electrical connections, but also their mutual location in space, we call this a space diagram in contradistinction to the time diagram representing merely the phase relation of the different vectors without reference to the position in space of those parts to which the vectors refer. In studying the working condition of D.C. dynamos we have already used such space diagrams to determine the direction of induced e.m.f., torque, flux and so on. In doing this we had to establish certain conventions as to windings and these we may retain also when drawing space diagrams of A.C. machines, but as in this case

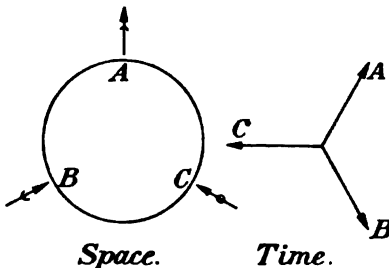


FIG. 152.

we have to deal with alternating vectors some further conventions become necessary as to sign and the growing or waning of the quantities at the moment to which the space diagram refers*. As an example illustrating the difference between time and space diagrams take the case of a three phase generator with

sequence of phases *ABC*. On the right of Fig. 152 is the time diagram of the vectors which may represent either current or e.m.f.; on the left is the space diagram consisting simply of a circle to represent the armature or stator of the machine with its terminals *ABC* so placed as to correspond with the sequence of phases. Following the convention adopted in D.C. machinery we call, in a generator, that terminal positive at which the current leaves the machine, that is to say, flows outwards. In a motor the positive terminal

* See the author's article on "Uniformity in drawing electrical diagrams" in *The Electrician*, No. 1973, p. 802.

is that at which the current flows inwards. Thus in the present case, which is that of a generator, positive currents are directed outwards and negative currents inwards. In the time diagram current A is positive and must in the space diagram be drawn radially outward from the circle; B is negative and must be drawn radially inwards. C is at the moment passing through zero to become negative. In the space diagram we draw it inwards, but place a little 0 in the shaft of the arrow to indicate that this vector is zero at the moment. The little half moons in the other two vectors indicate the direction in which the magnitude of each alters. A is still growing and the half moon is placed with its convex side to the arrow head, B is diminishing and the half moon is placed with its convex side towards the tail.

Fig. 153 is another example of a space diagram. It refers to a series motor supplied with A.C. Since both flux and current change simultaneously the direction in which the torque acts is independent of the direction in which the current flows and any D.C. series motor can therefore be used with A.C., but its power factor will be bad and the iron losses will be great because of eddies in the field magnets. The latter defect can be remedied by laminating the field system, the former can be reduced by certain methods which will be discussed presently. For the moment we are only concerned with the general method of representing the working of such a motor by a space diagram. Fig. 153 refers to the moment when the impressed e.m.f. has positive crest value. Current and flux are co-phasal and lag behind the e.m.f. by the same angle ϕ . The induced e.m.f. has two components, one is that due to the rotation of the armature in the field and the other is that due to the reactance of the machine as a whole. The numerical values of these components are $\epsilon\Phi\omega$ and ωLI respectively. The time diagram gives crest values, the space diagram instantaneous values. As in this diagram there are two axes, namely, a vertical axis for the flux produced by the stator and a horizontal axis joining the brushes we must define the sign of the vector for both directions. We call a vector positive if it is directed either vertically upwards or horizontally from left to right

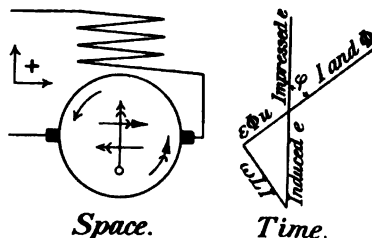


FIG. 153.

as shown in the little key diagram. The flux vector must therefore be shown with the arrow pointing upwards and as still growing; the current vector in the brush axis must be drawn with the arrow head on the right and the half moon so drawn as to indicate that it is still growing. The vector of rotational e.m.f. lies in the time diagram in the lower left hand quadrant and has therefore at the moment a negative value; hence in the space diagram it must be drawn to the left and since the e.m.f. is proportional to the flux which at that moment is still growing the half moon must be added with the convex side to the arrow head.

The Single Phase Series Motor. A D.C. series motor of usual proportions, even if it has a laminated field system, would not be practicable for A.C. because its power factor would be extremely low. In a D.C. motor the ampereturns of the field may be two or three times those of the armature and hence the whole machine if used in A.C. would have a very large reactance, to overcome which the impressed e.m.f. would have to contain a very large wattless component. To make such a motor practicable the inductance must be reduced and that may be done in two ways. We may compensate the armature ampereturns by a compensating winding and we may at the same time reduce the number of turns on the field by reducing the air gap to the mechanically possible minimum. By these means the inductance may be reduced to a small fraction of that which would obtain in a motor of D.C. proportions and if such a motor is used on a circuit of small frequency a fairly good power factor will be obtainable. The most important application for such motors is electric traction and the standard frequency for this purpose is 16 with a latitude of a few per cent. up and down. This frequency has been chosen to facilitate the interlinking of ordinary electricity works with railway power stations. The usual frequency of electricity works is in the neighbourhood of 50. By using a frequency changer having a ratio of 1 to 3 the railway power house can interchange power with the electricity works so that either may act as a stand-by for the other. In order to compensate as nearly as possible the inductance of the armature the compensating winding must be a counterpart of the armature winding. It must therefore not be placed on salient poles but must surround the whole circumference of the armature. Fig. 154 is the space diagram of a series A.C. motor with compensating

winding. The latter is shown as a circle surrounding the armature. The vectors are drawn to represent the moment when the current passes through zero to become positive. To avoid complication the vector of back e.m.f. of rotation has not been inserted. Otherwise the diagram is self-explanatory.

There remains the question of commutation. This must obviously be more difficult than in a D.C. motor, not only because of the reduced air gap which increases the inductance of the coils under commutation, but also because the alternating main flux sets up what may be called a transformer e.m.f. in the coils short circuited by the brushes and may thus cause a large short circuit current super-imposed on the working current. This additional current passing between toe and heel of the brush causes additional heating. To eliminate this effect and generally improve the commutation the expedient is used of producing a slight phase difference between the armature flux and that due to the compensating winding. For this purpose the latter is shunted by an inductionless resistance.

The space diagram Fig. 154 refers to the moment when the current passes through zero to become positive. S is the coil producing the working flux Φ , A represents the armature and C the compensating winding. R is the shunt resistance. The time diagram is the same as in Fig. 153, but turned clockwise so as to make the current vector horizontal. It is not repeated in Fig. 154. The small time diagram below this figure merely represents the armature flux Φ_A , the flux produced by the compensating coil Φ_c , and the resultant flux F , which being positive is shown as directed from left to right in the space diagram. With counter-clockwise rotation this flux produces a downward e.m.f. in the wires under the right hand brush. At that moment, when the main field is zero and about to become positive, the coils short circuited by the brushes are the seat of a self-induced (or so-called transformer) e.m.f. which must oppose the change of sign and growth of the main field; that is to say, the transformer e.m.f. under the right hand brush is upwards. It is

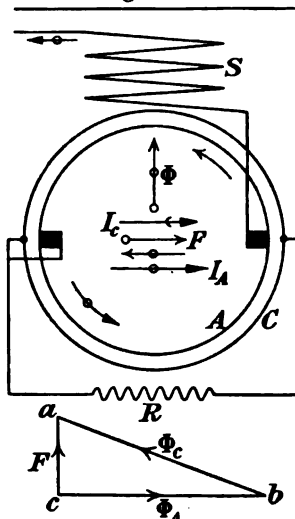


FIG. 154. Single phase series motor.

therefore opposed by the e.m.f. of rotation and thus at a particular speed we obtain perfect balance. At higher and lower speeds there will be some short circuit currents. During starting the speed is too low to produce any appreciable compensating effect by means of the flux F and for this reason it is necessary to restrict as much as possible the transformer e.m.f. and to minimise the short circuit current. The first condition is obtained by low frequency and a commutator with the maximum possible number of segments, namely one for every two active wires, and the second by hard brushes. The reactance voltage of commutation can be compensated in the ordinary way by interpoles, but the transformer e.m.f. at starting and the excess over it of the rotational e.m.f. due to F at high speed impose certain limitations on the design if the motor is of large power and intended for electric traction. Experience has shown that even hard brushes cannot deal with more than about seven volts transformer e.m.f. between toe and heel. The length of the commutator is limited by mechanical reasons so that to obtain sufficient brush contact surface it is necessary to employ brushes covering about three sections of the commutator. Hence the transformer voltage of one armature turn cannot be much more than two volts and this with 15 frequency means that the limit of flux out of one pole is about $3\frac{1}{2}$ megalines. The tangential force due to one pole is proportional to the product of flux and linear current density on the armature. The latter is restricted by the permissible temperature rise so that the total tangential force exerted by the armature is proportional to the number of poles and the power to the product of number of poles and circumferential speed. The latter quantity again cannot be raised indefinitely for obvious mechanical reasons. We thus arrive at the result that the maximum power obtainable from a motor of this type is about proportional to the number of poles. The exact relation between maximum power and number of poles must obviously depend on a variety of details such as efficacy of cooling, time rating, quality of the brushes, length of air gap and so on; it is therefore not possible to give a definite figure as generally applicable, but taking as an example the highly successful motors of the Loetschberg electric locomotives designed by Dr Behn-Eschenburg, we find that at the $1\frac{1}{2}$ hour rating these 16 pole motors develop each 1250 h.p., which is at the rate of 78 h.p. per pole. It should be noted that shunting the compensating winding by an ohmic resistance only compensates for transformer voltage,

but is no aid to commutation. The time at which such aid is most wanted is the moment when the armature current is a maximum, but at that moment the flux F passes through zero and cannot aid commutation. The usual interpoles must therefore be provided in addition to the compensating winding.

Speed regulation in these locomotives is obtained by varying the impressed voltage. Since a step down transformer is necessary anyhow to reduce the standard line voltage of 15,000 to the working voltage of the motor which is of the order of some hundred volts, the variation in the latter can most easily be obtained by providing the transformer with tappings on its secondary.

The Circle Diagram of a Series Motor. The working of a single phase series motor can be represented by a circle diagram in a similar way to that which is adopted to represent the working conditions of an induction motor. In adapting the circle diagram for this particular case we shall assume that the motor has no iron losses and no mechanical losses; also that the reluctance of its magnetic circuit is constant, which means that we neglect the effect of saturation. These assumptions are made for the sake of rendering the graphic treatment as simple as possible. They do not detract from the practical value of the diagram because it is always possible to correct results by finally taking the losses and the effect of saturation into account. Thus if we find that in the diagram the length of a certain line represents induced torque we can correct for net torque by subtracting from the length an amount corresponding to friction and hysteretic loss. The correction for saturation may be made by finding from the magnetisation characteristic how the flux and value of inductance change with the current.

Assuming then a constant value of the inductance L and neglecting frictional and iron losses we may represent the working

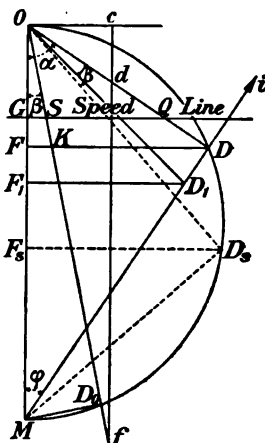


FIG. 155. Circle diagram of single-phase motor.

of the motor by the circle diagram Fig. 155. Let MO be the vector of the impressed e.m.f. e and MD its watt component which is in phase with the current i . The wattless

component due to inductance is DO and this is proportional to the current so that $i = \frac{DO}{\omega L}$. With a suitable change of scale DO represents therefore the current as to magnitude, but not as to phase position. The watt component of the e.m.f. has to supply the ohmic loss in the winding and the rest must balance the counter e.m.f. of rotation. We thus find $D_1D = \rho i$ and $MD_1 = \epsilon \Phi u$. Since the reluctance is constant $\Phi \equiv i$. The power input (being the product of watt component of e.m.f. and current) is proportional to the area of the triangle MDO , that is to the length DF . The power output is reduced by the internal loss which is proportional to the area of the triangle D_1DO . The mechanical power is therefore given with the power scale by the length D_1F_1 . At stand-still the output is zero. The point D goes to D_0 and this point is found by drawing OD_0 under an angle β to the vertical so that $\tan \beta = \rho/\omega L$. OD_0 is then the starting current and the line OD_0 is the output line, which means that the mechanical power corresponding to any working point D is found by measuring the horizontal distance from the working point to the output line, that is the length DK . That DK represents mechanical power also follows from the geometry of the diagram if we can prove that $DK = D_1F_1$. We have

$$\frac{DD_1}{DO} = \frac{FK}{FO} \text{ and } \frac{DO}{DM} = \frac{OF}{DF}$$

By multiplication $\frac{DD_1}{DM} = \frac{FK}{DF}$ or $\frac{D_1M}{DM} = \frac{DK}{DF}$

and since $\frac{D_1M}{DM} = \frac{D_1F_1}{DF}$ we find $D_1F_1 = DK$

The torque being proportional to Φi , that is to i^2 or $(OD)^2$ we find from the geometry of the circle that the torque may be considered as proportional to $(OF)(OM)$ and since OM is constant the length OF represents to a certain scale the torque corresponding to the working point D . To get the speed we use the formula: counter e.m.f. = $\epsilon \times \text{flux} \times \text{speed}$, where ϵ is a constant of the design. Since flux is proportional to current, that is to OD , we have for the speed the expression

$$u \equiv \frac{D_1M}{DO} \text{ or } u \equiv \frac{DM}{DO} - \frac{DD_1}{DO} \text{ or } u \equiv (\tan \alpha - \tan \beta)$$

The intercept SQ of the two rays OD and OD_0 on a horizontal through any point G represents therefore the speed to a particular scale. The scale for speed like the scales for all the other quantities is found algebraically for any definite working point D . The efficiency irrespective of mechanical and hysteretic losses is obviously the ratio D_1M/DM , or D_1F_1/DF . This may be also represented by a scalar quantity. Draw any vertical such as cf and make it 100 length units. By similarity of triangles we have

$$\frac{OF}{DF} = \frac{cd}{cO} \text{ and } \frac{KF}{OF} = \frac{cO}{cf}$$

Multiplying we find $\frac{KF}{DF} = \frac{cd}{cf}$ and deducting 1 from each side gives

$\frac{KD}{FD} = \frac{df}{cf}$. Since KD is the output and FD the input the ratio is the efficiency and since we have made $cf = 100$ units the length df is the efficiency in per cent.

If the motor were to be switched on when at rest the starting current would be OD_0 , in reality even greater because of the reduced inductance at saturation, so that for safe starting a resistance should be inserted or the impressed voltage lowered. If the former means is adopted we can determine by the diagram how great the starting resistance must be in order to limit the starting current to a definite value, say to OD_s . The length MD_0 represents the product of the excessive starting current OD_0 and the resistance ρ of the motor itself. In the same way MD_s represents the product of the permissible starting current OD_s and the total resistance then in circuit, which is the starting resistance R plus that of the motor ρ . The starting resistance which will limit the current to OD_s is therefore found from

$$R = \rho \left(\frac{MD_s}{MD_0} \frac{OD_0}{OD_s} - 1 \right)$$

The Repulsion Motor. Since in the straight series motor the rotor is part of the supply circuit this type is limited to such a supply voltage as may be safely used on a revolving armature; say up to 700 v. for large and less for small motors. If the supply is given at a higher voltage a transformer becomes necessary, but it is possible to save the cost of a separate transformer by making the motor itself a transformer. The compensating winding forms the primary of this transformer and the armature with short circuited

brushes forms the secondary. The working flux is impressed by a coil S in geometric quadrature to the brush axis as shown in Fig. 156.

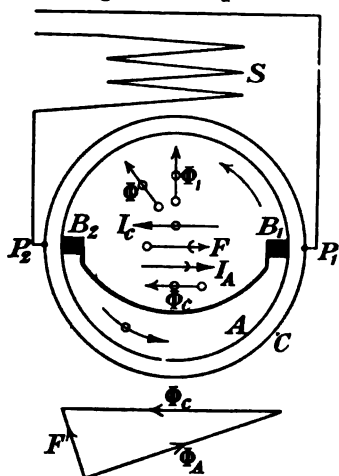


FIG. 156. Single phase series motor with short-circuited rotor.

A is the armature winding and C the compensating coil. Also in this case, as will be seen from the time diagram below the figure, the resultant F of the armature and compensating flux acts in the same way as in Fig. 154; it produces a rotational e.m.f. in the armature coils short circuited by the brushes in opposition to the transformer e.m.f. In this arrangement the stator produces two fluxes, namely the main flux Φ_1 and the compensating flux Φ_c , and as these are due to the same current, they coincide in time and may therefore in the space diagram be replaced by their resultant Φ . We may produce this resultant flux directly as in Fig. 157 by the compensating coil after suppressing the main winding S by shifting the points P_1P_2 in a clockwise direction, or, what comes to the same thing, we may shift the brush axis with respect to the P_1P_2 axis in a counter-clockwise direction. We thus obtain the "repulsion motor" invented by Prof. Elihu Thomson. In this figure Φ is the main flux produced by the coil C . It has two components $\Phi_t = \Phi \sin \alpha$ and $\Phi_r = \Phi \cos \alpha$. The former produces the transformer e.m.f. which causes the armature current; the latter is the

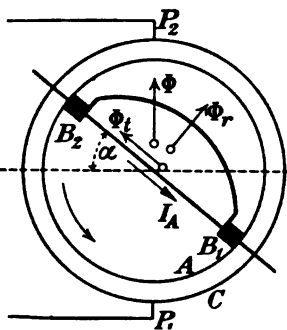


FIG. 157. Repulsion motor.

field which combined with the armature current gives torque. If the brushes are set in the dotted line, that is at right angles to the stator axis, there is no transformer e.m.f. and therefore no current through the armature. If the brushes are shifted in a clockwise direction there is armature current and a torque is exerted in a counter-clockwise direction; hence the name repulsion motor. If the brushes are shifted by 90 electrical degrees the motor becomes a short circuited

transformer. The armature current is very large, but as there is no field component in geometrical quadrature with it, there cannot be any torque. The stand-still position is for $\alpha = 0$. Shifting the brushes from this position in either sense causes rotation in the opposite sense; the motor is reversible. There is no starting resistance required and the regulation of torque and speed is perfectly gradual. When the brushes are in the stand-still position no current flows through the armature as a whole, but a strong short circuit current flows through the few turns short circuited by the individual brushes. To stop the motor it is therefore better to switch off completely and not merely to put the brushes in quadrature with the main flux. Another inconvenience of the ordinary repulsion motor is that its working condition is rather sensitive to even small variation in the brush angle, requiring careful setting. In both respects the alteration devised by Mr Deri is an improvement. Mr Deri employs two sets of brushes connected as shown in Fig. 158.

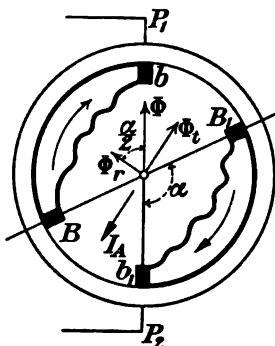


FIG. 158. Repulsion motor with double brushes.

The brushes bb_1 are fixed and the brushes BB_1 are movable. The flux Φ produced by the stator has two components, one Φ_t produces the transformer e.m.f. which drives current through the winding of the armature included between b and B on the one side and b_1 and B_1 on the other. It will be noticed that the conductors lying between b and B_1 and also those between b_1 and B are free from current. This is indicated in the figure by the thinner line. The other component of Φ which produces rotational e.m.f. is Φ_r . At stand-still b and B on one side and b_1 and B_1 on the other coincide, and as the plane of the short-circuited coils is also that of the flux no current passes through the brush contacts. If the movable brushes are shifted counter-clockwise the torque due to armature current I_a and flux Φ_r acts in a clockwise direction as shown. But the angle of shift is now twice as great and therefore a more accurate setting of the brushes is possible. The fact that at each of the four brushes the change of current is only from full value to zero or from zero to full value as compared with full value in one sense to full value in the opposite sense as in the original repulsion motor improves the commutation. The heating of the armature is less because only

that part of the winding most efficacious in producing torque carries current. As an offset to these advantages must be considered the greater complication of brush gear and the fact that if the motor is to be reversible the commutator must be made longer so as to permit the movable brushes passing the fixed brushes to either side.

The Latour-Winter-Eichberg Motor. In all the commutator motors above described the working flux is generated in a fixed part of the machine and this having necessarily inductance, the power factor can never be unity. If the working flux could be generated in a rotating winding then it would be possible to so arrange the connections that the e.m.f. generated in such a winding by rotation compensates for the e.m.f. of self-induction and thus a motor with unity power factor might be obtained. The first to recognise this fundamental principle was Ernest Wilson who showed in his British Patent 18525 of 1888 how this principle can be applied to a two phase motor. Later Maurice Latour* has shown that the same principle may be applied to a single phase motor and Goerges† applied it to a three phase motor. This was shown at the Frankfurt Exhibition in 1901. In Latour's motor the rotor was connected to the primary supply and its application was therefore restricted to low voltage; moreover there was no provision for economic speed regulation; only at one particular working condition could unity power factor be obtained. These defects have been removed later by Latour himself and other inventors‡ so that the commutator motor both for single and polyphase currents has become a practical machine. The space diagram Fig. 159 shows how the e.m.f. of self-induction may be balanced by an e.m.f. of rotation. The left side of the figure applies to the moment when the working current i is a positive maximum, the right side when it passes through zero to become negative. Below each figure is shown the time diagram. The symbols are the same as used before and a direction from left to right or upwards signifies a positive value in the space diagram. Let an armature not surrounded by a stator be rotated counter-clockwise and let an alternating current i be sent through it by the brushes b_1 and b_2 . In addition to these apply two other brushes B_1 and B_2 in quadrature which are short-circuited as shown. The working flux Φ generates an e.m.f. of rotation e_r which at the moment

* *Industrie Electrique*, 1901 and 1902.

† German Patent 61951 of 1901.

‡ Winter and Eichberg, German Patent 153730 of 1901; Roth, French Patent 325250, 1902; Blondel, French Patent 327414, 1902. See also *E.T.Z.* 1904, p. 75.

to which the left space diagram refers has crest value and being directed to the left is negative. In the time diagram the vector of e_r must therefore be directed downwards. This e.m.f. generates the short circuit current through the brushes B_1B_2 and as the armature has inductance there must be a time lag of 90° , so that the vector of this short circuit current I must be drawn to the left (ohmic and iron losses neglected). We indicate I in the space diagram by an arrow drawn to the left and a little circle in the shaft to indicate that it is zero at the moment, but on the point of becoming negative. The current I generates a flux F co-phasal with it and this with counter-clockwise rotation generates the e.m.f. E in the vertical axis. This at the moment passes through zero to become negative.

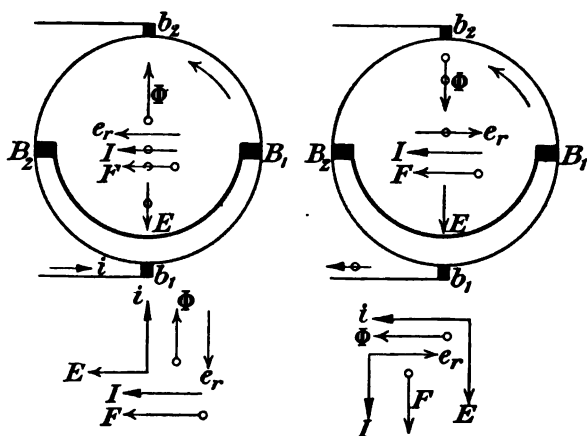


FIG. 159.

In the time diagram it will therefore have to be represented by the vector E directed to the left. We thus find that the e.m.f. generated by rotation in the axis of the b_1b_2 brushes leads by 90° over the current i . The lagging wattless component of the e.m.f. which is the source of the current i is therefore $\omega Li - E$ and since E is proportional to i and the speed of rotation, there is a definite speed at which the wattless e.m.f. component vanishes and the rotating armature behaves merely as an inductionless resistance. At a lower speed it acts as a choking coil; at a higher speed it acts as a condenser.

The flux F , changing sign at the moment, generates a transformer e.m.f. in the coils short-circuited by the brushes b_1b_2 . This is directed upwards under b_2 and downwards under b_1 . On the other

hand the rotation of these conductors in the field Φ generates e.m.f.'s in the opposite sense and if these balance we obtain good commutation. The transformer e.m.f. of one short-circuited turn is $2\pi fF$ and the rotational e.m.f. is $2\pi u\Phi$, if u is speed of rotation. The two e.m.f.'s balance if $fF = u\Phi$.

Let $c = \frac{z}{4\pi}$ represent the number of turns comprised in unit angle of armature winding and $\Phi \sin \alpha$ the flux interlinked with an elementary group of coils making the angle α with the Φ axis. The crest value of self-induced e.m.f. in the elementary group is $de = 2\pi f\Phi \sin \alpha \left(\frac{z d\alpha}{4\pi}\right)$ and this integrated from $\alpha = 0$ to $\alpha = \pi$ so as to include all the $z/2$ turns of the winding gives the total e.m.f. of self-induction $f\Phi z$

The rotational e.m.f. is uFz and $\Phi z = uF$ is therefore the condition for unity power factor.

The diagram on the right of Fig. 159 refers to the moment when the current i goes through zero to become negative. The diagram is self-explanatory, the symbols and method of notation being the same as in the previous case. It will be noted that now there is a transformer e.m.f. in the coils under the B brushes due to Φ and this may also be balanced by a rotational e.m.f. provided the relation $uF = f\Phi$ holds good. The condition for unity power factor will insure sparkless commutation at the short-circuited brushes, whilst the relation $fF = u\Phi$ will insure sparkless commutation under the working brushes. It is obvious that both conditions can only be fulfilled simultaneously if the armature is driven at synchronous speed when Φ and F will naturally become equal. This follows from $e_r = u\Phi z$ and also $e_r = fFz$. A machine of the kind shown in Fig. 159

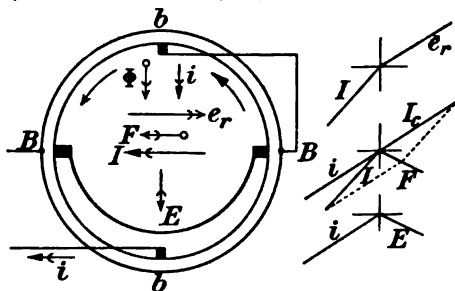


FIG. 160.

cannot work as a motor because the supply current i and impressed e.m.f. E are in time quadrature; the diagram is here merely used to show the general principle of compensating a self-induced e.m.f. by a rotational e.m.f. and thus improving the power factor.

To make the machine work as a motor we must introduce in the B_1B_2

axis a working current as near as possible co-phasal with the exciting current i and that is done according to Latour by arranging the armature as the secondary of a transformer, the primary of which is formed by the stator. In Fig. 160 the outer circle represents the stator, the inner circle the armature. Both are in series as far as the bb axis of the armature is concerned, but the brushes on the BB axis are simply short-circuited. Let i be the armature current in the bb axis, I_s be the current through the stator (in this case $I_s = -i$ because of the series connection) and I the current through the armature along the BB axis. The time diagram for the currents is shown on the right of the figure for the moment when I_s is positive and still growing. I may be considered the secondary and I_s the primary current of a transformer consisting of stator and armature winding and the two currents together produce the resultant flux F which leads over I by a little more than a quarter period. This flux is negative and decreasing at the moment. A strong counter-clockwise torque is produced by I and Φ and a weak clockwise torque by i and F . The armature rotates therefore counter-clockwise. The e.m.f. e_r produced by rotation in the flux Φ opposes the current I and this is the condition that the machine works as a motor. From the upper time diagram on the right it will be seen that e_r has a small wattless leading component with respect to I . The lower diagram shows that E , which is produced by rotation in the flux F , has a large leading component with respect to i . Thus both these leading components operate to improve the power factor. At the same time they also improve the commutation as explained with reference to Fig. 159. This is the principle of phase compensation discovered independently of each other by Latour and by Winter and Eichberg, but the last-named inventors have gone a step further in its practical application by making currents and fluxes adjustable so as to obtain the best working conditions at different speeds and loads. They also recognised that a motor of the type shown in Fig. 160 cannot be used on a high voltage circuit because the armature is directly connected to the line. Since transformation is therefore necessary anyhow they use transformers with tappings so as to vary the voltage impressed on the bb and that impressed on the BB axes independently. Fig. 161 is a diagram of the Winter-Eichberg motor. A transformer T_1 brings down the voltage of the supply to a value suitable for the stator winding and this voltage is again reduced by the transformer T to a value suitable

for the rotor winding. The forward torque being due to the armature

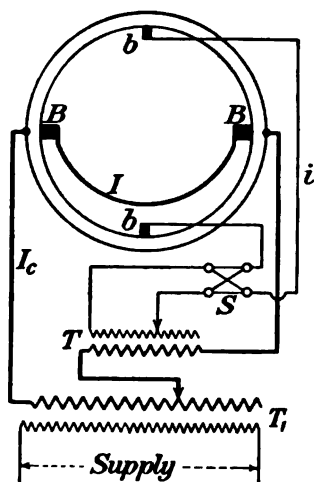


FIG. 161. Winter-Eichberg motor.

current I and the stator field Φ it is obvious that by using a step down transformer and thus increasing I_c and therefore I we obtain a larger output than in the case of Fig. 160. At low speeds (and also at the start) the regulating switch of the first transformer is put to the left so as to increase I and therefore also the flux F . The condition for good commutation $u = f \frac{\Phi}{F}$ at the brushes B , through which much the largest current passes, is therefore automatically approached and can be more accurately fulfilled by adjusting the regulating switch over the tapplings of the second transformer T . A reversing switch S is added if the motor is to be reversible. It will be noted that the working of this motor depends mainly on transformer action; a reasonably high frequency of the order of 40 or 50 cycles per second is therefore advantageous.

The Three Phase Series Commutator Motor. In mechanical arrangement this type of motor resembles an induction motor, but with this difference that the rotor instead of having slip rings is provided with a commutator and three sets of brushes. The winding is that of a D.C. armature. The stator may be wound for salient poles, though a continuous winding is preferable and we shall here assume that the field produced by such a winding is sinusoidal. The rotor winding forms necessarily a mesh connection and as this is to be in series with the stator the latter must have three distinct windings forming a star. This leads to some complication in the winding (though quite feasible) and a somewhat simpler arrangement is obtained by the use of a three phase series transformer which makes it possible to use mesh connection also for the stator. If the motor is to be designed for high voltage a transformer becomes necessary anyhow and by putting the primary in series with the stator and connecting the secondary to the brushes we obtain proportionality between rotor and stator ampereturns

at all times as if stator and rotor were directly series coupled. Whether the rotor is standing or running the flux produced by the rotor ampere-turns revolves in space with an angular speed corresponding to the frequency; the flux produced by the stator ampere-turns also revolves in space with the same angular speed and both fluxes preserve their angular displacement, which is determined by the position of the brushes relatively to the terminals on the stator winding. The flux actually existing is the resultant of stator and rotor flux. The flux due to the rotor current alone cannot produce torque, nor can it produce an e.m.f. on the brushes having a watt component. It does produce a wattless e.m.f., leading if the speed exceeds synchronism and lagging if it falls short of it. Since the rotor flux has no effect on either torque or watt component of induced e.m.f. these quantities may be determined by reference to the stator flux alone. Under the convention as to sense of winding explained before, flux and the current which produces it have the same direction. Thus if the current in the A phase at a particular moment has crest value the axis of the stator flux must be drawn in the space diagram from the centre to the A terminal for a negative and from the A terminal to the centre for a positive crest value of current. The magnitude of this flux is 1.5 times the crest value of the flux produced by one phase alone. The same applies to the rotor flux, the direction of which is given by the corresponding brush. The two fluxes being constant and revolving at the same speed, it follows that the resultant flux is also constant and preserves its relative position to the current vectors in the time diagram. In addition to the resultant flux interlinking with stator and rotor windings there are also small fluxes interlinked only with one or the other winding due to slot and head leakage which inject a lagging e.m.f. at all times.

Let in Fig. 162 the outer circle represent the mesh connected three phase winding of a two pole stator and the inner circle the armature. ABC are the terminals of the stator and abc are the brushes. For the sake of simplicity we assume that the transformer has a one to one ratio so that stator and rotor current are the same, and the fluxes produced are then proportional to the number of turns in each circuit. We neglect iron losses, assume the magnetic reluctance to be constant and we also assume that the flux distribution is sinusoidal, which means that we neglect upper harmonics. The moment for which the space diagram is drawn is that at which

with a sequence ABC the current in the A phase has crest value. Let S be stator and R rotor turns, then the resultant flux Φ is found by drawing the parallelogram with sides S and R . With a brush displacement α we have the relation $\tan \theta = ed/Oe$ and since $ed = R - S \cos \alpha$ and $Oe = S \sin \alpha$ we can write $\tan \theta = \frac{R - S \cos \alpha}{S \sin \alpha}$. Let $a = R/S$, the ratio between rotor and stator windings, then

$$\tan \theta = \frac{a - \cos \alpha}{\sin \alpha}$$

The angle θ is therefore only dependent on the brush position, whatever may be the working condition of the motor. The total rotational e.m.f. is proportional to the flux Φ and the speed and

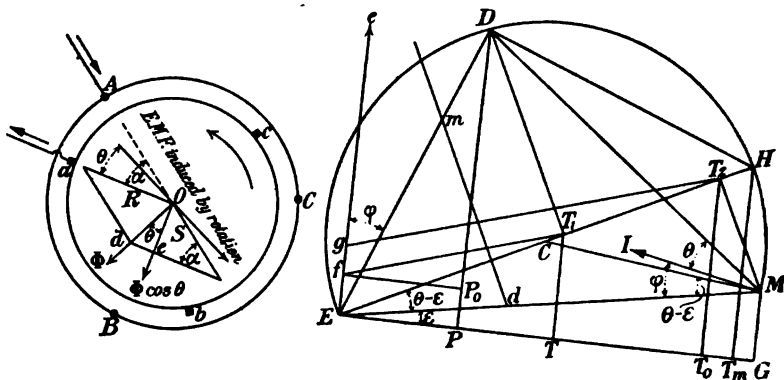


FIG. 162. Circle diagram of series motor.

since, by reason of constant reluctance, current and flux are proportional, the rotational e.m.f. is proportional to current and speed. The curved arrow in the space diagram on the left of Fig. 162 shows the direction of rotation and the straight arrow that of the total e.m.f. induced by rotation. Of this only the component which is due to $\Phi \cos \theta$ is a watt component, the other due to $\Phi \sin \theta$ is wattless and leading. In the time diagram on the right the arrow marked I is the vector of the current in the A phase when this has crest value and the line MD represents that component of the total impressed e.m.f. ME which is required to balance the e.m.f. of rotation. The impressed e.m.f. ME must also have another component which balances the impedance e.m.f. If ρ is the resistance of phase A (stator and rotor) and L the inductance at the particular

angle at which the brushes are set, DE forms with the current vector the angle $90 + \epsilon$, where ϵ is defined by $\tan \epsilon = \rho/\omega L$. If no current were passing through the rotor the self-induced flux in the stator would be

$$\Phi_s = \frac{0.58}{2\delta} \frac{S}{2p} \tau I$$

This expression is obtained by a method similar to that used in Chapter VIII and Fig. 93 and the calculation is so simple that it need not be given in detail. δ is the air gap augmented by an appropriate amount to take into account the reduction of the gap surface due to its being interrupted by slots and if desired the increased reluctance of the magnetic circuit due to saturation can also be represented by an increase in the air gap. If we put for the stator

$$c_1 = \frac{0.58 \cdot \tau l}{2\delta \cdot 2p}$$

then the self-induced flux may be expressed by

$$\Phi_s = c_1 SI$$

Similarly we have for the rotor, if the stator carries no current,

$$\Phi_R = c_1 RI$$

The flux actually existing when both members carry current is Φ , the vectorial resultant of these two as shown in the space diagram. To find the inductance for any particular setting of the brushes we must combine the vectors of self-induced e.m.f. in the two members, for which purpose it is necessary to find the inductance of each member separately. Φ_s rotating with a speed corresponding to the frequency cuts the stator wires and produces the e.m.f. of self-induction which is $\omega L_s I$. The actual value of this e.m.f. can be easily calculated from the flux induced, the total number S of stator wires and the type of winding. Since the coil side is $\frac{2}{3}$ the pitch the coefficient k is 1.84 and the e.m.f. is in absolute measure

$$1.84 \frac{\omega}{2\pi} \Phi_s \frac{S}{3}$$

Or we may determine the crest value of the e.m.f. from the fundamental equation $e = Blv$. The calculation in either case is so simple that it need not be given in detail. Let

$$c = \frac{0.0573}{2\delta \cdot 2p} \tau l 10^{-8}$$

then the inductance of the stator taken separately is in henrys

$$L_s = cS^2$$

and that of the rotor is

$$L_r = cR^2$$

the inductance of the two in series is therefore

$$L = k \sqrt{S^2 + R^2 - 2SR \cos \alpha}$$

where k is a constant.

This is, however, not the whole inductance of the machine since slot and head leakage also increases the e.m.f. of self-induction. These leakages are proportional to the current and do not alter with the position of the brushes. The total inductance has therefore two components, one is a constant and the other and larger depends on the brush angle. The larger α the greater is the inductance and the smaller the current at stand-still. For $\alpha = \text{zero}$ the inductance is a minimum and the short circuit current a maximum. An objection might perhaps be raised against the above argument on the ground that we have treated the rotor as if it were at stand-still, whereas in reality it rotates and its wires are cut by the self-induced field at a slower rate than that on which the formula for L is based. The answer is that we calculate the e.m.f. of rotation as if the field were at stand-still and that therefore the two errors cancel. The advantage of this method of treatment is that the impedance e.m.f. is made independent of speed so that the length of the impedance vector ED is simply proportional to the current and may with a suitable change of scale be made to represent the magnitude of the current. The triangle EDM can now be drawn. DE represents impedance volts and is inclined to I by the angle $90 + \epsilon$ and therefore to MD (rotational volts) by the angle $90 + \epsilon - \theta$ which is constant. The point D must therefore have its locus on a circle of which EM is a chord. The centre C is found by drawing lines from E and M under the angle $\theta - \epsilon$. The current I lags behind the impressed e.m.f. ME by the angle ϕ . If we wish to represent the current by the line ED not only in magnitude but also in direction, we must swivel the current vector clockwise through the angle $90 + \epsilon$ and the e.m.f. vector by the same amount. ME thus takes the position Ee and the current vector takes the position ED . The torque being given by the product: flux, current and $\cos \theta$ may be represented in the diagram by a scalar quantity. The flux is proportional to the current ED and the product of flux and current is therefore proportional to \overline{ED}^2 . Drop a perpendicular from the working point D on to the diameter EH and obtain the point T_1 . By a well-known

law of the circle $ET_1 \times EH = (\overline{ED})^2$. Since EH is a constant ET_1 represents the square of the current and if we drop from T_1 a perpendicular on to the line EG we find $ET = ET_1 \cos \theta$. ET represents therefore to a certain scale the torque in kgm. At stand-still the point D goes to M and the construction for the torque gives ET_0 . At some low speed the working point goes to H and the torque is ET_m . It will be seen that $ET_m > ET_0$. Maximum torque is not exerted at stand-still, but at some low speed. This means that the motor is unstable at very low speeds. The speed for any given torque beyond the unstable condition may also be represented by a scalar quantity. Draw from any point such as d on the chord EM a straight line at right angles to the diameter EH . Angle $Edm = \text{angle } EDM$. The two triangles EDM and Edm have the angle at E in common and the angles at D and d are equal. The triangles are therefore similar and we have the relation $\frac{md}{Ed} = \frac{MD}{ED}$.

MD is the rotational e.m.f. and proportional to the product of flux and speed and, since flux is proportional to current, MD is also proportional to current and speed, whilst ED is proportional to current. The ratio MD/ED is therefore a measure of the speed and since Ed is constant dm gives the speed to a certain scale. To find the speed for any given torque ET we erect a vertical in T to the line EG . Where this meets the diameter as in T_1 draw a vertical to EH and prolong to the circle thus finding the point D . Join D to E and where this cuts the speed line from d is the point m and dm is a measure for the speed. The input is obviously proportional to the height of D over the line EG . Thus DP is a measure for the input. To get the output we reason as follows: Output is the difference between input and loss. At stand-still the output is zero and the whole of the input is loss. This is given by the line MG . Since the loss is proportional to the square of the current, that is to the length ET_2 , we may take this line or any line proportional to it as the measure of the loss. Make $Eg = MG$ and draw T_2g . From T_1 draw a line parallel to T_2g and find the point f ; then Ef is the loss corresponding to the torque ET and to the working point D . Draw fP_0 parallel to EG and thus find P_0 . The output is then given by the scalar quantity DP_0 to the same scale in which DP represents the input. To summarise:

EM = stand-still or short circuit current,	ET = torque,
ED = working current,	DP = input,
MD = rotational e.m.f.,	DP_0 = output, dm = speed.

The diagram is only valid for one particular setting of brushes. A new diagram is required for every brush position. With a fixed brush position the speed increases as the torque is diminished and

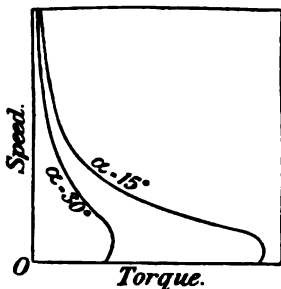


FIG. 163. Torque-speed characteristic of three-phase series commutator motor.

for a given torque the speed is reduced as the brush angle is increased. There is a torque-speed characteristic for every brush angle so that by an appropriate adjustment of the brushes the motor may be so regulated as to give a definite torque at a definite speed. The machine has the same character as a D.C. series motor, but with this difference that it has an indefinite number of torque-speed characteristics according to the brush setting. Two such curves are shown in Fig. 163

for brush angles of 15 and 30 degrees*. The A.C. motor has an advantage over the D.C. type inasmuch as no starting resistance is required. By shifting the brushes so as to make $\alpha = 180^\circ$ (electrical) the inductance is so much increased as to keep the first rush of current quite small. With this brush position there is of course no torque. If now the brush angle is slowly diminished the starting current will slightly increase, but still remain moderate, whilst a strong torque is developed. On the other hand the commutation is more difficult than in a D.C. motor. To get a good power factor the flux component in the brush axis must have the same direction as the armature current which means a great reactance voltage. To get good commutation the stator should have the stronger winding, but then the power factor is not so good. The use of interpoles is impossible with movable brushes, but if we keep the brushes fixed and provide the stator winding with tapping points we may still have the advantage of adjusting the brush angle to suit various working conditions and at the same time employ a strong rotor winding so as to get a good power factor. In this case interpoles become possible and therewith an improvement in the commutating conditions.

* Reproduced in part from N. Shuttleworth's paper "Polyphase commutator machines and their application," *Journal Inst. El. Eng.*, No. 244, p. 439, to which publication the reader is also referred for an account of the shunt type of motor.

CHAPTER XV

PHASE ADVANCING

Commercial importance of a good power factor—Methods of obtaining a good power factor—Static condensers used as phase advancers—Economical limit of increasing the power factor of a system as a whole—Improving the power factor of individual induction motors—Advantage of injecting the leading kva. into the rotor—Phase advancers of the rotary type—Working diagram of induction motor fitted with phase advancer—Vibratory type of phase advancer—Economical limit of compensation of an individual induction motor—Phase advancing in connection with speed regulation—Tariffs and power factor.

Commercial Importance of a Good Power Factor. The plant capacity of generators, transformers and lines depends on the kva., their earning capacity on the actual kw. transmitted and sold. Thus if a certain undertaking has a power factor of 0.6 and it were possible to raise it to 0.9 the same plant could deliver 50 per cent. more energy during the year with a corresponding increase in earnings. Since the same electrical plant is used there is no increase of working cost on account of interest, sinking fund and labour, the only extra expenditure being that for fuel.

The loss of power by ohmic resistance being proportional to the square of the current it follows that the efficiency of the whole system is improved if the power factor is raised and there is the further advantage that the regulation is also improved.

Methods of Obtaining a Good Power Factor. An obvious method is to use only such consuming devices which in themselves have a good power factor. Thus if all the current is used for incandescent lighting the power factor of the whole system will be very nearly unity. This, however, is an exceptional case. Generally the bulk of the current is used for motive power, thermal or electro-chemical purposes and such consuming devices together with the regulating apparatus cannot always be arranged so as to take current at unity power factor. With motors of the synchronous type this is possible and, as has been shown in Chapter X, it is also possible with converters, but for industrial work the motor most in use is of the asynchronous type and thus the power factor of

the whole system must fall considerably short of unity. This also is the case where arc lamps are on the circuit, because such lamps must have some electromagnetic regulating gear and consequently a sensible inductance. Although large motors can be built having a power factor of 0.85 to 0.90 at full load it is unlikely that so high a power factor can be obtained over the system as a whole because at any given time some of the motors will work underloaded, so that cases of a 0.7 or even 0.6 power factor over the whole system are not impossible.

To improve this condition there are two ways; we may either add some phase advancer to each individual motor or we may connect to the system some apparatus which takes current with a pronounced leading component. The right place for such an additional apparatus is obviously at the far end of the line and as near as possible to the centre of delivery, for if it were put at the home end near the generators it would only relieve them, but not the line and the step down transformers of wattless current. A synchronous over-excited and idle-running motor may be used for this purpose, but it will be easily seen that such an arrangement can hardly be justified on financial grounds*. The permanent losses in such a motor will be at least 5 and may reach 10 per cent. of its kva. capacity; its cost per kva. will be of the same order as that of a generator and may considerably exceed it in the case when the generator is a large turbo alternator, so that the money value of the additional output obtainable from the generator may be actually less than the cost of the apparatus to which this additional output is due. There is further the loss of energy which, with an apparatus requiring 5 to 10 per cent. of its kva. for its own working, must be a fairly large item in comparison with the energy actually sold during the year. An idle running motor as a means of correcting the power factor may therefore be dismissed as not very promising. The case is better if the motor may at the same time be used as a source of mechanical power and this is done with converters for traction services, but as a general proposition this method if applied to industrial supply will be found rather expensive and wasteful. To make phase advancing as applied to the system as a whole a commercial proposition the apparatus should have a very small internal loss, require little attention and its cost should be appreciably

* It is nevertheless used in certain cases, but then rather with a view to obtaining better regulation than setting free saleable power.

less than the financial value of the kva. of generating and distributing plant set free for sale as kw. to additional customers. On circuits of the usual commercial frequency of about 50 cycles and 500 to 600 volts on the secondary of the transformers these conditions are admirably fulfilled by

Static Condensers used as Phase Advancers. Thanks to the method of construction invented by Mr Mansbridge in 1900 for telephonic work and subsequently developed by the British Insulated and Helsby Cables, Ltd., for power circuits (Patent 14817 of 1914), condensers are now obtainable at a reasonable price per mf. and able to stand a working pressure up to 650 volts. Since the kva. output of a condenser varies as the square of the voltage this is the most economical voltage for use*. I am indebted to this firm for the following particulars as to price (at the present time, viz. July, 1917) per kva. at 50 cycles and different voltages. The prices quoted are for the condensers only in their frames and do not include switch gear.

	£	s.	d.
Cost per kva. at 600 volts 50 cycles	1	15	0
„ „ 550 volts 50 cycles	2	15	0
„ „ 440 volts 50 cycles	3	18	0

The internal loss of a nest of condensers is given by the makers as 0.3 to 0.4 per cent. of the kva. at 16° C. Since it is generally possible to arrange the condensers so that they are switched on and off with the motor, the loss is quite insignificant if condensers are used with individual motors. Even if they are used on a system the power factor of which has to be improved as a whole, the loss of less than $\frac{1}{2}$ per cent. of the kva. to be injected during the time that the system has to be kept alive is not a very serious matter.

Economical Limit of Increasing the Power Factor of a System as a whole. The question how far it is advisable to go in the improvement of the power factor of a system as a whole is of considerable commercial importance. Let P represent the plant capacity in kva. of generators, transformers and line, φ_0 the phase angle before and φ that after improvement of power factor, then $P (\cos \varphi - \cos \varphi_0)$ is the gain in saleable power due to the installation of condensers of a total kva. capacity of $P (\sin \varphi_0 - \sin \varphi)$.

* Where the supply voltage is lower a reactance coil may be put into the condenser circuit to bring up the voltage to this figure by "partial resonance."

Let K be the capital outlay per kva. of the electrical part of the original plant, k that of the condensers per kva.; and $\alpha = \frac{k}{K}$, the ratio between the two, then phase advancing will be a commercial proposition if

$$\alpha < \frac{\cos \varphi - \cos \varphi_0}{\sin \varphi_0 - \sin \varphi}$$

It will be seen from this inequality that the extent to which the improvement of the power factor is commercially justified depends

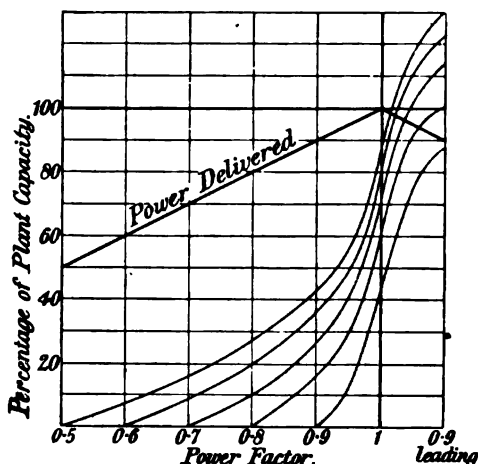


FIG. 164. Relation between gain in saleable power and kva. of phase advancer.

on two things, namely the ratio of the cost of the phase advancer to that of the electrical plant per kva. and the original power factor. The value of α depends on the particular features of each plant. Knowing this we can determine the relation between gain in power and cost of the condenser. To facilitate the commercial investigation the curves in Fig. 164 may be used. The sloping

lines represent power delivered with a plant of 100 kva. at power factors varying from 0.5 lagging to 1 and 0.9 leading. The ordinates of the curves are kva. to be injected. The use of this diagram may be explained by an example. Let the original power factor be 0.7 and assume that this is to be improved to 0.9. By following the curve starting at the abscissa 0.7 until the ordinate through 0.9 is met we find that the kva. capacity of the condenser must be 27. To add 20 kw. to the saleable power we must provide a condenser of 27 kva. If we wished to bring the power factor up to unity, that is to add 30 kw. to the saleable power, we would have to provide a condenser of 70 kva. plant capacity. In this case the 10 additional kw. of power would have to be purchased at the capital expenditure of an additional 43 kva. of condenser plant. If a power factor of 0.7 lagging is to be changed into one of 0.9 leading, the condenser kva.

would have to be 114. It is obvious that it is commercially unsound to reduce the phase angle to a negative value and it is even questionable whether a reduction to zero is advisable. In practice it is customary to be content with an improvement of the power factor to 0.9 or 0.95.

Improving the Power Factor of Individual Induction Motors. The circle diagram shows that the power factor of an induction motor is reduced if the load is lessened. It also shows that with a reduction of impressed voltage the whole diagram becomes smaller and a reduced load can be carried with about the same maximum power factor as the full load at full voltage. The power factor at reduced load may therefore be improved by reducing the impressed voltage. There is no difficulty in doing this, since a transformer must be used in any case and the only addition required is a regulating switch in conjunction with tappings on the low pressure side of the transformer. This method of improving the power factor at reduced load is simple, and may effect a reduction in the bill for electrical energy taken by individual consumers, but it does only imperfectly meet the condition that the plant as a whole shall be able to deliver more power because it is only effective in underloaded motors, which are not taking much current. The fully loaded motors take the maximum of wattless current and the improvement of the general power factor is therefore merely due to a small reduction in the wattless component of current taken by motors which do not load the plant to any great extent. What we require is that the fully loaded motor shall have a good power factor.

Several inventors have endeavoured to solve this problem by constructing the induction motor in such a way that after it has been started it is transformed into a synchronous motor, the rotor being excited with D.C. and becoming the field magnet, whilst the stator acts as armature. Apart from the complication in switch gear, the provision of a large, low voltage existing current and the difficulty of making the motor jump into step, there is the fundamental objection that in order to obtain a good power factor the ampereturns in the rotor must be two or three times as many as in the stator. In an ordinary induction motor the ampereturns in both circuits are about equal. To make such a motor suitable for synchronous operation it is therefore necessary

to provide for a very much heavier electrical loading of the rotor and this means a large increase in rotor copper and, to provide space for this, a sensible increase in the diameter. A motor of this kind must therefore be considerably larger and more expensive than an ordinary induction motor.

Advantage of Injecting the Leading kva. into the Rotor. In discussing the injection of kva. dynamically into the supply circuit it has been pointed out that this method is wasteful of power and that a better way is the employment of condensers. The condenser in this case supplies the magnetising current required by the stator, thus relieving the line from the wattless component. Since the condenser is worked at the full frequency of the supply (generally about 50) and the full voltage of the motor, the capacity of the condenser is not excessive and phase advancing by condenser becomes a commercial proposition. Now it is obviously immaterial into what part of the apparatus the leading kva. are injected. If we select for this purpose the rotor, that is if we magnetise the rotor from an external source, we relieve the line equally from the duty of supplying wattless magnetising current and thus improve the power factor, but it will easily be seen that a condenser is quite unsuitable for this purpose. Both frequency and e.m.f. in the rotor are exceedingly low and to supply the large magnetising current required in the rotor a condenser of prodigious dimensions and prohibitive cost would be necessary. On the other hand these conditions are all in favour of some dynamic source of leading kva., that is of some moving apparatus supplying exciting current to the rotor. The phase advancer in this case becomes simply a small exciter supplying A.C. to the rotor and taking the place of a D.C. exciter in a synchronous motor. The advantage of supplying the exciting current to the rotor instead of to the stator lies in this, that it has to be supplied at a very much lower voltage and consequently the total kva. becomes only a small fraction of the amount that would have to be injected by a condenser into the stator. An example will make this point clear. Let the input at full load be 300 kw. and the natural power factor of the motor be 0.8 which corresponds to a phase angle of 37° . To bring the power factor to unity by injection of kva. into the stator circuit the amount required would be $300 \times \tan 37^\circ = 225$ kva. By impressing an e.m.f. of slip frequency on the rotor the slip will be increased since the slip

e.m.f. must now contain components to cover the additional ohmic loss of the exciter, the leading e.m.f. produced by it and there may also be a slight increase of rotor current due to the magnetising component. With a natural slip of two per cent. at full load this may thus be increased to 3 or 3.5 per cent. The total kva. to be injected into the rotor will therefore be 3.5 per cent., or, making a liberal allowance for the increased rotor current, 4 per cent. of $225 = 9$ kva. The kva. capacity of the exciter is therefore only 3 per cent. of the power of the motor. The kva. capacity of an equivalent condenser connected to the stator would be 75 per cent. of the power of the motor and its cost would be very much greater. But the dynamic phase advancer has also another advantage over the condenser. By attaching a condenser to the line we merely improve the power factor of the line, but leave the motor unaltered. Its overload capacity is therefore not improved. If we excite the rotor by a dynamic phase advancer, we not only improve the power factor of the line, but we also increase the overload capacity of the motor, and reduce the primary copper heat at full load. The restrictions imposed on the designer as regards temperature rise apply mainly to the stator and if we can relieve the stator winding of the magnetising component we reduce stator losses and improve the efficiency. The reduction of loss in stator copper is of the same order as the internal losses in the exciter, so that phase advancing is obtained without sensible expenditure of energy. Where a motor is subjected to occasional overloads of short duration advantage may be taken of the increase of stalling torque which results from rotor excitation and a smaller size may be used. This means a saving in capital outlay which may partly or wholly compensate for the cost of the exciter and it also means a reduction in iron losses.

Phase Advancers of the Rotary Type. In the last chapter it has been shown that a three phase commutator motor can be built so as to have a good power factor. The lower the frequency the shorter becomes the vector DE in Fig. 162 and thus with the very low frequency of the rotor current it becomes possible to so construct the machine that the angle ϕ is not only made zero but negative. The motor which acts as a phase advancer will then take power from the slip rings and give back part of it mechanically to the shaft of the main motor if it is geared to it. The phase

advancer may be either a shunt or series machine; the latter arrangement is diagrammatically shown in Fig. 165. M is the

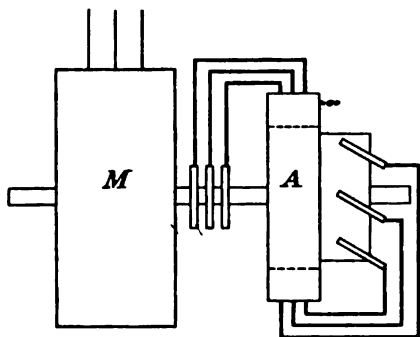


Fig. 165. Cascade connection of induction and commutator motor.

main motor and mounted on its shaft is the advancer A , which is a commutator machine. It is obvious that by adjustment of brush position (angle α in Fig. 162) the working condition of the advancer may be adapted to call for a certain amount of electrical power from the slip rings which means that the slip e.m.f. and therefore the slip may be increased, thus

providing for a certain reduction of speed without the wasteful expedient of inserting resistance into the rotor of M . In its widest application the arrangement shown in Fig. 165 provides not only for a good power factor, but also for a certain amount of speed regulation. Confining, however, the discussion at present merely to the problem of phase advancing it will be clear that the advancer need not give out mechanical power and then the slip will only be increased by a small amount, namely that corresponding to the ohmic drop in the advancer and the wattless e.m.f. component injected. If an increase of slip is undesirable this condition can be met by so constructing the advancer that it acts as a generator taking mechanically from the main motor the small amount of power which corresponds to the increase of slip voltage, but the slip cannot be reduced to zero because a certain slip is necessary for the main motor to exert torque.

Professor Miles Walker has developed two types of rotary phase advancer*. In the earlier type the armature had an open coil winding and very wide brushes were used. In the later type the armature has the usual re-entrant multipolar lap winding and the field winding is distributed between the three phases in such a manner as to compensate not only for the armature ampereturns, but also to provide a commutating field to ensure sparkless working. For details of this ingenious winding the reader is referred to the original paper; here we are only concerned with the general application

* Miles Walker, *Journal Inst. El. Eng.*, Part 195, p. 599 and Part 218, p. 327.

of the broad principle that by means of a rotating armature a three phase leading e.m.f. can be injected into the rotor circuit if the field is produced by the rotor current and the speed exceeds that corresponding to synchronism.

Suppose that the angle α in Fig. 162 is increased to 180° ; this means that the resultant field Φ is the sum of the fields produced by stator and rotor winding. With the sequence A, B, C this field revolves counter-clockwise with a speed corresponding to the slip ring frequency. Since current and flux are in line no torque is exerted and at stand-still the machine acts simply as a choking coil. The e.m.f. axis in a rotating armature is in quadrature with the field axis and the e.m.f. leads in the direction of rotation of the winding. If the winding stands still and the field rotates as in this case, the e.m.f. is also in quadrature, but lags. Now let the armature be driven by external mechanical power. The e.m.f. induced in the armature winding, being proportional to the relative speed between winding and field, will now decrease and if the speed be augmented to synchronism vanish alto-

gether, but the e.m.f. induced in the stationary stator winding will not be affected. If the armature be driven with a speed greater than synchronism the e.m.f. induced in the brush axis will be in quadrature and leading and therefore opposed to the e.m.f. induced in the stator winding. The latter is thus always undesirable and had better be suppressed altogether which simply means that there should be stator iron, but no stator

winding. This is the principle on which the Le Blanc phase advancer is constructed; it is shown diagrammatically in Fig. 166. R is the armature rotated by externally supplied power in the stator S . The slip rings of the motor are connected to the three brushes A, B, C so that the armature forms the start point of the rotor winding. To facilitate commutation the stator iron is cut out in the region of the short circuited coils.

If this expedient of facilitating commutation need not be used, that is in machines of small power, then the advancer may have a continuous ring of iron round the armature. In this case it becomes possible to dispense altogether with a stator by extending the

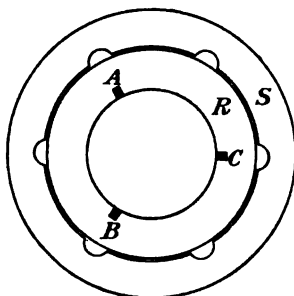


FIG. 166. The Le Blanc phase advancer.

armature iron beyond the winding. This modification is due to Dr Scherbius, whose phase advancer is shown in Fig. 167. The

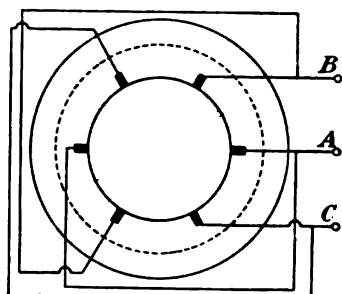


FIG. 167. The Scherbius phase advancer.

armature has a larger diameter than the winding which is indicated by the dotted circle. The winding is put through tunnels in the armature iron and there is no stator. The flux produced by the armature current closes through the external iron ring and as the magnetic reluctance of an air gap is thus avoided we obtain for the same magnetising current a stronger field. On the other hand commutation is made

more difficult and must be provided for by hard brushes. The figure shows a four pole armature with lap winding. The brushes are fixed and the flux rotates in space with a speed corresponding to the slip frequency. The armature may be driven by an independent source of mechanical power, or geared to the motor or fixed on the same shaft. Let Φ be the flux in megalines produced by the rotor current I , and z the number of armature conductors, then the crest value of e.m.f. over an electrical diameter of the winding is for a lapwound armature $\Phi \frac{z}{100} u$, where u is revs. per second difference between the mechanical speed of the winding and the speed of the rotating flux. The relation between current and flux is given by the magnetisation characteristic of the phase advancer. The speed u depends on the frequency of the rotor current; it decreases slightly as the load increases, but at the same time the flux increases with the load so that the injected leading e.m.f. increases with the load.

Let f_1 be the frequency of supply, $2p$ the number of poles in the motor, $2p_a$ that in the advancer and σ the slip, then with the advancer mounted on the motor shaft it runs at $(1 - \sigma) \frac{f_1}{p}$ r.p.s. The frequency of the rotor current is $f = \sigma f_1$ and the rotating field Φ makes $\frac{\sigma f_1}{p_a}$ r.p.s. Thus

$$u = (1 - \sigma) \frac{f_1}{p} - \frac{\sigma f_1}{p_a}$$

The star value of the injected e.m.f. is half the diametral value and the effective value is 0.71 of the crest value. We thus find for the star value of effective volts injected

$$e = 0.355 \Phi \frac{z}{100} \left((1 - \sigma) \frac{f_1}{p} - \frac{\sigma f_1}{p_a} \right). \quad (144)$$

and if the advancer is driven by a separate motor at the constant speed of u r.p.s.

$$e = 0.355 \Phi \frac{z}{100} \left(u - \frac{\sigma f_1}{p_a} \right). \quad (145)$$

It will be seen from these equations that the e.m.f. injected depends on two things, the flux which is produced by the current I and the slip which is primarily due to the load, but is slightly augmented by the action of the advancer. The slip again influences the current so that the relations between current and injected e.m.f., or load and phase angle are rather complicated and cannot be expressed in simple algebraic formulae. But by making certain assumptions it is possible to solve the problem graphically as a first approximation. These assumptions are that the flux is proportional to the current, which means neglect of saturation, and that the slip (which is a very small fraction anyhow) is constant. Under these assumptions we get simple proportionality between current and injected e.m.f., or $e = kI$. Having found k in a first approximation we can then by a trial and error method correct for saturation and the actual slip*.

Working Diagram of Induction Motor fitted with Phase Advancer. It has been shown in Fig. 128 that the locus of the vector representing the current supplied to a "general transformer" and therefore also to an induction motor is a circle, provided saturation may be neglected. In Fig. 128 we assumed that the secondary current has a constant lag behind the induced secondary e.m.f.; in the special case of the induction motor we assumed this lag to be zero and now, since a leading e.m.f. is injected, we must assume this lag to be changed into a lead. This is the only alteration to be made in Fig. 128 and we thus obtain the circle diagram of the "compensated motor" as shown in Fig. 168. No detailed proof is required since the same arguments as used in Chapter XII also apply in this case, but it is useful to note the physical significance of the quantities. OC is the rotor current, OA the flux linked with the rotor winding,

* For the trial and error method may be substituted a direct graphic method as explained on p. 375.

OE that linked with the stator winding, OV the watt-component of the slip voltage OS and VS the injected voltage. Since $\frac{OV}{VS}$ under our preliminary assumptions is constant we can calculate the angle α from

$$\tan \alpha = \frac{e}{I(R_2 + \rho)}$$

where e is calculated from either of the above formulae, R_2 is the star value of the rotor resistance and ρ is that of the advancer including slip ring and brush contacts and connecting cable. Since

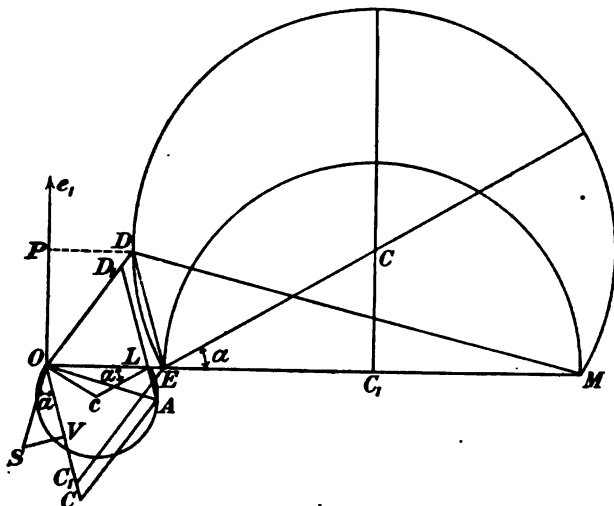


FIG. 168. Ideal circle diagram of induction motor compensated by phase advancer.

the advancer forms the star point of the system only half the diametric value of the resistance of its winding must be taken in calculating ρ . The advancer, although driven by mechanical power, cannot supply electrical power to the rotor circuit because brush and flux axes are in line, but it absorbs electrical power from the rotor to cover losses. OV is greater than the ohmic drop in the rotor and OS is greater than OV . Hence there must be an increase of slip, especially since for good commutation fairly hard brushes are necessary. The mechanical power to drive the advancer is small; it is merely that required to cover friction, windage and hysteresis. When compensating a large slow speed motor it is therefore good practice not to mount the advancer on the motor shaft, but to

drive it by a separate little motor, generally a squirrel cage induction motor.

The diagram Fig. 168 is drawn out of scale so as to give room for the lettering. In reality the circle ratio is much larger than shown in that diagram. Fig. 169 is the diagram for a circle ratio of 12. The inner circle EF is that of the non-compensated motor, the larger outer circle EG refers to the compensated motor and would correctly represent the locus of the primary current vector if there were no saturation and the slip were constant. This however not being the case, the true locus is a curve such as EH between the two.

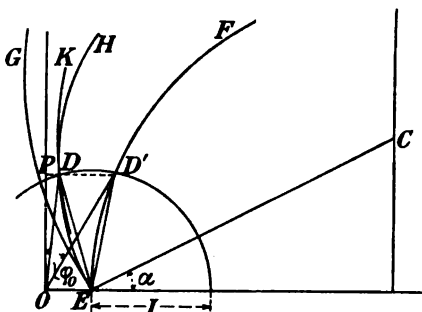


FIG. 169. Working diagram of induction motor and phase advancer.

To find it proceed as follows: Assume a certain current and find from the characteristic the corresponding flux. Assume also a certain slip and find from (144) or (145) the injected e.m.f. e . Since the resistances are known we find $\tan \alpha$ and can draw the triangle OVS of Fig. 168. OS should equal the product of open slip ring voltage and assumed slip. If this is not the case we correct our first assumption as to slip and get a second and if necessary a third approximation. Having thus found α we find the centre C of the circle K which is the locus for this constant slip and constant proportion between I and Φ . The working point must lie somewhere on this circle. Its exact position is easily found by describing with E as centre a circle with radius I . Where the two circles intersect is the working point D . We thus find the curve EH as the true locus of the primary current vector. The primary current neglecting iron loss is OD and the secondary is ED . For the non-compensated motor, taking the same power input OP , the primary current is OD' and the secondary is ED' . The natural power factor $\cos \phi_0$ has been raised to nearly unity. It will be noted that the increase in rotor current is small, but the decrease in stator current is appreciable, hence the internal losses of the motor have decreased. This advantage is lost if phase advancing is carried so far as to make ϕ negative. The diagram also shows that the overload capacity is sensibly increased.

Vibratory Type of Phase Advancer. In the author's phase advancer commonly known under the name "Vibrator" the leading e.m.f. is produced by the interchange of electrodynamic and mechanical energy. Imagine a two pole D.C. armature of small diameter and great length (usual ratio 1:2) placed without any external control into a D.C. field and traversed by the rotor current from one phase. A torque is produced alternating with slip frequency and this makes the armature vibrate, that is, rotate alternately backward and forward. By this motion an e.m.f. is induced which must obviously be in quadrature with the current since apart from the very small amount of energy to cover mechanical and iron losses no power can be given off. That the e.m.f. must be leading follows from the following consideration. At the moment the current goes through zero the speed and e.m.f. are maxima and when the current has reached crest value the speed is zero. During this quarter period the armature has therefore given to the circuit electrically the energy which was stored kinetically in its moving mass. Whilst the current was growing energy was given to its circuit, which means that the growth was accelerated. During the next quarter period, when the current declines from its crest value to zero it produces mechanical acceleration of the armature, that is to say, the circuit loses electrical energy by transferring an equivalent mechanical amount to the armature. The current must therefore die off sooner than it otherwise would, or in other words the phase of the current relatively to the slip e.m.f. which produces it is advanced.

The phase advancer has as many armatures as there are rotor phases*. With the usual arrangement of a three phase rotor the three armatures may be coupled in star or mesh, the latter arrangement being preferable if the rotor current is large, since the size of the commutator and brush contact surface may be reduced to 58 per cent. of what would be required for star connection. The percentage loss in brush contact drop is also reduced by reason of the higher value of the slip e.m.f. The arrangement is diagrammatically shown in Fig. 170. *A, B, C* are the three armatures mesh connected to the slip rings *AB, AC, BC*; *a, b, c* are the D.C. exciting coils and *S* is the starter. The phase advancer need not be taken out of circuit during starting. Its inductance is so very small as to have no influence on the starting torque. At the start the armatures

* By using a "Scott" connection an advancer with two armatures may be used on a three phase rotor.

merely quiver, but as the motor gets up to speed they begin to swing more and more, making several revolutions either way during a period. The vibrator acts equally well whether the induction machine is used as a motor or as a generator and no change of connections is required as with the rotary type. This point is of some importance in three phase electric locomotive work when recuperation of energy on down grades and automatic limitation of speed is required*.

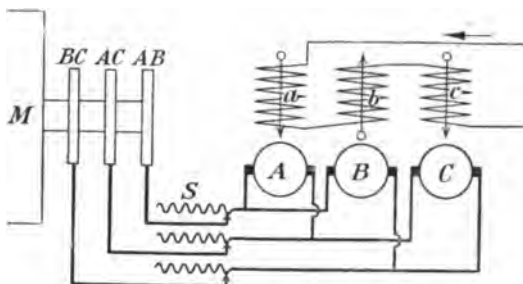


FIG. 170. The author's phase advancer. The three vibrating armatures are in mesh connection.

In establishing a theory of the vibrator it is convenient to assume that the loss through brush contact and friction and hysteresis may be represented as an electrical loss by an additional ohmic resistance in the rotor circuit. Bearing friction may be neglected since the armature runs in ball bearings and with the very small diameter (about 8 cm. for the smallest and 16 cm. for the largest vibrator for a 2000 H.P. motor) windage may also be neglected. If R_2 is the mesh value of the rotor resistance and ρ that of one armature with brushes and connections to the slip rings, including an appropriate amount for the mechanical losses, we have for the watt component of slip e.m.f. RI , where

$$R = R_2 + \rho$$

Let Φ be the flux in megalines produced by the exciting current,

* In this connection it is interesting to note that a phase advancer applied to the rear motor of a cascade procures the incidental advantage of reducing copper heat in the front motor since this is thereby relieved from the duty of passing exciting current to the stator of the rear motor. In a cascade not provided with phase advancer the front motor must therefore be heavier than corresponds to its actual H.P. rating, and since for obvious reasons one would not make the two motors of a cascade different, both must be heavier than corresponds to their rating as single motors. The saving of weight in a main line locomotive made possible by relieving the stators of magnetising current is of the order of some tons, that is largely in excess of the additional weight of the phase advancer.

z the number of conductors on the armature, D its diameter and m its mass (in units of 9.81 kg.) reduced to the circumference. From the expression for the torque (Formula (5), Chapter I) we find the tangential force for the crest value of the current

$$F = \frac{2z}{61.6D} \Phi I \sqrt{2} \text{ kg.}$$

if D is in cm. and I is the effective value of the current. This expression may be simplified by introducing the distance τ between neighbouring conductors. Inserting for D the equivalent $\frac{z\tau}{\pi}$ and transforming we obtain

$$F = 0.143 \left(\frac{\Phi}{\tau} \right) I \quad . \quad . \quad . \quad . \quad (146)$$

The tangential force causing the armature to vibrate is therefore simply proportional to the current and as the latter is a simple harmonic function the tangential force is also such a function, namely $F \sin \omega t$, the time being counted from the moment when the current goes through zero. ω is the angular speed of the vector representing rotor current. We have therefore $\omega = 2\pi f$ where $f = sf_1$ is the slip frequency. Let v be the instantaneous circumferential speed in metres per second. The differential equation of the motion is

$$m \frac{dv}{dt} + F \sin \omega t = 0$$

From this we find for the crest value of the speed

$$v = \frac{F}{m\omega} \quad . \quad . \quad . \quad . \quad . \quad (147)$$

corresponding to $u = \frac{100v}{\pi D}$ r.p.s.

The crest value of injected e.m.f. is $\Phi \frac{z}{100} u = v \left(\frac{\Phi}{\tau} \right)$ and the effective value is

$$e = 0.71v \left(\frac{\Phi}{\tau} \right)$$

Combining with (146) and (147) we obtain

$$e = \frac{0.1}{m} \left(\frac{\Phi}{\tau} \right)^2 \frac{I}{\omega} \quad . \quad . \quad . \quad . \quad . \quad (148)$$

The armature of a vibrator is always fairly saturated and the polar angle is small (100 to 120°) so that the cross turns have practically no weakening effect on the field. We may therefore consider Φ as a constant at all loads, so that the injected e.m.f. is simply

proportional to the ratio I/ω . Since the slip decreases with the load, the ratio I/ω does not decrease quite as markedly as the load, which means that the injected e.m.f. is relatively greater to the current at small than at large loads. But the natural power factor at small loads is smaller than at large loads so that the two effects have a tendency to compensate with the result that with a correctly designed vibrator a power factor very near unity may be obtained over a working range from about one-third of full load to about 30 per cent. overload.

Write B for the constant factor of (148), then we have

$$e = B \frac{I}{\omega} \quad \dots \quad (149)$$

If E is the open slip ring voltage we have

$$\sigma E \cos \alpha = RI \text{ and in Fig. 168 } \tan \alpha = \frac{B}{R} \frac{1}{\omega}$$

Since $\omega = \sigma \omega_1$ (where $\omega_1 = 2\pi f_1$) we have $\sigma = \frac{B}{R} \frac{1}{\omega_1} \frac{1}{\tan \alpha}$ or as a percentage

$$\sigma \text{ per cent.} = \frac{100}{\omega_1} \frac{B}{R} \frac{1}{\tan \alpha} \quad \dots \quad (150)$$

From the impedance triangle, the sides of which are $\omega \frac{E}{\omega_1}$, RI and

$B \frac{I}{\omega}$ we find

$$I \sqrt{R^2 + \frac{B^2}{\omega^2}} = \omega \frac{E}{\omega_1}$$

$$I \sqrt{\omega^2 R^2 + B^2} = \omega^2 \frac{E}{\omega_1}$$

and since $B^2 = R^2 \omega^2 \tan^2 \alpha$ we have also

$$I \omega R \sqrt{1 + \tan^2 \alpha} = \omega^2 \frac{E}{\omega_1} \text{ or } IR = \frac{\omega}{\omega_1} E \cos \alpha$$

Inserting for ω the value $\frac{B \cos \alpha}{R \sin \alpha}$ we obtain finally

$$I = \frac{E}{\omega_1} \frac{B \cos^2 \alpha}{R^2 \sin \alpha} \quad \dots \quad (151)$$

This gives the relation between current and the angle α in Fig. 169, so that the working point D may be found for any current. To facilitate the calculation of $\tan \alpha$ a graph may be used of which the following table gives a few points:

$\frac{\cos^2 \alpha}{\sin \alpha}$	0.37	0.71	0.82	1.17	1.43	1.8	3.2
$\tan \alpha$	1.5	1	0.9	0.7	0.6	0.5	0.3

It should be noted that R in (151) is not absolutely constant, but depends to some extent on the current, increasing somewhat as the current decreases. The reason is that the ρ component of R contains values to represent brush contact drop, mechanical brush friction and iron losses. The power absorbed by these cannot correctly be represented by the product of square of current and resistance, so that in practically applying (151) a trial and error method must be used. We estimate as nearly as possible the correct value of R for a given I , find from (151) the corresponding angle α and from (150) the slip. We can now calculate the speed v and revs. per second. This enables us to find frictional and iron losses. The brush contact loss is known for the particular current density with the assumed current I . These data are now used to find the correct

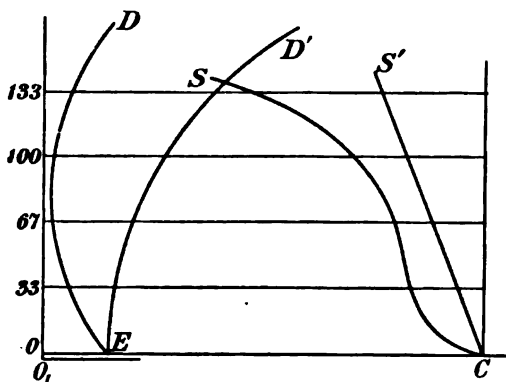


FIG. 171. Working diagram of induction motor compensated by vibrator.

value of R and by inserting this in (151) we get a second approximation for the relation between I and α . In the same way a third approximation may be obtained, though it does not differ materially from the second, because the corrections that have to be made in the small value of ρ are very small as compared with the resistance R_2 of the rotor. The working diagram of an induction motor compensated by a suitably designed vibrator is of the character shown in Fig. 171. In the non-compensated motor the working point lies on the circle ED' ; in the compensated motor it lies on the curve ED . The figures on the left are percentages of normal full load input. The corresponding slip is shown by the horizontal distance of the line CS' from the vertical through C , for the non-compensated

motor. For the compensated motor the slip is greater and is shown by the horizontal distance of the curve CS from the vertical through C .

In Fig. 168 we have tacitly assumed that EM represents the ideal rotor short circuit current when the phase advancer is inserted and motor and advancer are at rest. Since the phase advancer must have some inductance the short circuit current of a motor compensated by such an apparatus must be smaller than that of a non-compensated motor. The question is, how much smaller? To find the answer we may establish the relation between circle ratio θ and inductance on open and closed circuit. Let the two values as before be L_1 and L . We found on p. 318 the leakage factor $\lambda = \frac{F}{\Phi}$ and the

circle ratio $\theta = \frac{1}{\lambda} - 1$. This may also be written

$$\lambda = \frac{L}{L_1} \text{ and } \theta = \frac{L_1 - L}{L}$$

All these values are known for a given motor in its natural, that is non-compensated, condition. If we find them for the compensated condition we can determine the ratio in which the short circuit current has been reduced by the insertion of the phase advancer. Let the suffix 0 refer to the natural values then the rate in which the short circuit current (that is the diameter of the semicircle) has been reduced by the addition of the phase advancer is given by θ/θ_0 . The inductance of the phase advancer may be expressed as a fraction α of that of the rotor itself, so that $L = L_0(1 + \alpha)$. We thus find

$$\frac{\theta}{\theta_0} = \frac{1 - \lambda_0 - \alpha\lambda_0}{(1 + \alpha)(1 - \lambda_0)} \text{ and } \theta = \frac{\theta_0 - \alpha}{1 + \alpha}$$

If the inductance of the phase advancer is only a few per cent. of that of the rotor, the third term in the numerator becomes negligibly small and we have

$$\frac{\theta}{\theta_0} = \frac{1}{1 + \alpha}$$

and the reduction in the circle ratio is inappreciable. If, however, the phase advancer itself has a sensible inductance, say of the order of 10 to 20 % of that of the rotor, then the circle ratio is reduced in a slightly greater measure; with the result that the locus curve shifts a little nearer to the semicircle and this means that the improvement in the power factor is not quite so great as would be the case if the phase advancer itself had a negligible inductance. This involves a

slight disadvantage of the rotary as compared with the vibratory type of phase advancer. In the rotary type we have not only the inductance of the armature and compensating windings which are of the same character as, though much smaller in magnitude than, that of the rotor and stator windings of the motor, but we must also provide the flux by the rotor current. The interlinkage of the flux with the exciting winding adds considerably to the total inductance. In the vibratory type there is no field inductance, because the field is produced by an external source of D.C. It can be made exceedingly strong thus producing high saturation in the teeth of the armature and consequently an almost negligible inductance in the armature winding. This means that the locus curve may be constructed on the basis of the natural circle ratio, a conclusion fully borne out by practical experience.

Economical Limit of Compensation of an Individual Induction Motor. It has already been pointed out that the use of an idle running, overexcited synchronous machine as a phase advancer is not a commercial proposition and that even if the machine can be

simultaneously used as a motor, the injection of a leading current into the system by this means can only under special conditions be justified. If instead of a synchronous machine we use a compensated induction motor the case is a little more favourable because the loss due to over-excitation is avoided, but we are faced with

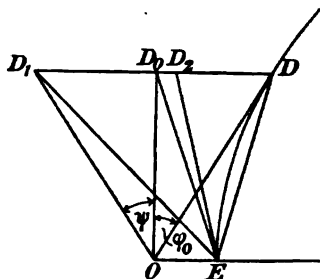


FIG. 172.

the difficulty that the internal losses of this motor are increased and its efficiency lessened. This will be clear from Fig. 172 where D is the working point of the non-compensated motor at full load and D_1 that of the over-compensated motor taking current at the leading phase angle ψ . The primary current OD has become OD_1 and may have been slightly increased, or at any rate not materially lessened; so that there is no saving in primary copper heat. The rotor current has considerably increased namely from ED to ED_1 . This means a greater loss in copper heat in the rotor, so that on the whole the efficiency of the motor has decreased. If the motor is designed for a very small slip, that is to say, with ample copper in the rotor,

this drawback is not very serious and the plan of over-compensating an induction motor to make up for the bad powerfactor of some other appliance on the system may be a commercial proposition. No general rule can be laid down but each case must be decided on its own merits.

Restricting, however, the investigation to a single motor, the question arises whether it is wise to carry the compensation always to unity power factor. The answer must depend on two things, namely, the natural power factor of the uncompensated motor and the extent to which we desire to reduce internal losses. As far as the stator is concerned the saving in copper heat will be greatest at unity power factor. If the natural power factor is very high there will be no increase in secondary copper heat if the motor is compensated to unity power factor. If the natural power factor is low, then unity power factor involves a somewhat larger rotor current and, since copper heat is proportional to the square of the current, a sensibly larger loss in the rotor. If on the other hand we do not wish to increase copper heat in the rotor, the power factor can only be improved to something less than unity. We may now simplify the problem to two conditions, (a) what will be the copper heat in stator and rotor if the power factor is always improved to unity and (b) what will be the power factor and copper heat in the stator if the rotor copper heat is to remain constant. In Fig. 172 condition (a) is represented by the working point D_0 and condition (b) by the working point D_2 . Let p be the percentage of rotor copper heat in the non-compensated motor referred to full power input and p_1 the corresponding percentage of stator copper heat and let the corresponding values for the compensated motor be p_c and p_{1c} . Let further ϕ_0 be the natural phase angle and ϕ the improved phase angle, then the effect of compensation in relation to reduction of total loss can be obtained from the following two tables, the first referring to case (a) and the second to case (b). In both cases it is supposed that the stalling torque of the non-compensated motor is about 1.8 of full load torque. The calculation is so simple that it need not be given in detail.

The use of these tables may be shown by an example. Let the power factor of a 500 kw. motor constructed for very low speed be 0.7 and assume that the stator copper takes three and the rotor copper two per cent. of the input, making the total ohmic loss 25 kw. If the power factor is improved to unity we shall have $0.49 \times 15 = 7.35$ kw. stator loss and $1.49 \times 10 = 14.9$ kw. rotor loss. The total loss is therefore 22.25 kw. or 2.75 kw. less than in the

CASE (a). NATURAL POWER FACTOR $\cos \Phi_0$ IMPROVED TO UNITY.

$\cos \Phi_0$	0.6	0.65	0.7	0.75	0.8	0.85
$\frac{P_{1c}}{P_1}$	0.36	0.42	0.49	0.56	0.64	0.72
$\frac{P_c}{P_1}$	2.03	1.74	1.49	1.32	1.14	1.06

CASE (b). NATURAL POWER FACTOR $\cos \Phi_0$ IMPROVED TO $\cos \Phi$ WITH THE SAME LOSS OF COPPER HEAT IN ROTOR.

$\cos \Phi_0$	0.6	0.65	0.7	0.75	0.8	0.85
$\cos \Phi$	0.77	0.85	0.91	0.95	0.98	1
$\frac{P_{1c}}{P_1}$	0.58	0.59	0.61	0.63	0.67	0.72

non-compensated motor. If we are satisfied with an improvement of the power factor to 0.91 we shall not reduce the stator loss to the same extent, but we shall not have any additional rotor loss. For the stator we have $0.61 \times 15 = 9.15$ kw. and for the rotor 10 kw. or in all 19.15 kw. showing a saving of 5.85 kw., which is more than sufficient to cover the losses in the phase advancer, so that the better power factor is obtained without sacrifice of efficiency.

Phase Advancing in Connection with Speed Regulation. A well-known method of reducing the speed of an induction motor is by inserting resistance into the rotor circuit. This is wasteful and cannot improve the power factor above its normal full load value. To avoid waste, some sort of cascade arrangement is necessary and if the cascade consists of two induction motors the power factor at reduced speed can be improved by fitting a phase advancer to the rear motor. This expedient has been used on three phase locomotives, the phase advancer being preferably a vibrator, because then the set acts as a generating unit on down gradient and gives power back to the line without sacrificing the advantage of a good power factor. For railway work a few definite speeds suffice, but for many industrial drives it is necessary to have means

of adjusting the speed within limits to any intermediate value and then it is obvious that the rear motor cannot be of the asynchronous type, but must be a commutator machine. It might be asked, why if speed adjustment is required we should not use simply one commutator machine only? For small or moderate power this is actually the solution often adopted, but when the power required is large, the difficulties of commutation become rather formidable and to avoid them it is preferable to limit the power with which the commutator machine has to deal by making it the rear motor of the cascade. In this way two advantages are secured; the machine has only to deal with the power corresponding to the difference between full and reduced speed and furthermore it has to deal with an alternating current at reduced frequency, namely, that of the slip. A variety of methods for speed adjustment of a cascade combination are now available* and in most of these some improvement of power factor is obtained concurrently with speed regulation. The adjustment is made by shifting the brushes, or altering the ratio of a transformer which supplies the exciting current to the rear motor, or both. If the rear motor is a shunt machine the set has the characteristic of a shunt motor, that is to say, it will give variable power without excessive change in speed; if the rear motor is a series machine the set will slow down when an increased torque is called for by the tool it drives and such an arrangement is to a certain extent self-regulating in the same way as a D.C. series motor adapts its working condition to the load.

An important case where drop of speed at increased load is required occurs in connection with a flywheel motor used for driving a rolling mill. Here a drop in speed of some 20 per cent. or more must be provided for in order to allow the flywheel to give up stored energy. This is most commonly done by the action of a relay in the rotor circuit controlling a servo motor which lifts the plates of the liquid starting resistance or by contactors and solid resistances. In this case the working of the mill depends on some delicate relay apparatus and a series of controlling appliances which must act very rapidly since the time of the pass is a matter of seconds; but more important still is the waste of energy entailed and the failure to improve the power factor just at the time a

* Heyland, *Elektr. Tech. Z.*, 1908, p. 353; Feigl, *Elektr. u. Masch.*, 1908, p. 743; Kubler, *Elektr. Kr. u. Bahnen*, 1907, p. 521; Shuttlesworth, *Journal Inst. El. Eng.*, Part 244, p. 439.

maximum of current is taken from the line. To avoid waste the energy should not be dissipated in heating the resistance in the rotor circuit, but should be in part at least returned to the system, and this is the fundamental idea of the various methods dealt with in the publications cited in the footnote. One system, namely, that devised by Krämer, is not included and as I have seen it in operation in an English rolling mill I give here a description. The difficulty of commutation in the rear motor is overcome by making this machine a converter taking the slip power from the front motor

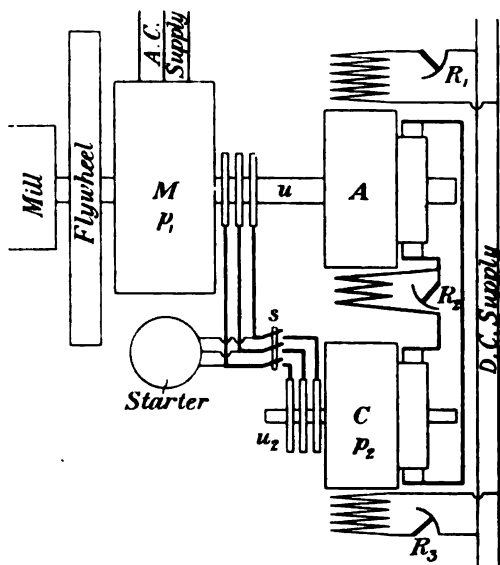


FIG. 173. Krämer system of speed variation and power factor improvement.

and delivering D.C. to an auxiliary motor mounted on the shaft of the main motor. Fig. 173 represents the arrangement. M is the main motor of $2p_1$ poles and receives A.C. power at any suitable voltage from the mains. It is geared, or directly coupled to the mill, and carries a flywheel to act as a store of energy. C is a six phase converter (though for simplicity it is shown as a three phase) which can be connected to the rotor circuit by closing the switch s . The D.C. generated in the converter of $2p_2$ poles is led to the auxiliary motor A which is compound wound, the series winding being so connected that it strengthens the field. Rheostats R_1 , R_2

and R_3 are provided to regulate the working of the plant, but once set the action is automatic.

With f_1 primary frequency the synchronous speed of the main motor is $u_1 = \frac{f_1}{p_1}$. At the working speed u the frequency in the converter circuit is $f_1 - up_1$ and the speed of the converter is

$$u_2 = \frac{f_1}{p_2} - u \frac{p_1}{p_2}$$

From these equations we find

$$u = u_1 - u_2 \frac{p_2}{p_1}$$

If the main motor is running light, that is to say, at synchronous speed, the converter is standing still. As load comes on and the main motor runs slower, the converter speeds up. Its maximum speed depends on the slip σ . For slip = 0 we have $u_2 = 0$ and for a slip = σ we have $u_2 = \sigma \frac{f_1}{p_2}$. The main motor is started in the usual way, the switch s being open and the D.C. exciting current being off. When running light and at nearly synchronous speed s is closed, and the starter switched off. Since the frequency is extremely low there is no difficulty in making the converter self-starting. If now the exciting current is put on, C generates D.C. driving A as a motor. If the mill is loaded the speed of M drops and the converter speeds up increasing the power supplied to A and thus assisting the main motor. The speed of A drops naturally since its field is strengthened by the series winding and it is now merely a question of suitably adjusting the three rheostats to get stable conditions, speeds and loads within reasonable limits. The saving of power at maximum slip as compared with the usual resistance regulation may be roughly estimated as follows. Let η_1 be the efficiency of the main motor and η^2 the combined efficiency of the other two machines. At maximum slip σ the mill receives directly from M the power $P_1\eta_1(1 - \sigma)$ and indirectly through C and A the power $P_1\eta_1\sigma\eta^2$, where P_1 is the power taken from the supply. The total power given to the mill (irrespective of that given out by the flywheel) is therefore

$$P = P_1\eta_1 [1 - \sigma(1 - \eta^2)]$$

To give the same power to the mill with resistance regulation requires an increase of supplied power to P_2 so that

$$P = P_2\eta_1(1 - \sigma)$$

We thus find

$$P_1 = P_2 \left(\frac{1 - \sigma}{1 - \sigma + \sigma\eta^2} \right)$$

and

$$P_2 - P_1 = P_2 \left(\frac{\sigma\eta^2}{1 - \sigma + \sigma\eta^2} \right)$$

as the reduction in the call for power from the line as a function of slip and efficiency. Thus for a combined efficiency of C and A of $\eta^2 = 0.7$ and a 20 per cent. slip the saving in the peak demand is 15 per cent. and with a slip of 25 per cent. the saving is 19 per cent. There is further the advantage of a good power factor, since by suitably adjusting the excitation of the converter it can be made to take a leading current from the rotor of M .

The plant capacity required for this improvement can be roughly estimated by an example. Let the main motor have a continuous rating of 1000 kw. at 40 frequency and a synchronous speed of 300 r.p.m. With a maximum slip of 20 per cent. the converter will have to take an A.C. at 8 frequency and 200 kw. If we make it a four pole machine its maximum speed will not exceed 240 r.p.m. The auxiliary motor will also be a 200 kw. machine, running light at 300 and loaded at 240 r.p.m. In both machines the speed is low for the output. Assuming the converter to be so excited as to have 0.85 p.f. leading, we find from Fig. 114 that its dimensions will be that of an ordinary D.C. generator having an output of $\frac{200}{1.25} = 160$ kw.

The speed of a commercial machine of that output would be about 600 r.p.m. As the actual speed is only 240 r.p.m. a larger standard machine having a commercial rating of, say, 300 kw. at 450 r.p.m. would have to be provided. The auxiliary motor must also be rated up because of its low speed. Its commercial rating would be about 350 kw. This means an addition of machinery of 650 kw. calculated on the rating of standard D.C. dynamos. We thus find that the extra plant capacity required to replace the wasteful method of resistance regulation and at the same time improve the power factor is roughly 65 per cent. of the rating of the main motor.

Tariffs and Power Factor. In the contracts for the supply of electricity there is generally a clause stipulating that the consumer shall take current at a power factor not less than a definite value. This is intended for the protection of the supply company so that they shall not be called upon to supply an excessive amount of

kva. for which according to the registration of the meter in kw. hours they get no payment. The disadvantage to the supply company in having to deliver wattless current is partly an increased coal bill by reason of greater ohmic losses and the running of under-loaded sets, but mainly in the inability of making full use of the plant capacity. The cost may therefore be considered to have two components, one that of the energy actually delivered and the other that of the kva. hours delivered. Professor R. Arno has suggested* that the formula

$$\cos t = k \int_0^t (\frac{2}{3} \text{ kw.} + \frac{1}{3} \text{ kva.}) dt$$

might be used as a basis for the tariff and has devised a simple alteration of the compensating coils in the electricity meter by which it registers with near approximation according to his formula. The meter then registers not the true energy, but a slightly greater amount which he calls the "complex charge" (*carico complesso*). For unity power factor the registration is that of an ordinary energy meter, but the worse the power factor the more does the meter register and thus it is to the interest of the consumer to employ apparatus having a good power factor. Another and very simple method of encouraging the user of power to take current at a good power factor is the following. A recording instrument which registers the highest kva. (or which comes to the same thing, the highest current) taken by the installation at any time during a quarter is placed next to the ordinary energy meter. Obviously such an instrument must not be affected by a rush of current incidental to a short circuit or a stalling load which will bring the circuit breaker out. It must however record the highest load sustained over a certain time, say 10 or 20 minutes. An instrument fulfilling these requirements is the Lincoln Maximum Demand Indicator. The essential feature is a bi-metal strip which curves under the influence of hot air and pushes a pointer into recording position. The air is heated by a series and shunt resistance and thus the maximum kva. are recorded. The customer is charged a fixed sum per maximum kva. and in addition a small amount for the kw. hours actually taken and registered on the ordinary meter. Under this system the inducement to improve the power factor held out to the consumer is restricted to moderate and large loads, since he can never be penalised on a light

* *Journal Inst. El. Eng.*, 1913, p. 271.

load power factor. But it is precisely a bad power factor at heavy loads which burdens the plant, for however bad a motor may be its wattless current must be smaller at light than at full load. The policy of penalising the customer not directly for a bad power factor at any load, but merely on an excessive wattless current taken at any time is therefore commercially sound from the supply company's point of view and at the same time fair to the consumer.

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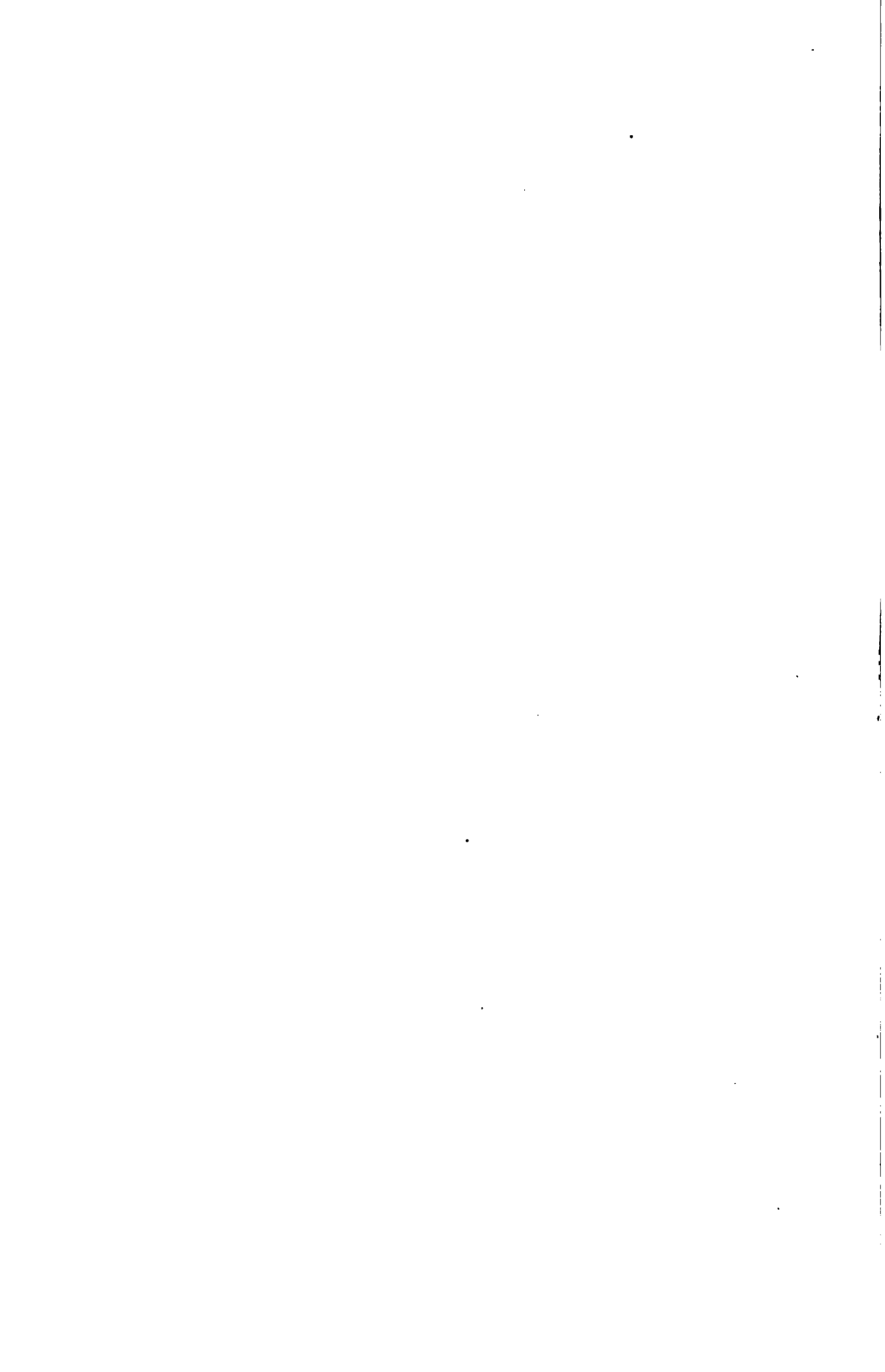
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